Novel mechanism of sound amplification in a weakly ionized gas

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A novel mechanism of sound amplification in a weakly ionized gas, caused by the friction between "hot" electrons and neutral particles, is proposed and investigated theoretically. The amplification can occur in propagation of the sound both along and transverse to an electric current. In the latter case, it is anomalously large, provided that the electron drift velocity depends weakly on the reduced electric field intensity. The new mechanism turns out to be competitive with the familiar mechanisms of amplification and damping of sound in weakly ionized gases.

1. INTRODUCTION

A weakly ionized gas is a system in which, in contrast to a neutral gas, the presence of external electric and magnetic fields leads to additional forces and channels of energy dissipation. A thermodynamically nonequilibrium system is easily obtained in this case, in which not only is the temperature of the electrons considerably greater than that of the gas, but also the energy distribution of the electrons can be non-Maxwellian. These features of a weakly ionized gas can lead to amplification brought about by an external source, of a sound wave traveling along it, or to the development of an acoustical instability because of fluctuations of the gasdynamic parameters of the medium.[1–3]

The universal mechanism of sound amplification in a weakly ionized gas is connected with the bulk thermal emission, which depends on the density of the charged particles. It is equivalent to the sound amplification in a medium with a negative second viscosity.[4] In a nonequilibrium system, this mechanism is most effective if the sound wave propagates in the direction of the electric current. In the perpendicular direction, the sound is attenuated. This case has been studied in greatest detail both experimentally and theoretically[5–7] (see also Refs. 1–3). In an electrically negative gas, amplification can take place even in the direction perpendicular to the electric current if the annihilation of the electrons is determined by the dissociative sticking to the molecule.[8] The latter is brought about by the strong dependence of the rate of sticking, and consequently of the density of the electrons, on the parameter E/N (E is the electric field intensity in the gas, N is the density of neutral particles). In the presence of an external magnetic field, another mechanism of buildup of acoustic oscillations is possible, connected with the interaction of the magnetic field with the currents induced by the vibrational motion of the medium.[1] In a molecular vibrational-nonequilibrium gas there exists an additional reason for sound amplification.[9,10] It is due to the possibility of conversion of the excess of vibrational energy into the energy of the sound wave.

Although the question of sound propagation in a weakly ionized gas already has a history, great interest nevertheless attaches to it today. For example, the question arises in quantum electronics and plasma chemistry in the study of the limiting powers in a gas discharge. This question is also important for stimulated Brillouin scattering in a gas, the efficiency of which depends on the attenuation coefficient of the sound waves.[11,12] Recently, anomalous properties of propagation of a shock wave in a weakly ionized plasma have been discovered experimentally. They are manifested especially in acceleration of the wave and diffusion of its front.[13,14] This phenomenon is not yet completely understood and needs further study.

In the present paper we suggest (and investigate theoretically) for sound amplification in a weakly ionized gas a novel mechanism in which, in spite of the small degree of ionization, the electrons play the fundamental role. The reason for the amplification is the friction between the electrons and the neutral particles, which causes the density fluctuations and the average energy of the electrons to be accompanied by corresponding changes in the thermodynamic characteristics of the gas. This mechanism is in its nature very close to the case of sound amplification in a magnetic field, since, in contrast to other known mechanisms, it is not connected with the bulk heat release in the gas. At the same time, it does not require the presence of a magnetic field. It is shown below that the friction between the electrons and the neutral particles leads to amplification of the sound waves in a direction perpendicular to the electric current. The maximum amplification takes place at frequencies that are comparable with the electron-annihilation frequency. At the same time, situations are possible in which a sound wave propagating along the electric current will also be amplified. Upon satisfaction of a number of conditions, which can be met in practice, the gain growth rate becomes anomalously large. The reason is that the small oscillations of the electron density along the electric field leads (because of the continuity of the current) to large oscillations of the mean energy of the electrons, with a phase shift $\pi/2$. The new amplification mechanism turns out to be fully competitive with the well-studied thermal mechanism.

2. BASIC EQUATIONS

We consider a weakly ionized gas placed in an external electric field. We shall assume that positive ions of a single type are present in the gas, along with electrons, both of which are created by an external ionizer, and both of which are annihilated by electron-ion recombination. The electric field heats the electrons, but leaves the ions "cold" with a temperature assumed equal to that of the gas. Moreover, because of the low degree of ionization, the frequency ν_{ee} of electron-electron collisions is significantly lower than the frequency ν_u of relaxation of the energy of the electrons. In this situation, the energy distribution of the electrons is non-

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equilibrium, and is determined both by the relative electric field E/N and by the cross section for interaction of the electrons with neutral particles. All these conditions are easily satisfied in practice.[2]

We shall use the traditional hydrodynamic approach to the description of neutral particles for the study of sound-wave propagation in a weakly ionized gas. We shall assume that the wavelength of the sound is large in comparison with the mean free path of the particles, while the perturbations of the parameters of the gas are assumed to be small in comparison with their mean values. For simplicity, we shall neglect viscosity and shall not take into account the known mechanisms of amplification or attenuation of the sound.

We shall consider charged particles in the framework of the nonequilibrium hydrodynamics of an electron gas, developed by the authors of Refs. 15, 16 (see also Refs. 2, 17, 18). In this approach, information on the momentum and mean energy of the electrons is contained in the nonequilibrium energy distribution of the electrons. The distribution function itself is controlled by the electron density n_e, by the relative electric field E/N, and so one. It is precisely for these quantities that the equations of hydrodynamics are obtained from the electron kinetic Boltzmann equation with the help of perturbation theory in small parameters—the ratios of the times and lengths of relaxation of the energy distribution of the electrons to the characteristic macroscopic scales of nonstationarity and inhomogeneity of the problem (in the given case, these are the period and length of the sound wave). In such a hydrodynamic system the coefficients (the drift velocity of the electrons, the diffusion coefficient, the recombination coefficient, *et al.*) are themselves functions of the parameter E/N and should be found from the locally homogeneous, stationary kinetic equation.

The equations of continuity and of motion of a gas, with account of friction between neutral and charged particles, have the form

$$\partial\rho/\partial t+\operatorname{div}(\rho\mathbf{V})=0, \tag{1}$$

$$\rho(d\mathbf{V}/dt)=-\nabla p+\mathbf{S}_e+\mathbf{S}_i, \tag{2}$$

where ρ, p, and $\mathbf{V}$ are the density, pressure and hydrodynamic velocity of the gas, while $\mathbf{S}_e$ and $\mathbf{S}_i$ are terms that describe the momentum transfer from the electrons and ions to the neutral particles. The quantity

$$\mathbf{S}_e=mn_e\langle\nu_m\mathbf{v}\mathbf{e}\rangle\mathbf{e},$$

where e and m are the absolute values of the charge and mass of the electron, n_e is the density of electrons, $\mathbf{v}$ is their velocity, $\nu_m = Nv\sigma_m$ is the transport frequency of collisions of electrons with neutral particles (σ_m is the transport cross section), $\mathbf{e} = \mathbf{E}/E$, E is the external electric field intensity. The angular brackets denote averaging over the electron distribution function $f(\mathbf{v})$.

In most cases of practical interest, the fraction of the energy transferred by the electron to an atom or molecule in a collision is small ($\nu_u \ll \nu_m$). Therefore, the anisotropy of the electron distribution function with respect to the velocity is small, which allows us to use the two-term approximation of Ref. 19. Calculation of $\mathbf{S}_e$ in this case yields

$$\mathbf{S}_e=e\mathbf{E}n_e-\nabla p_e,$$

$p_e = n_e T_e$ is the pressure of the electrons, T_e is their effective "temperature":

$$T_e=\frac{4\pi}{3}m\int_0^\infty v^4 f_0\,dv, \tag{3}$$

where $f_0(v)$ is the isotropic part of the electron distribution function over the velocities, normalized by the condition

$$4\pi\int_0^\infty f_0 v^2\,dv=1.$$

Because of the low temperature of the ions in comparison with T_e, their pressure can be neglected. Then

$$\mathbf{S}_i=-e\mathbf{E}n_i$$

(n_i is the ion density), and Eq. (2) reduces to

$$\rho(d\mathbf{V}/dt)=-\nabla(p+p_e)+e\mathbf{E}(n_e-n_i). \tag{4}$$

Let the period of the sound oscillations be large in comparison with ν_u^{-1} and with the relaxation time of the electron charge $\tau_M = (4\pi\sigma)^{-1}$ (σ is the electrical conductivity of the plasma). The charged-particle densities are determined by the ion-density balance equation of:

$$\partial n_i/\partial t+\operatorname{div}(n_i\mathbf{v}_i)=q_0N-\beta_r n_e n_i \tag{5}$$

and by the equation of conservation of electrical current

$$\operatorname{div}\mathbf{j}=0. \tag{6}$$

Here $\mathbf{v}_i = \mu_i\mathbf{E}$ is the drift velocity of the ions (μ_i is their mobility), q_0N is the rate of formation of electron-ion pairs per unit volume and per unit time, β_r is the electron-ion recombination coefficient. The expression for j, neglecting the ion current, has the form[2,17,18]

$$\mathbf{j}=-en_e\Big[\mathbf{v}_{e0}-\varkappa_1\mathbf{e}(\mathbf{e},\nabla\ln n_e)-\varkappa_2\mathbf{e}\frac{\operatorname{div}\boldsymbol{\gamma}}{\gamma} -\varkappa_3\mathbf{e}(\mathbf{e},\nabla\ln\gamma)-\frac{1}{n_e}\nabla(D_T n_e)\Big], \tag{7}$$

where $\boldsymbol{\gamma} = \mathbf{E}/N$, D_T is the coefficient of transverse diffusion of the electrons. The coefficients $\varkappa_i$ have the dimension of the coefficients of diffusion and describe the effect of the nonlocality of the electron distribution function in first order in the parameter λ_u/L, which is assumed to be small. Here λ_u is the relaxation length of the electron energy, L is the plasma-inhomogeneity characteristic dimension of the plasma and is, in sound propagation, of the order of its wavelength. The quantities $\varkappa_i$ are found from perturbation theory in the small parameter λ_u/L for the electron distribution function, which in this case is equal to

$$f_0(v)=f_{00}(v)\Big[1+b_1(v)(\mathbf{e},\nabla\ln n_e) +b_2(v)\frac{\operatorname{div}\boldsymbol{\gamma}}{\gamma}+b_3(v)(\mathbf{e},\nabla\ln\gamma)\Big].$$

The formulas for the determination of the coefficients $\varkappa_i$ from the functions $b_i(v)$ and the equations for $b_i(v)$ were obtained in Ref. 15.

The nonlocality of the electron energy distribution function leads to nonlocality of the quantity

$$T_e = T_{e0}(\gamma)\left[1+\varphi_1(\gamma)(\mathbf{e}, \nabla \ln n_e)+\varphi_2(\gamma)\frac{\operatorname{div}\boldsymbol{\gamma}}{\gamma} + \varphi_3(\gamma)(\mathbf{e}, \nabla \ln \gamma)\right], \quad (8)$$

where

$$T_{e0}=\frac{4\pi}{3}m\int_0^\infty v^4 f_{00}\,dv, \quad \varphi_i=\frac{4\pi}{3}m\int_0^\infty v^4 f_{00} b_i\,dv, \quad i=1-3.$$

The addition of the usual electrodynamic equations

$$\operatorname{rot}\mathbf{E}=0, \quad (9)$$

$$\operatorname{div}\mathbf{E}=4\pi e(n_i-n_e) \quad (10)$$

allow us to close the investigated set of equations.

3. SOUND WAVES IN THE CASE $\nu_u\tau_m \ll 1$

The obtained set of equations is rather complicated for the analysis of sound propagation in the general case. It can be simplified in individual specific situations.

We shall assume the unperturbed weakly ionized gas to be homogeneous and quasi-neutral. Transport of perturbations of the charged-particle density along the electric field is possible for two reasons: because of the violation of quasineutrality and as a result of diffusion currents. The importance of either cause is determined by the parameter $\nu_u\tau_M$.[17,18] At $\nu_u\tau_M \ll 1$, diffusion currents become dominant, while the excitations of the gas can be considered as quasi-neutral. If $\nu_u\tau_M \gg 1$, we should neglect diffusion along the electric field, but must take into account the violation of quasineutrality.

To begin, we consider the first case, $\nu_u\tau_M \ll 1$, and set $n_e = n_i = n$. We linearize the set of equations (1), (4)–(6), (9), and (10) with respect to the perturbations of $\tilde{\rho}$, $\tilde{\mathbf{V}}$, $\tilde{p}$, $\tilde{E}$, $\tilde{n}$, $\tilde{N}$, $\tilde{\gamma}$ near the stationary homogeneous state. We use the standard method of expansion of the excitations in plane waves $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, where the wave vector is real and the frequency is complex ($\omega = \Omega + i\Gamma$). As a result we obtain

$$k\tilde{V}/\omega-\tilde{N}/N=0,$$

$$\frac{\omega\tilde{V}}{k}=c_0^2\frac{\tilde{N}}{N}+\frac{nT_{e0}}{\rho}\left[(1+ik\eta\varphi_1)\frac{\tilde{n}}{n}+(\hat{T}_{e0}+ik\eta\varphi_3)\frac{\tilde{\gamma}}{\gamma} -ik\eta\varphi_2\frac{\tilde{N}}{N}+i\varphi_2\frac{(\mathbf{k}\tilde{\mathbf{E}})}{E}\right],$$

$$\left(-i\frac{\omega}{\nu_r}+2\right)\frac{\tilde{n}}{n}=\frac{\tilde{N}}{N}-i\frac{v_i}{\nu_r}\frac{(\mathbf{k}\tilde{\mathbf{E}})}{E}-\hat{\nu}_r\frac{\tilde{\gamma}}{\gamma},$$

$$(i\eta v_{e0}+kD_T+\varkappa_1 k\eta^2)\frac{\tilde{n}}{n}+[\eta v_{e0}(\hat{v}_{e0}-1)+kD_0+\varkappa_3 k\eta^2]\frac{\tilde{\gamma}}{\gamma} -(i\eta v_{e0}+\varkappa_2 k\eta^2)\frac{\tilde{N}}{N}-\varkappa_2 k\frac{(\mathbf{e}\tilde{\mathbf{E}})}{E}-i\frac{v_{e0}}{k}\frac{(\mathbf{k}\tilde{\mathbf{E}})}{E}=0. \quad (11)$$

Here c_0 is the velocity of sound in the non-ionized gas, $\eta = (\mathbf{k}\cdot\mathbf{e})/k$ is the cosine of the angle between the electric field and the direction of propagation of the excitations,

$$\hat{T}_{e0}=\frac{d\ln T_{e0}}{d\ln\gamma}, \quad \hat{v}_{e0}=\frac{d\ln v_{e0}}{d\ln\gamma},$$

$$D_0=\gamma\frac{\partial D_T}{\partial\gamma}, \quad \hat{\nu}_r=\frac{d\ln\nu_r}{d\ln\gamma}.$$

In the derivation of (11) is was assumed that $v_i \ll c_0$. In the resultant system, we must add two other equations, namely,

$$\tilde{\gamma}/\gamma=(\mathbf{k}\tilde{\mathbf{E}})/kE-\tilde{N}/N, \quad \tilde{\mathbf{E}}=-i\mathbf{k}\tilde{\varphi},$$

which follow from the definition of γ and Eq. (9).

In the study of the system of equations (11), account must be taken of the smallness of the perturbations introduced into the sound propagation by the ionization of the gas: $\Gamma \ll \Omega$, $\Gamma \ll \nu_r$, and $|c-c_0| \ll c_0$, where $\nu_r = \beta_r n$ is the frequency of electron-ion recombination. In the general case, the solution of the system (11) is very complicated. We shall consider separately the propagation of the sound transverse to ($\eta = 0$) and along ($\eta = 1$) the electric field. For simplicity, we set $|\hat{\nu}_r| \ll 1$. Account of this term does not lead to new qualitative effects, since in most of the cases of practical interest, at effective electron temperatures ~ 1 eV, we have $-0.3 < \hat{\nu}_r < 0^{20}$.

At $\eta = 0$, we get

$$\frac{c^2}{c_0^2}=1+\frac{p}{\gamma_a p}\left\{\frac{2}{\alpha^2+4}-\hat{T}_{e0} +\frac{\varphi_2 k^2 D_T}{v_{e0}}\frac{[D_0(\alpha^2+4)/D_T-2](\alpha^2-2\beta+4)+\alpha^2\beta}{(\alpha^2-2\beta+4)^2+\alpha^2\beta^2}\right\},$$

$$\frac{\Gamma}{\Omega}=\frac{p_e}{\gamma_a p}\left\{\frac{\alpha}{\alpha^2+4} +\frac{\varphi_2 k^2 D_T}{v_{e0}}\frac{\alpha(2\beta-\alpha^2-4)+[D_0(\alpha^2+4)/D_T-2]\alpha\beta}{(\alpha^2-2\beta+4)^2+\alpha^2\beta^2}\right\}. \quad (12)$$

Here γ_a is the adiabatic exponent of the neutral gas,

$$\alpha=\frac{\Omega}{\nu_r}, \quad \beta=\alpha\frac{v_i}{c_0}\frac{kD_T}{v_{e0}}.$$

We have

$$\varphi_2 k^2 D_T/v_{e0}\sim(k\lambda_u)^2\ll 1, \quad \beta\sim\alpha(v_i/c_0)k\lambda_u\ll\alpha.$$

Therefore, it follows from (12) that the maximum growth rate of sound amplification is obtained at $\alpha \approx 2$ and is equal to $\Gamma/\Omega = p_e/8\gamma_a p$. The change in the square of the sound velocity is of the same order. If $\alpha \ll 1$, then $\Gamma/\Omega \sim \alpha p_e/p$. At $\alpha \gg 1$ we have $\Gamma/\Omega \sim p_e/(\alpha p)$.

In the case of sound propagation along the electric field ($\eta = 1$) and $\hat{v}_{e0} \sim 1$, the maximum value of the modulus of Γ is also observed at $\alpha \sim 1$:

$$\frac{\Gamma}{\Omega}=\frac{p_e}{\gamma_a p}\frac{\alpha}{(\alpha^2+4)}\frac{(\hat{v}_{e0}-1)}{\hat{v}_{e0}}.$$

At $\hat{v}_{e0} > 1$, there should be sound amplification, and at $\hat{v}_{e0} < 1$, sound attenuation. If $\alpha \ll 1$, then

$$\frac{|\Gamma|}{\Omega}\sim(p_e/p)\max(\alpha, k\lambda_u).$$

In the opposite case ($\alpha \gg 1$)

$$\frac{|\Gamma|}{\Omega}\sim(p_e/p)\max(\alpha^{-1},k\lambda_u).$$

The sign of Γ is determined by the magnitudes and signs of the electron kinetic coefficients φ_i, $D_L=D_T+\varkappa_1$, and $D_E=D_0+\varkappa_2+\varkappa_3$.

An anomalously high value of the modulus of Γ is obtained in the case considered if $\hat{v}_{e0}=0$. Then, at $\alpha\lesssim 1$,

$$\frac{\Gamma}{\Omega}=-\frac{p_e}{2\gamma_a p}\frac{\hat{T}_{e0}(\alpha^2+4)\,(2kD_E/v_{e0}+\alpha v_i/c_0)}{\alpha^2 v_i^2/c_0^2+[kD_E(\alpha^2+4)/v_{e0}+2\alpha v_i/c_0]^2}. \tag{13}$$

The condition $\hat{T}_{e0}>0$ is virtually always satisfied. According to (13), an anomalously high attenuation of the sound along the electric current should be observed at $D_E>0$, with a decrement $|\Gamma|/\Omega\gg p_e/p$. If, however, $D_E<0$, then at

$$2k(-D_E)/v_{e0}>\alpha v_i/c_0 \tag{14}$$

sound amplification will occur with increment

$$\frac{\Gamma}{\Omega}\sim\frac{p_e}{p}\frac{1}{k\lambda_u}\gg\frac{p_e}{p}.$$

In this case, the change in the sound velocity and also its dispersion are both anomalously large in the weakly ionized gas.

The value of Γ falls off with increase of α, as follows from (13). Moreover, at $\alpha\gg 1$, other terms, not taken into account in (13), become important, terms which lead to additional attenuation.

4. SOUND WAVES AT $\nu_u\tau_M\gg 1$

We take into account the violation of quasi-neutrality in the perturbations of a weakly ionized gas ($\tilde{n}_e\neq\tilde{n}_i$), neglecting diffusion along the electric field. In this case, the set of equations (11) is replaced by

$$k\tilde{V}/\omega-\tilde{N}/N=0,$$

$$\frac{\omega\tilde{V}}{k}=c_0^2\frac{\tilde{N}}{N}+\frac{nT_{e0}}{\rho}\hat{T}_{e0}\frac{\tilde{\gamma}}{\gamma}+\left(T_{e0}+i\eta\frac{eE}{k}\right)\frac{n}{\rho}\frac{\tilde{n}_e}{n}-i\eta\frac{eEn}{\rho k}\frac{\tilde{n}_i}{n},$$

$$\left(1-i\frac{\omega}{\nu_r}\right)\frac{\tilde{n}_i}{n}+\frac{\tilde{n}_e}{n}=\frac{\tilde{N}}{N}-i\frac{v_i}{\nu_r}\frac{(\mathbf{k}\tilde{\mathbf{E}})}{E}-\hat{\nu}_r\frac{\tilde{\gamma}}{\gamma}, \tag{15}$$

$$(i\eta v_{e0}+kD_T)\frac{\tilde{p}_e}{n}+[\eta v_{e0}(v_{e0}-1)+kD_0]\frac{\tilde{\gamma}}{\gamma}-i\eta v_{e0}\frac{\tilde{N}}{N}-i\frac{v_{e0}}{k}\frac{(\mathbf{k}\tilde{\mathbf{E}})}{E}=0,$$

$$i\mathbf{k}\tilde{\mathbf{E}}=4\pi e(\tilde{n}_i-\tilde{n}_e).$$

We now consider sound propagation transverse to the electric current ($\eta=0$). Here the solution of the set (15) is substantially simplified upon satisfaction of the conditions

$$\frac{k^2D_T}{\Omega}\frac{v_i}{v_{e0}}\sim k\lambda_u\frac{v_i}{c_0}\ll 1,\quad k^2D_T\tau_M\sim k\lambda_u\frac{v_{e0}}{c_0}\Omega\tau_M\ll 1,\quad |\hat{\nu}_r|\ll 1,$$

which are valid in cases of practical interest. In this case

$$\frac{c^2}{c_0^2}=1+\frac{p_e}{\gamma_a p}\left[-\hat{T}_{e0}+\frac{v_i\alpha/v_{e0}\Omega\tau_M+2}{(v_i\alpha/v_{e0}\Omega\tau_M+2)^2+\alpha^2}\right],$$

$$\frac{\Gamma}{\Omega}=\frac{p_e}{2\gamma_a p}\frac{\alpha}{(v_i\alpha/v_{e0}\Omega\tau_M+2)^2+\alpha^2}.$$

Thus, as in the case $\nu_u\tau_M\ll 1$ analyzed earlier, at $\eta=0$ sound amplification is observed and

$$(c^2/c_0^2-1)\sim p_e/p.$$

The ratio Γ/Ω at $\alpha\sim 1$ has the same order and

$$v_i\alpha/v_{e0}\Omega\tau_M\ll 1.$$

If the last two conditions are not satisfied, then the value of Γ/Ω is even smaller.

Analysis of the system (15) in sound propagation along the electric field ($\eta=1$) and $\hat{v}_{e0}\sim 1$ shows that sound amplification is observed here with a growth rate

$$\frac{\Gamma}{\Omega}=\frac{p_e}{2\gamma_a p}\frac{\alpha}{\hat{v}_{e0}(\alpha^2+4)}$$

if $\alpha\sim 1$ and $\Omega\tau_M v_{e0}/c_0\ll k\lambda_u$. In the opposite case, sound attenuation occurs.

A special situation arises, as before, if $\hat{v}_{e0}=0$. Then sound attenuation results with

$$\frac{\Gamma}{\Omega}=-\frac{p_e}{2\gamma_a p}\frac{eE}{kT_{e0}}\frac{\Omega\tau_M(\alpha v_i/v_{e0}+\Omega\tau_M)}{(\alpha v_i/v_{e0}+\Omega\tau_M)^2+(\alpha\Omega\tau_M)^2}.$$

If $\alpha\ll\Omega\tau_M(v_{e0}/v_i)$ and $\alpha\lesssim 1$, then the attenuation is anomalously large:

$$\frac{|\Gamma|}{\Omega}=\frac{p_e}{2\gamma_a p}\frac{eE}{kT_{e0}}\frac{1}{(1+\alpha^2)}\sim\frac{p_e}{p}(k\lambda_u)^{-1}\gg\frac{p_e}{p}.$$

The relative change in the square of the sound velocity in the gas and also its dispersion both increase in this case. For other values of α the attenuation is weaker.

5. DISCUSSION OF RESULTS

It was shown above that the friction between the electrons and the neutral particles leads to sound amplification in a direction perpendicular to the electric current, with growth rate $\Gamma/\Omega\lesssim p_e/p$. A special situation arises in sound propagation along the electric current if $\hat{v}_{e0}\approx 0$. In this case, both anomalously large sound amplification is observed (at $\nu_u\tau_M\ll 1$), with a growth rate $\Gamma/\Omega\sim p_e/pk\lambda_u$, and also anomalously large attenuation (at $\nu_u\tau_M\gg 1$). We shall consider these phenomena in more detail as applied to specific cases.

The buildup of the sound oscillations in the direction perpendicular to the electric current takes place in the following way. The growth of the density $\tilde{N}$ of the medium produces an increase in the rate of ionization and, consequently, in the electron density $\tilde{n}_e$. The latter leads to an increase in the electron pressure $\tilde{p}_e$. Because of the friction between the electrons and the neutral particles, the velocity $\tilde{\mathbf{V}}$ of the gas changes and together with it (because of the continuity of the medium) also the value of $\tilde{N}$. At frequencies that are comparable with the frequency of electron-ion recombination ($\alpha \sim 1$), the oscillations of $\tilde{N}$ and $\tilde{n}_e$ are shifted in phase by an amount ~ 1. This shift leads to sound amplification.

The case $\nu_{ee} < \nu_u$ was considered above. It is easy to show, in analogous fashion, that sound amplification will also take place in the opposite limit, when the energy distribution of the electrons is Maxwellian.

The sound-amplification mechanism connected with the bulk heat release does not work in a direction perpendicular to the current. The dissipative processes here lead to sound absorption. The decrement Γ^* of the absorption, produced by the viscosity and thermal conductivity, is directly proportional to Ω^2. In most cases, at $p = 1$ atm, $\Gamma^*/\Omega^2 \approx 2\cdot 10^{-10}$ s.[21] Under conditions that are favorable to amplification, $\Gamma/\Omega \sim p_e/p$. Therefore, setting as an estimate $T_e/T \sim 10^2$ and $\Omega \sim 10^4$ s^{-1}, we obtain from the requirement $\Gamma > \Gamma^*$ a lower bound for the degree of ionization $n/N > 2\cdot 10^{-8}$. Thus, sound amplification in a weakly ionized gas in a direction perpendicular to the electric current can be observed at a sufficiently high degree of ionization. It is earlier to obtain amplification at low frequencies.

Sound amplification along the electric current is explained on the basis of the following chain. An increase in the density of the medium $\tilde{N}$ at sufficiently low frequencies ($\alpha \lesssim 1$) leads to an increase in $\tilde{n}$. The latter, because of the conservation of electric current density (at $\nu_u \tau_M \ll 1$), changes the relative electric field γ:

$$\frac{\tilde{\gamma}}{\gamma} = \left(\frac{ikD_E}{v_{e0}} - \hat{v}_{e0}\right)^{-1} \frac{\tilde{n}}{n}.$$

If $\hat{v}_{e0} \sim 1$, then the quantities γ and n oscillate in counterphase, since $kD_E/v_{e0} \sim k\lambda_u \ll 1$. Change in γ produces oscillations of T_e and p_e. The latter, because of the friction between the electrons and the neutral particles, leads to a change in $\mathbf{V}$, and consequently to a change in N. Significant sound amplification for the stated mechanism does not occur here. The situation changes if $\hat{v}_{e0} = 0$. Then the oscillations of γ turn out to be shifted relative to n by $\pi/2$ forward or backward, depending on the sign of D_E. The amplitude of the oscillations of γ is $(k\lambda_u)^{-1}$ times greater than n. If $D_E < 0$, then the chain described above produces sound amplification.

The condition $\hat{v}_{e0} = 0$ can be satisfied for a number of gases and gas mixtures, a list of which is given in Ref. 22. This applies, in particular, to CH_4, N_2:Ar, CO:Ar, CO_2:Ar,[22] H_2:Ar,[23] and HCl:Ar.[24] Argon can be replaced by other inert gases in which there is a Ramsauer effect—krypton and xenon. The mechanism of the appearance of regions where $\hat{v}_{eo} \lesssim 0$, is currently well understood.[3,22] It is particularly characteristic of a weakly nonequilibrium plasma, in which the energy distribution of the electrons differs from Maxwellian. For this, in addition to the Ramsauer effect, an effective vibrational excitation of molecules by electron impact is necessary. In the case considered above the main basic channel of electron annihilation is the dissociative recombination. This is usually realized in practice when $\hat{v}_{eo} \lesssim 0$.

Another necessary condition for sound amplification along the electric field is the requirement that $D_E < 0$. This coefficient is analogous to the thermal diffusion coefficient in a gas at weak nonequilibrium, since the value of γ determines the mean energy of the electrons. Calculations of D_E for a number of specific gases are given in Refs. 25 and 26.

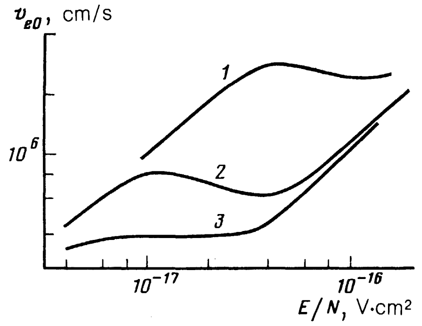

FIG. 1. Dependence of v_{e0} on $\gamma = E/N$ and the composition of the gas in the mixture N_2:Ar: 1—$\delta = 2\cdot 10^{-2}$, 2—10^{-3}, 3—10^{-4}.

Like the thermal diffusion coefficient, the quantity D_E can change sign, depending on the collision frequency of the electrons with neutral particles.

In the present work, a calculation was carried out, by the method of Refs. 25 and 26, for the determination of D_E in N_2:Ar mixtures. The energy distribution of electrons in a nonequilibrium, weakly inhomogeneous gas was found numerically from the Boltzmann equation, and the electron transport coefficients were determined. We took into account elastic scattering of electrons from neutral particles, as also their rotational, vibrational, and electronic excitations, and also ionization. The scattering cross section of electrons from Ar atoms and N_2 molecules was taken to be the same as in Ref. 25. It was shown earlier that this choice of cross sections allows us to obtain good agreement of the results of the calculation in pure gases with the experimental data on the drift velocity of electrons, their coefficients of transverse and longitudinal diffusion, the reaction rate constants, and the ionization of neutral particles by electron impact.

Figures 1 and 2 show the dependences of v_{e0} and D_E on γ and δ (δ is the fraction of the molecules of N_2 in the mixture) for various N_2:Ar mixtures. It is seen that at all points where $\hat{v}_{eo} = 0$, the condition $D_E < 0$ is satisfied, i.e., sound amplification can be observed. It should be noted that small additions of N_2 in Ar are sufficient for the validity of the condition $\hat{v}_{eo} = 0$. Therefore, sound amplification can take place even in not very pure Ar that contains few impurities, for example, of air. The latter obviously holds also for Kr and Xe.

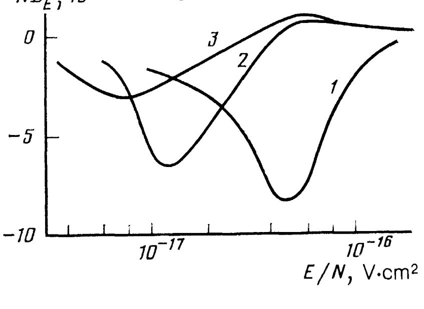

FIG. 2. Dependence of ND_E on $\gamma = E/N$ and on the composition of gas in the mixture N_2:Ar: 1—$\delta = 2\cdot 10^{-2}$, 2—10^{-3}, 3—10^{-4}.

The condition $\nu_u \tau_M \ll 1$, which is necessary for amplification of the sound waves, can be satisfied without difficulty in a weakly ionized gas.[3,18] The condition (14), which is also necessary for sound amplification, can be rewritten in the form $v_i < \lambda_u \nu_r$. Estimates of this inequality for the mixture N_2:Ar = 2:98, under the assumptions $v_i \sim 10^3$ cm/s, $\beta_r = 2\cdot 10^{-7}$ cm^3/s, gives $n/N \geqslant 2\cdot 10^{-7}$. Consequently, there should be a sufficiently high degree of ionization of the gas. The assumption $\hat{\nu}_r = 0$ is also satisfied here, since recombination of the electrons in the mixtures N_2:Ar is determined basically by the ions N_2^+, N_4^+ because of the recharging of the ions Ar^+ and Ar_2^+ on the N_2 molecules. The recombination coefficient on nitrogen ions in the region of effective electron temperatures considered (~ 1 eV) is practically independent of γ.[27]

Account of Joule heating of the gas does not change those results of the present research, which pertain to sound amplification in a direction perpendicular to the current. Actually, the thermal mechanism of sound amplification does not operate in this direction. (An exception is the case in which the annihilation of the electrons is determined by the dissociative sticking to the molecule[8].) In sound propagation along the current, the traditional thermal mechanism of amplification and the method proposed in the present research operate jointly. Their growth rates are additive. Upon satisfaction of the condition $\hat{v}_{e0} \ll k\lambda_u \ll 1$, depending on the quantity $\nu_u \tau_M$, the mechanism shown above leads either to anomalously large amplification or to attenuation of the sound along the current. We now compare the effectiveness of this mechanism with that of the traditional thermal mechanism. The growth rate of the amplification in the second case is of the order of the rate of heating of the gas at constant pressure[3]:

$$\nu_T = \frac{\gamma_a - 1}{\gamma_a}\delta_T \frac{jE}{p},$$

where δ_T is the fraction of the electron energy converted into heat. If the V–T relaxation rate is small, then the value of δ_T is connected with the excitation of the rotational levels of the molecules and with the elastic scattering of the electrons. The ratio Γ/ν_T is equal to

$$\frac{\Gamma}{\nu_T} \sim \frac{c_0}{\delta_T v_{e0}} \frac{v_{e0}\lambda_u}{D_E}.$$

We now estimate this quantity, using the results of the calculation that we have carried out, for the mixture N_2:Ar = 2:98. According to the calculation, $\hat{v}_{e0} \approx 0$, at $\gamma \approx 10^{-16}$ W·cm^2, when $\delta_T \approx 0.05$, $v_{e0} \approx 2.4\cdot 10^6$ cm/s, $v_{e0}\lambda_u/|D_E| \approx 10$. Using these data, we obtain $\Gamma/\nu_T \sim 2$. Thus, the mechanism of sound amplification along the electric field that we considered gives a greater efficiency than the traditional mechanism.

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