Parametric intracavity conversion of squeezed light

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The conversion of initially squeezed laser light to another frequency by inserting a transparent parametric crystal is examined. Two kinds of parametric processes are discussed: degenerate downconversion and second harmonic generation. In the case of second harmonic generation, the initial laser light generated without the parametric crystal is assumed to be in the amplitude-squeezed state produced by regular pumping of the active medium. On insertion of the crystal, the amplitude squeezing, depending on the parameters of the system, either converts entirely to the second harmonic or remains partially at the fundamental frequency and converts partially to the harmonic frequency. In the case of downconversion the initial laser light is assumed to be in the phase-squeezed state. The theory shows that after insertion of the crystal the laser becomes a source of two perfectly squeezed optical fields (at the laser and subharmonic frequencies). Their photocurrent noise in the low-frequency range can be suppressed almost completely.

The experimental realization of nonclassical states of the electromagnetic field which possess sub-Poisson photon statistics or squeezed states presents a number of new problems, one of which is the need for a study of the interaction of these fields with various physical systems. These questions have only begun to be considered,[1–5] but it is already well known that nonclassical states can be easily destroyed. This is so for at least two reasons. One is the internal system noise with which the field interacts. One physical consequence of this turns out to be a limitation of the amplification possibilities of the nonclassical fields.[5,6] The second reason is light losses, which necessitates a high efficiency of the optical recording circuits. A less obvious manifestation of these losses is the destruction of the nonclassical states—or, at least, their inefficient generation—in some optical systems with nonlinear absorption.[7]

The main purpose of this paper is to consider the possibility of converting squeezed light without destroying its statistics. This question is discussed for the two cases of parametric frequency conversion—second harmonic generation (SHG) (also known as frequency doubling) and subharmonic generation (SG) (also known as downconversion). Parametric systems are now, it seems, the most popular objects of investigation in quantum optics; however, so far for the most part they have not been considered or used as sources of nonclassical fields. The results of the analysis presented below show that parametric systems show themselves to be promising for the conversion of nonclassical fields, for example, in problems where it is required to transform squeezed light of one frequency into squeezed light of another frequency.

Since an exact solution of the quantum problem of the evolution of two parametric coupled modes is not known, approximate solutions have been pursued by various authors. Thus, for example, a number of papers on SHG[8] using the approximation of small propagation time in the parametric medium have predicted weak squeezing of the fields. In papers on SG the pump field is usually taken to be external and classical,[9–11] i.e., no use is made of its statistics.

In the present paper we consider a scheme for parametric conversion inside the laser cavity. We know of only one example in which quantum fluctuations during parametric conversion inside the laser cavity were considered.[12] The authors of that paper studied the influence of SHG on the laser field near the generation threshold with SG adiabatically excluded, and did not detect any quantum features in the fluctuations at the laser frequency. We will analyze here the statistics of both parametrically coupled modes. The main approximation which we will use is the approximation of small fluctuations, with the exception of the last section (generation of squeezed vacuum during SHG), in which the pump field is taken to be classical and the approximation of small fluctuations is not used.

The general picture of the formation of the statistical properties of the fields is defined by two parametric processes. These are two-photon (nonlinear) absorption, in which a pair of photons combine into one, and phase capture. In the latter case, as is well known, the difference phase of the fields is rigidly fixed while undergoing small fluctuations around its mean value.

The first of these processes is manifested in SHG, in which two-photon absorption takes place in a laser which generates radiation at the frequency ω. Its statistical properties therefore repeat the case of a laser with a two-photon absorption cell.[7] Thus, due to the nonlinear absorption the photons turn out to be antibunched, but over times significantly shorter than the lifetimes in the cavity. Therefore even though the state of the field becomes nonclassical, the spectral characteristics of its noise are practically Poisson. The second-harmonic light also turns out to be nonclassical. This has to do with the nature of the antibunching of the photons of frequency ω: not only are the photons of a pair antibunched, but so are the pairs themselves. Because each such pair is converted into a second-harmonic photon, the latter are also antibunched.

One feature of parametric interactions which is associated with phase capture is manifested in SHG. If a squeezed state of the field of the initial source is characterized by anomalously small (in comparison with the field in the coherent state) phase fluctuations, then as a result of capture the phase fluctuations of the second-harmonic will also be small. The state of the second harmonic turns out to

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be squeezed. Since the presence of the crystal in the cavity is associated with losses, the squeezed state of light at the frequency ω will be destroyed. In this regard, it is of interest to consider the SHG regime with a small number of photons, which corresponds to small losses which leave the squeezed state of the field at frequency ω almost intact.

1. BASIC EQUATIONS

The optical schemes which we will consider are shown in Fig. 1. The active medium generates light at frequency ω which is converted in the nonlinear crystal located in the cavity into radiation with frequency 2ω (the second harmonic) or $\omega/2$ (the subharmonic). The cavity is assumed to be high-Q only for the frequencies ω and 2ω or $\omega/2$, which allows us to limit our analysis to the simple two-mode situation. The initial source (the laser without the crystal) generates squeezed light. For the scheme with SHG (Fig. 1a) we used the model of a source with regular pumping to the upper working level of the active medium.[13] For the scheme with SG (Fig. 1b) the pumping of the active medium is ordinary and the squeezing of the initial light is realized by a parametric cell which is controlled by an external field.[14] The nonlinear crystal is assumed to be transparent, but can have dissipation losses, which, however, should be much less than the losses which accompany the exit of the radiation from the cavity.

Let ρ be the density matrix corresponding to the state of the electromagnetic field in the cavity. We write the basic equation for ρ in the form

$$\dot{\rho}=(L_1+L_2+L_3)\rho, \tag{1}$$

assuming that the variation of the field in the cavity occurs as a result of its interaction both with the active medium (the term $L_1\rho$) and with the crystal ($L_2\rho$). The last term describes the departure of radiation from the cavity. For $L_2=0$ Eq. (1) describes the initial source. Its explicit form is given in the Appendix. We will model the interaction with the transparent crystal by the customary effective Hamiltonian, which describes the elementary act of the merging of two photons into one and the decay of one photon into two:

$$L_2\rho=-i[V_{\rm eff},\rho], \quad V_{\rm eff}=ka_1^2a_2^+ + \text{h.c.}$$

Here $a_{1,2}^+$ and $a_{1,2}$ are the creation and destruction operators of photons with frequencies $\omega_{1,2}$ and k is the interaction constant and is determined by the nonlinear medium.

As a characteristic of the squeezed light, one usually introduces a quadrature operator of the form

$$X(\theta)=a^+\exp(i\theta)+\text{h.c.}, \tag{2}$$

whose normal variances

$$D=\langle(\Delta X(\theta=0))^2\rangle_N, \quad F=\langle(\Delta X(\theta=\pi/2))^2\rangle_N,$$
$$\langle(\Delta X)^2\rangle_N=\langle X^2\rangle-\langle X\rangle^2-1$$

are measured by a heterodyne detector, in which a reference wave interferes with the investigated wave. For a field in the coherent state we have $D=F=0$, whereas in the squeezed state either D or F is negative. For $D<0$ we speak of an amplitude-squeezed state of the field, and for $F<0$ we speak of a phase-squeezed state of the field. Along with the variances of quadrature operator (2), we will be interested in the spectrum of the photocurrent of the detector or the noise spectrum of the light $i^{(2)}(\Omega)$. This quantity depends on the field which is incident on the detector[15] ($\hbar=c=e=1$):

$$i^{(2)}(\Omega)=\frac{\eta}{\omega}\int d\mathbf{r}\langle E^+(\mathbf{r},t)E(\mathbf{r},t)\rangle + \left(\frac{\eta}{\omega}\right)^2\int d\mathbf{r}_1\,d\mathbf{r}_2 \times\int d\tau\, e^{i\Omega\tau}\langle E^+(\mathbf{r}_1,t)E^+(\mathbf{r}_2,t+\tau)E(\mathbf{r}_2,t+\tau)E(\mathbf{r}_1,t)\rangle. \tag{3}$$

Here

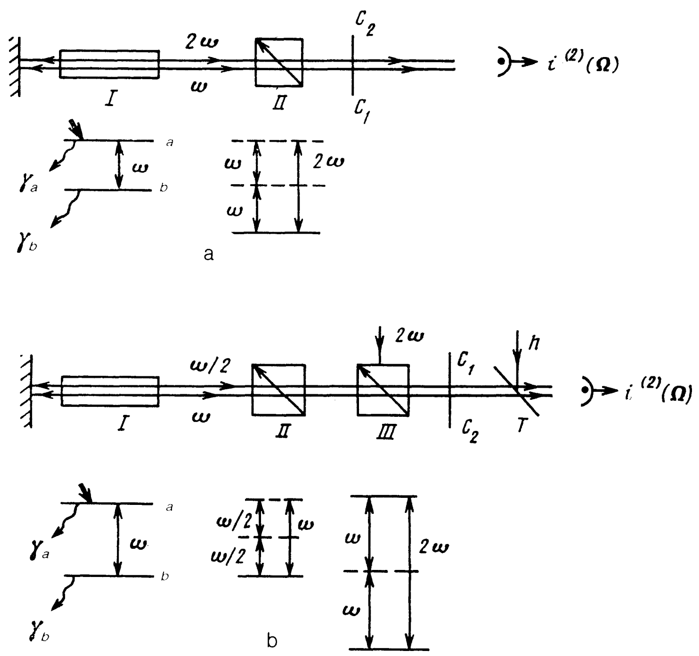

FIG. 1. I) active medium, II) parametric crystal. Under each element is shown the corresponding level scheme; ω is the laser frequency; the thick arrow indicates pumping of the active medium; a) second harmonic generation, b) subharmonic generation; III) parametric cell interacting with external coherent light 2ω; T) beam splitter; h) heterodyne wave.

$$E=i\sum_k\left(\frac{\omega_k}{2L^3}\right)^{1/2} a_k(\mathbf{r},t)\exp(-i\omega_k t+i\mathbf{kr})$$

is the Heisenberg operator of the electromagnetic field intensity, ω is the mean frequency of the radiation, η is the quantum efficiency of the detector. The integration in Eq. (3) is over the area of the photocathode. The first term in Eq. (3) is independent of frequency and is the photocurrent shot noise. The second is the excess noise. For a coherent field the excess noise component is equal to zero, and one speaks of the shot-noise or standard quantum limit of the sensitivity of the detector. For squeezed light the excess component can be negative, and the shot noise is either partially or completely compensated, so that the sensitivity of the detector can be substantially higher than the standard limit.

To calculate the correlations in Eq. (3), we use the characteristic functional (Ref. 16, p. 56):

$$C=\mathrm{Sp}\left\{T\exp\left[\int_0^t dt'\,W(t')\right]F\right\},$$

$$W(t)=\sum_k\left(y_k(t)\,\underline{a}_k^+(t)-y_k^*(t)\,\underline{a}_k(t)\right).$$

Here the arrows under the operators indicate their disposition with respect to the density matrix F of the complete "atoms + field" system, and the chronological operator T orders the time in the order of increase with respect to F. By functional differentiation we can obtain the entire set of normally ordered correlation functions of the field from C. Thus, for example,

$$\langle a_1^+(t_1)a_1(t_2)\rangle=\left\{-\frac{\delta^2 C}{\delta y_1(t_1)\,\delta y_1^*(t_2)}\right\}_{y_k=0}.$$

Using Eq. (1), it can be shown that

$$C=\mathrm{Sp}\,\{\Lambda^{-1}(t,0)K(t)\},$$
$$\dot K=[L_1+L_2+L_3+W_0(t)]K,$$
$$W_0(t)=\sum_k\left(y_k(t)\,\underline{a}_k^+-y_k^*(t)\,\underline{a}_k\right),\qquad (4)$$
$$K(0)=\rho(0),$$

where Λ is the evolution operator (in our case nonunitary) which propagates the density matrix of the field: $\rho(t)=\Lambda(t,0)\rho(0)$. Note that $K(t)=\rho(t)$ holds for $W_0=0$.

To solve Eq. (4), we make use of the representation proposed in Ref. 16 (p. 122):

$$K(t)=\int d^2\alpha\,\bar K(\alpha,t;s)\,\Delta(\alpha,-s),$$
$$\bar K(\alpha,t;s)=\mathrm{Sp}\,\{K(t)\Delta(\alpha,s)\},$$
$$\Delta(\alpha,s)=\frac{1}{\pi}\int d^2\beta\exp\left(\frac{s}{2}|\beta|^2+\beta a^+-\beta^* a\right)\exp(-\beta\alpha^*+\beta^*\alpha),\qquad (5)$$

where the parameter s characterizes the type of basis over which the expansion is carried out. Thus, $s=1$ corresponds to the expansion of K over the coherent states. Generalization of Eq. (5) to the two-mode case under consideration is trivial: $\alpha\to\alpha_k=\{\alpha_1,\alpha_2\}$, $s\to s_k=\{s_1,s_2\}$, etc. We write the equation for $\bar K$, which follows from Eq. (4), in the diffusion approximation, i.e., keeping only terms up to second order in α_1 and α_2:

$$\dot{\bar K}(\alpha_k,t;s_k)=\left\{\sum_{i,j=1}^{2}\left[\frac{\partial}{\partial\alpha_i}A_i(\alpha_k)+\frac{\partial^2}{\partial\alpha_i\partial\alpha_j}B_{ij}(\alpha_k)+\frac{\partial^2}{\partial\alpha_i\partial\alpha_j^*}c_{ij}(\alpha_k)+M(\alpha_k)\right]\right\}\bar K+\text{c.c.}\qquad (6)$$

We assume that the fluctuations of the field are small. This allows us to linearize the coefficients of the derivatives about their classical values $z_k=\{z_1,z_2\}$:

$$A_i(\alpha_k)=A_i(z_k)+\sum_{j=1,2}\left[\frac{\partial}{\partial\alpha_j}\bigg|_{\alpha_j=z_j}(\alpha_j-z_j)+\text{c.c.}\right]A_i,$$

$$B_{ij}(\alpha_k)=B_{ij}(z_k),\quad c_{ij}(\alpha_k)=c_{ij}(z_k).\qquad (7)$$

When we take Eq. (7) into account, Eq. (6) admits an exact solution. Here one obtains the equations of the semiclassical theory $\dot z_{1,2}=A_{1,2}(z_k,y\equiv 0)$. Using the approximation of small fluctuations means that the variances should be not too large: $|D_{1,2}|,|F_{1,2}|\ll|z_{1,2}|^2$, so that, for example, fields with Gaussian or super-Gaussian statistics cannot be analyzed within the framework of these approximations.

Solution of Eq. (6) gives rise to a closed system of equations for the variance $D_{1,2}$ and $F_{1,2}$. These quantities correspond to the amplitude and phase fluctuations, which in the standard generation regime turn out to be independent and satisfy the following equations:

$$\begin{cases}\dot D_1=-2d_1D_1+2b_dY_d+q_1,\\ \dot D_2=-2d_1D_2+2b_dY_d+q_2,\\ \dot Y_d=-(d_1+d_2)Y_d+b_d(D_2-D_1),\end{cases}$$

$$\begin{cases}\dot F_1=-2f_1F_1+2b_fY_f+p_1,\\ \dot F_2=-2f_2F_2-2b_fY_f+p_2,\\ \dot Y_f=-(f_1+f_2)Y_f+b_f(F_2-F_1).\end{cases}\qquad (8)$$

Here the quantities $Y_{d,f}$ describe the statistical coupling of the waves:

$$Y_{d,f}=(\langle a_1\rangle\langle a_2^+\rangle-\langle a_1a_2^+\rangle)\exp(i(\varphi_2-\varphi_1))$$
$$\mp(\langle a_1a_2\rangle-\langle a_1\rangle\langle a_2\rangle)\exp[i(\varphi_2+\varphi_1)]+\text{c.c.},\quad \varphi_{1,2}=\arg z_{1,2}.$$

The upper sign corresponds to Y_d, and the lower one to Y_f.

The solutions of Eq. (6), taking Eqs. (8) into account, allow one to determine the characteristic functional C and, consequently, to calculate all the necessary correlations of the fields. Note that the resulting means characterize the field inside the cavity. Their coupling with the extracavity means is discussed in detail for the case of SHG.

2. INTRACAVITY FREQUENCY DOUBLING

In this section we will consider the scheme shown in Fig. 1a. In accordance with the accepted terminology in nonlinear optics, we will call the field with frequency ω the reference radiation, indicating its variables by the subscript "1" and those associated with the second harmonic by the subscript "2". We will assume that the initial laser can generate amplitude-squeezed light.

2.1. Semiclassical equations: stationary generation regime

For the complex amplitudes of the reference radiation and the second harmonic

$$z_1=r_1e^{i\varphi_1},\quad z_2=r_2e^{i\varphi_2},$$

the semiclassical equations take the form

$$\dot r_1=\frac{Ar_1}{1+\beta r_1^2}-2|k|r_1r_2\sin\psi-{}^1/_2C_1r_1,$$
$$\dot r_2=|k|r_1^2\sin\psi-{}^1/_2C_2r_2,$$
$$\dot\psi=\frac{|k|}{r}(r_1^2-4r_2^2)\cos\psi,\qquad \psi=2\varphi_1-\varphi_2+\arg k.$$

Here A and β are respectively the linear gain coefficient and the saturation parameter of the laser transition, and C_1 and C_2 are the output fluxes of the reference radiation and the second harmonic from the cavity. The conditions of the stationary regime for the dimensionless field intensities $I_1=\beta r_1^2$ and $I_2=\beta r_2^2$ and the difference phase ψ reduce to the relations

$$2A/(1+I_1)=C_1+\Pi,\qquad \Pi=8|k|^2I_1/C_2\beta,$$
$$I_2=\Pi I_1/2C_2,\quad \psi=\pi/2, \tag{9}$$

where the value $\psi=\pi/2$ is found to be stable under the condition that

$$\Pi<C_2/2. \tag{10}$$

From Eqs. (8) it can be seen that the intensity of the second harmonic becomes proportional to I_1^2, and for the reference radiation, as in the case of a laser with a two-photon absorption cell, nonlinear losses arise [the term Π in the first of Eqs. (9)]. Under condition (10) the intensity of the second harmonic is bounded: $I_2<(1/4)I_1$. This means that inside the cavity the power coefficient of conversion to the second harmonic cannot exceed 50%.

2.2. Squeezing inside and outside the cavity; suppression of photocurrent noise

In this section we will be interested only in the amplitude fluctuations of the fields since for the frequency doubling scheme (the scheme with SHG) one can expect smoothing effects specifically for the amplitude fluctuations. The stationary values of the variances $D_{1,2}$ inside the cavity are calculated with the help of Eqs. (8). The values of the coefficients in Eqs. (8) are the following:

$$d_1=(C_1+\Pi)I_1(1+I_1)^{-1},\quad d_2=C_2/2,$$
$$b_d=(\Pi d_2)^{1/2},\quad q_1=2d_1(D_{10}-\Pi/2d_1),\quad q_2=0. \tag{11}$$

The quantity D_{10} which appears here is the value of the variance D_1 in the initial laser without the parametric crystal and depends on the nature of the pumping of the active medium. For an ordinary laser

$$D_{10}=I_1^{-1}\to 0,\quad I_1\gg 1.$$

For a laser with regular pumping to the upper level[13]

$$D_{10}=1/I_1-{}^1/_2\gamma_b/(\gamma_a+\gamma_b)\to-{}^1/_2,\quad I_1\gg 1,\quad \gamma_b\gg\gamma_a.$$

Here γ_a and γ_b are the relaxation constants of the upper and lower levels of the working transition.

From Eqs. (11) we obtain a relation between the stationary variances of the amplitude fluctuations of the reference radiation—D_1 and the second harmonic—D_2

$$D_2=tD_1,\quad t=\Pi(\Pi+d_1+d_2)^{-1}, \tag{12}$$

where

$$D_1=q_1(1-\Pi d_2/(d_1+d_2)(d_1+\Pi))/2d_1.$$

Since $0<t<1$, the value of D_2 is always closer to zero than that of D_1. The value $D=0$ corresponds to a coherent state of the mode and Poisson statistics of the photons in the cavity. Thus in the scheme depicted in Fig. 1a the photon statistics of the second harmonic will be closer to a Poisson distribution than the statistics of the reference radiation of the laser. Nonetheless, states of the field in the cavity are possible in which both modes are squeezed: D_1, $D_2<0$. Since a transparent nonlinear crystal does not have any internal noise, the antibunched photon pairs of the reference radiation are converted into antibunched photons of the second harmonic.

Using Eqs. (12), we will first find the values of the variances for an ordinary laser with chaotic pumping ($D_{10}=0$). The limiting value of the normal variance of the amplitude quadrature for the reference radiation is $D_1=-3/8$, and for the second harmonic, $D_2=-1/8$.

For a laser with regular pumping ($D_{10}=-1/2$) the corresponding limiting squeezing is $D_1=-3/4$, $D_2=-1/4$.

These limiting values of the squeezing inside the cavity are realized under the following conditions: the laser should operate in a substantially superthreshold regime $I_1\gg 1$, and the power of the second harmonic in the cavity should be close to the limiting stable value $I_2\lesssim(1/4)L_2$, but the intensity of the second harmonic leaving the cavity, due to transmission losses at the exit mirror of the resonator, should be much smaller than the output intensity of the reference radiation, i.e., $I_2^{\text{out}}\gg I_1^{\text{out}}$. It is found, however, that while such rigid conditions are necessary for intracavity squeezing, squeezing of light outside the cavity, which also determines the photocurrent noise under our conditions, can be achieved more simply.

The photocurrent noise spectrum of the jth harmonic of the field ($j=1,2$) in the scheme in Fig. 1a is calculated on the basis of Eq. (3)

$$i_j^{(2)}(\Omega)=i_{\text{shot}}\left\{1+2\eta C_j\,\mathrm{Re}\int_0^\infty e^{i\Omega\tau}\langle a_j^+(0)a_j^+(\tau)a_j(\tau)a_j(0)\rangle d\tau\right\}. \tag{13}$$

The correlation function entering into this expression is calculated with the help of Eqs. (4)–(7)

$$\langle a_j^+(0)a_j^+(\tau)a_j(\tau)a_j(0)\rangle$$
$$=r_j^4+r_j^2D_j\left(\frac{\gamma_1+d_n}{\gamma_1-\gamma_2}e^{\gamma_1 t}-\frac{\gamma_2+d_n}{\gamma_1-\gamma_2}e^{\gamma_2\tau}\right)$$
$$+(-1)^n r_j^2Y_d\frac{b_d}{\gamma_1-\gamma_2}(e^{\gamma_1\tau}-e^{\gamma_2\tau}); \tag{14}$$

where $n=2$ if $j=1$, and vice versa. The constants $\gamma_{1,2}$ are the temporal characteristics of the decay of the amplitude correlations in a cavity with two coupled quantum modes

$$\gamma_{1,2}=-{}^1/_2(d_1+d_2)\pm[{}^1/_4(d_1-d_2)^2-b_d^2]^{1/2}. \tag{15}$$

Substituting Eq. (14) into Eq. (13), we obtain

$$i_j^{(2)}(\Omega)=i_{\text{shot}}\left\{1+2\eta\,\mathrm{Re}\frac{C_j}{\gamma_1-\gamma_2}[-(D_j(\gamma_1+d_n)+(-1)^n Y_d b_d)(\gamma_1-i\Omega)^{-1}+(D_j(\gamma_2+d_n)+Y_d(-1)^n b_d)(\gamma_2-i\Omega)^{-1}]\right\}. \quad (16)$$

To analyze the statistics of the recorded light, we write the magnitudes of the spectral densities of the photocurrents in the low-frequency region

$$i_2^{(2)}(0)=i_{\text{shot}}\left\{1+\eta\frac{2C_2(d_1+d_2)}{\gamma_1\gamma_2}D_2\right\}=i_{\text{shot}}(1+\xi_2\eta),$$
$$i_1^{(2)}(0)=i_{\text{shot}}\left\{1+\eta\frac{2C_1 d_2}{\gamma_1\gamma_2}(D_1-D_2)\right\}=i_{\text{shot}}(1+\xi_1\eta). \quad (17)$$

The second term in braces (excluding the quantum efficiency η) is the sub-Poisson parameter ξ_i of the recorded light or in the approximation of small fluctuations the normal variance of the amplitude quadrature D_j^{out} of the jth mode outside the cavity, which characterizes the depth of the dip in the low-frequency region of the spectrum. Thus it can be seen that squeezing of the reference radiation outside the cavity depends not only on the squeezing of this mode inside the cavity, but also on the squeezing of the second harmonic inside the cavity. Squeezing of the second harmonic outside the cavity depends only on the squeezing of the same mode inside the cavity.

Let us find the values of $\xi_{1,2}$ which describe the noise spectrum of the detector in the low-frequency region. We assume, as before, that the laser is substantially above the threshold, $I_1 \gg 1$. In this case ξ_j will depend only on the one parameter C_1/Π, which is the ratio of the probability of the direct departure of a photon of reference radiation from the cavity to the probability of its departure via conversion to the second harmonic.

For an ordinary laser with chaotic pumping, calculation based on formulas (12), (15), and (17) gives for the second harmonic light

$$\xi_2=-2(2+C_1/\Pi)^{-2}.$$

The best suppression of second harmonic noise takes place when the cavity is almost completely blocked for light at the laser frequency ($C_1 \ll \Pi$). One then has $\xi_2 = -1/2$, i.e., the photocurrent noise close to zero frequency is suppressed twice as well as the shot noise limit.

For the reference radiation

$$\xi_1=-(2(\Pi/C_1)^{1/2}+(C_1/\Pi)^{1/2})^{-2}.$$

Its limiting value $\xi_1 = -1/8$ is reached at $C_1 = 2\Pi$.

For a laser with regular pumping

$$\xi_2=-2(2+C_1/\Pi)^{-1},\quad \xi_1=-(2\Pi/C_1+1)^{-1},\quad \xi_2+\xi_1=1.$$

It is clear that when the crystal is not present in the cavity ($\Pi = 0$), for a laser with regular pumping the photocurrent noise at the laser frequency is completely suppressed: $\xi_1 = -1$. In the second limiting case, when the cavity is blocked for the reference radiation ($C_1 \ll \Pi$), $\xi_2 = -1$. There exists, however, conditions for which the laser radiates both waves in the squeezed state with sub-Poisson detector noise, which for both waves is twice as small as the shot noise limit (Fig. 2a).

We show, finally the form of the photocurrent noise spectrum (16) for the second harmonic under the conditions $C_1 \ll \Pi$, $I_1 \gg 1$, $\Pi \lesssim C_2/2$ (Fig. 2b).

3. INTRACAVITY SG

In this section we consider the scheme shown in Fig. 1b. In accordance with the terminology of nonlinear optics we call the wave at frequency ω which is generated by the active medium the pump wave, and the wave with frequency $\omega/2$, the subharmonic. The characteristics of the pump wave will be labelled by the index "2" and those of the subharmonic, by the index "1". We now assume that the initial source generates phase-squeezed light.

3.1. Semiclassical equations; stationary regime of SG

For the complex amplitudes of the pump wave and the subharmonic

$$z_2=r_2e^{i\varphi_2},\quad z_1=r_1e^{i\varphi_1}$$

the semiclassical problem of intracavity SG has the form

$$\dot z_1=-2ik^*z_1^*z_2-C_1z_1/2,$$
$$\dot z_2=\frac{Az_2}{1+\beta|z_2|^2}+mz_2^*-ikz_1^2-C_2z_2/2. \quad (18)$$

Here the quantity m characterizes the mechanism which leads to squeezing of the phase of the initial source. For the chosen model this mechanism is intracavity parametric conversion provided by an intracavity parametric cell (see Appendix) with which the external classical monochromatic wave interacts.

For real amplitudes and phases system (18) takes the form

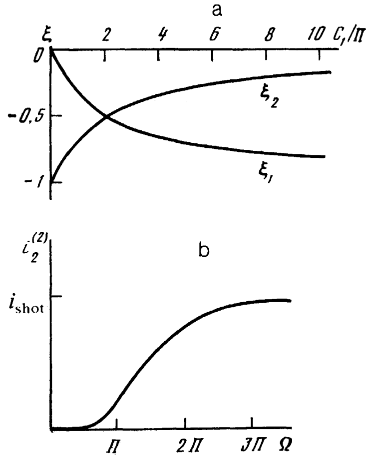

FIG. 2. Second harmonic generation; a) ξ_1 is the sub-Poisson parameter of the fundamental radiation, ξ_2 is the sub-Poisson parameter of the second harmonic; b) photocurrent noise spectrum for the second harmonic.

$$\dot r_1=-2|k|r_1r_2\sin\psi-C_1r_1/2,$$
$$\dot r_2=Ar_2/(1+\beta r_2^2)+|m|r_2\cos\psi_0+|k|r_1^2\sin\psi-C_2r_2/2,$$
$$\dot\psi=|k|(r_1^2-4r_2^2)\cos\psi/r_2-|m|\sin\psi_0,$$
$$\dot\psi_0=2|k|r_1^2\cos\psi/r_2-2|m|\sin\psi_0,$$
$$\psi=2\varphi_1-\varphi_2+\arg k^*,\quad \psi_0=\arg m-2\varphi_2. \tag{19}$$

Here C_1 and C_2 are the output fluxes of the subharmonic and the pump wave from the cavity. The stationary solution of the system (19) with nonzero pump-wave intensities $I_2=\beta r_2^2$ and subharmonic intensity $I_1=\beta r_1^2$ is given by the relations

$$2A/(1+I_2)=C_2+\Pi-2|m|,$$
$$I_1/I_2=2\Pi/C_1,\quad \Pi=8|k|^2I_1/C_1\beta, \tag{20}$$
$$\psi_0=0,\quad \psi=3\pi/2.$$

In this case it is necessary to satisfy the inequality

$$C_2+\Pi-2|m|>0.$$

Analysis shows that solution (20) is stable under the conditions

$$C_1-\Pi/2+|m|>0,\quad I_2>{}^1/_2\Pi(C_2+\Pi/2-2|m|)^{-1}.$$

Along with solution (20), there exists a regime which is stable for $C_1>4|k|r_2$ and for which the classical amplitude of the subharmonic is equal to zero ($r_1=0$). This regime will be considered in the final section.

3.2. Generation of two squeezed fields with suppression of the photocurrent noise

Of greatest interest for SG are phase fluctuations of the fields whose variances F_1 and F_2 satisfy system of equations (8) with the coefficients

$$f_1=C_1,\quad f_2=2|m|-\Pi/2,\quad b_f=(C_1\Pi/2)^{1/2},$$
$$p_1=-C_1,\ p_2=(\sigma+1)(C_2-\Pi-2|m|)$$
$$-2|m|,\ \sigma=2\gamma_a(\gamma_a+\gamma_b)^{-1}I_2+1.$$

Here γ_a and γ_b are the widths of the upper and lower working levels of the active medium.

Let us first consider how the fluctuations of the pump wave vary with location of the crystal in the cavity. From Eqs. (8) we obtain the following expression for the stationary variance of the pump wave F_2

$$F_2={}^1/_2\frac{[(\sigma+1)(C_2+\Pi-2|m|)-2|m|](C_1+2|m|)-C_1\Pi/2}{2|m|(C_1-\Pi/2+2|m|)}, \tag{21}$$

which for $\Pi=0$ corresponds to the variance of the phase of the initial source. The latter possesses maximum squeezing ($F_2\gtrsim-1/2$) [Ref. 14] for

$$C_2\gg 2|m|. \tag{22}$$

If the crystal is put into a source whose light possesses maximum phase squeezing, then the squeezed state of the pump wave will be destroyed. This happens because the generation of the subharmonic introduces losses for the pump wave. In fact, under condition (22) expression (21) takes the form

$$F_2=-\frac{1}{2}+\frac{(\sigma+1)\Pi(C_1+C_2)}{2C_2(C_1+C_2-\Pi/2)}$$

from which it is clear that $F_2=-1/2$ at $\Pi=0$. The latter condition, however, is fulfilled not only when the crystal is not present in the cavity, but also for the generation regime with $I_1\to0$.

The phase fluctuations of the subharmonic and the pump wave in the stationary regime are connected by the relation

$$F_1=-\frac{1}{2}\frac{1}{t+1}+\frac{t}{t+1}F_2,$$
$$t=\Pi/2(C_1-\Pi/2+2|m|)>0. \tag{23}$$

Here, in the absence of SHG [see Eqs. (12)], an intrinsic mechanism appears which leads to squeezing of the subharmonic independent of the state of the pump wave. Thus, for $t\ll1$ or

$$C_1+2|m|\gg\Pi \tag{24}$$

we have $F_1=-1/2$, but the quantity F_2 does not play any role here whatsoever. This mechanism proves to be most efficient if the intensity of the subharmonic is small: as $I_1\to0$ we have $t\approx0$ and $F_2\approx-1/2$.

The special features of the phase-squeezed states associated with an increase of photodetection sensitivity are manifested by heterodyne detection. Let there be placed in front of the detector a nonabsorbing mirror with transmissivity T (see Fig. 1b), mixing the coherent reference wave and the phase-squeezed light. If the phase difference in the recording channel is equal to $\pi/2$, then for the noise spectrum in the low-frequency region $i_j^{(2)}(0)$, $j=1,2$, in the case of a strong reference wave we obtain the following expressions:

$$i_j^{(2)}(0)=i_{\text{shot}}\{1+2\eta C_j|T|^2(\gamma_1\gamma_2)^{-1}(F_jf_n+(-1)^jY_fb_f)\},\quad n\neq j.$$

Here the variances $F_{1,2}$ are determined according to Eqs. (21) and (23), and the time constants of the phase correlations, as in the case of SHG, are given by the roots of the equation

$$\gamma^2+(f_1+f_2)\gamma+(f_1f_2+b_f^2)=0.$$

We present expressions for the noise spectra of the pump wave and the subharmonic under condition (22), i.e., in the case in which the initial source possesses maximum squeezing. Thus, for the pump wave

$$i_2^{(2)}(\Omega\to0)=i_{\text{shot}}\{1-\eta|T|^2(1-\sigma\Pi/C_2)\},$$

and for the subharmonic

$$i_1^{(2)}(\Omega\to0)=i_{\text{shot}}\{1-\eta|T|^2[1-{}^1/_4(\Pi/C_2)^2$$
$$+{}^1/_2\Pi(1+\sigma\Pi/C_2)/C_2]\}.$$

If

$$\sigma\Pi/C_2\ll1 \tag{25}$$

the shot noise component of both fields can be almost completely suppressed during efficient photodetection ($\eta|T|^2\approx1$). This case is of the greatest practical interest since the sensitivity of the detector is now higher than the standard limit.

Thus, for the SG process a regime can arise in which both generating fields—both the pump wave and the subharmonic—turn out to be squeezed and their noise components are almost completely suppressed. The main condition for this [see Eqs. (24) and (25)] is that the quantity Π be small, in which case the crystal introduces almost no losses to the pump wave and, consequently, does not destroy the squeezed state, and the squeezing of the subharmonic is formed under the action of an intrinsic mechanism.

3.3. Generation of squeezed vacuum

Strictly speaking, the regime with $I_1 = 0$ cannot be considered within the framework of the considered approximations of small fluctuations. The point is that the variance D_1 of the subharmonic here turns out to be larger than for the case of a Gaussian wave. In order to consider the special features of this interesting regime, let us begin with the following physical situation. Let the pump wave generated by the active medium be strong and have only small fluctuations: $I_2 \gg \langle n_1 \rangle$, $|\alpha_2 - z_2| \ll |z_2|$, where $\langle n_1 \rangle$ is the mean number of subharmonic photons in the cavity, and $I_2 = \beta |z_2|^2$ is the dimensionless intensity of the pump wave. We can then neglect the influence of the weak subharmonic wave on the pump wave, assuming the latter to be given by the classical wave. Under these conditions the following equation follows from Eq. (6) for the subharmonic

$$\dot{K}_1(\alpha_1, t; s_1) = \Big\{ \frac{\partial}{\partial \alpha_1} (2ik\alpha_1^* z_2 + C_1\alpha_1/2) + \frac{\partial^2}{\partial \alpha_1^2} (-iks_1 z_2) + \frac{\partial^2}{\partial \alpha_1 \partial \alpha_1^*} \Big(-\frac{s_1-1}{2}\frac{C_1}{2} \Big) + \text{c.c.} + y_1(t) \times \Big(\alpha_1^* - \frac{s_1-1}{2}\frac{\partial}{\partial \alpha_1} \Big) - y_1^*(t) \Big(\alpha - \frac{s_1-1}{2}\frac{\partial}{\partial \alpha_1^*} \Big) \Big\} K_1, \tag{26}$$

$$K_1(\alpha_1, 0; s_1) = \Phi_1(\alpha_1, 0; s_1).$$

Here the complex amplitude of the pump wave is determined from Eqs. (18) for $z_1 = 0$, and the complex conjugate operation does not act on s_1. The function $\Phi_1(\alpha_1, 0; s_1)$ is the s_1-ordered quasiprobability, which corresponds to the initial state of the subharmonic. For $y_1 = 0$ $K_1(\alpha_1, t; s_1) = \Phi_1(\alpha_1, t; s_1)$. This equation corresponds to the SG process with prescribed classical pumping. It is described by the effective Hamiltonian

$$V_{\text{eff}} = kz_2^* a_1^2 + \text{h.c.},$$

but with damping taken into account since the formation of the subharmonic takes place in the cavity and the light then leaves the cavity.

Equation (26) admits an exact solution, with the help of which it is possible to calculate the characteristic functional

$$C(y_1(t), y_1^*(t)) = \exp\Big\{ \frac{1}{4} \int_0^t dt_2 \int_0^{t_2} dt_1 [(D_1 \exp(-\Gamma_1 |t_2 - t_1|) - F_1 \exp(-\Gamma_2 |t_2 - t_1|)) (y_1(t_2) y_1(t_1)) \times \exp(-2i\varphi_2) + \text{c.c.}) - (D_1 \exp(-\Gamma_1 |t_2 - t_1|) + F_1 \times \exp(-\Gamma_2 |t_2 - t_1|)) (y_1(t_2) y_1^*(t_1) + \text{c.c.})] \Big\}.$$

Here

$$\varphi_2 = \arg z_2, \quad \Gamma_{1,2} = C_1(1 \mp \varkappa^{-1})/2, \quad D_1 = (\varkappa - 1)^{-1}, \quad F_1 = -(\varkappa + 1)^{-1}, \quad \varkappa = C_1/4|kz_2|.$$

In this case the stationary generation regime with zero complex field amplitude of the subharmonic $\langle a_1 \rangle = \mathrm{Sp}\{\rho(t) a_1\} = 0$ is established. The regime becomes stable for $\varkappa > 1$. Since $\langle a_1 \rangle = 0$ but $F_1 < 0$ for the subharmonic, this state is called the squeezed vacuum. We note some of its properties. Calculating the variance of the photon number, we find

$$\langle (\Delta n_1)^2 \rangle = \langle n_1 \rangle (1 + \xi_1),$$
$$\xi_1 = 2\langle n_1 \rangle - D_1 F_1 / 4 \langle n_1 \rangle, \quad \langle n_1 \rangle = \tfrac{1}{4}(D_1 + F_1).$$

Hence it is clear that phase squeezing ($F_1 < 0$), like amplitude squeezing, leads to super-Gaussian photon statistics: $\xi_1 > 2\langle n_1 \rangle$. Recall that for Gaussian statistics $\xi = \langle n_1 \rangle$.

From the practical point of view, the squeezed vacuum of the subharmonic is of interest because by mixing it with a strong pump wave it is possible to lower the photocurrent shot noise. For mixing of the subharmonic with a strong coherent wave, we obtain the following expression for the photocurrent spectrum

$$i_1^{(2)}(\Omega) = i_{\text{shot}} \Big[1 + 2\eta |T|^2 \frac{F_1 C_1}{\Gamma_2} \frac{\Gamma_2^2}{\Gamma_2^2 + \Omega^2} \Big],$$

where the shot-noise component i_{shot} is determined by the power of the pump wave. In the low-frequency region $\Omega \ll \Gamma_1$

$$i_1^{(2)}(\Omega) = i_{\text{shot}} \{ 1 - \eta |T|^2 [1 - (\varkappa - 1)^2 / (\varkappa + 1)^2] \}.$$

Thus, for example, for $\varkappa = 2$

$$i_1^{(2)}(\Omega) = i_{\text{shot}} (1 - {}^8/_9 \eta |T|^2),$$

i.e., under conditions of ideal photodetection the shot noise is suppressed by almost a factor of ten.

In Refs. 9–11 SG was considered in a cavity irradiated by a strong external pump wave, and it was demonstrated that the subharmonic leaving the cavity can be found in the squeezed vacuum state. The fact that such a regime also exists in our case is not surprising, since in the classical pump field the general nature of the fluctuations of the subharmonic should not depend on where the pump source is located—whether it be inside or outside the cavity. However, in comparison with the extracavity scheme, obviously the characteristics of the stationary regime and the conditions of stability will be substantially different.

APPENDIX

The equation for the initial laser (in the absence of the transparent parametric crystal) according to the results of Refs. 13 and 14 is written as follows:

$$L_1\rho = (u - \zeta u^2/2)\rho - i\mu[\varepsilon B^{+2} + \varepsilon^* B^2, \rho], \quad B = ga,$$
$$u\rho = -BB^+ R\rho + B^+ R\rho B + \text{h.c.},$$

$$R = \frac{N_a}{\gamma_{ab}} \Big[1 + \frac{1}{\gamma_b \gamma_{ab}} (\overleftarrow{BB^+} - \overrightarrow{BB^+}) \Big] \times \Big[1 + \frac{1}{\gamma_{ab}} \Big(\frac{1}{\gamma_a} + \frac{1}{\gamma_b} \Big) (\overleftarrow{BB^+} + \overrightarrow{BB^+}) + (\gamma_a \gamma_b \gamma_{ab}^2)^{-1} (\overrightarrow{BB^+} - \overrightarrow{BB^+})^2 \Big]^{-1}.$$

Here g is the constant of interaction with the working transition of the active medium, γ_a and γ_b are the longitudinal decay constants of the upper and lower levels of the working transition, γ_{ab} is the transverse relaxation constant, N_a is the stationary population of the upper working level, the population of the lower level is set equal to zero, a^+ and a are the creation and destruction operators of the field at the frequency ω generated by the active medium, and we have written $\varepsilon = im/2$, where the quantity m is determined by the interaction of the parametric cell with the external field.[14] The arrows under the operators indicate their disposition with respect to ρ.

The case $\zeta = 1, \mu = 0$ corresponds to a laser with sub-Poisson photon statistics which generates an amplitude-squeezed field. Such a source is used in the study of SHG. The case $\zeta = 0, \mu = 1$ corresponds to a laser with a parametric cell whose field can be phase-squeezed. This type of source is used in the study of SG. The outflow of radiation from the cavity is modeled in the following way:

$$L_3\rho = \sum_{j=1,2} {}^1/_2 C_j (a_j^+ a_j \rho - a_j \rho a_j^+) + \text{h.c.}$$

The saturation parameter β arising in the defintion of the dimensionless field intensities is given by

$$\beta = 2|g|^2 (\gamma_a + \gamma_b)(\gamma_a \gamma_b \gamma_{ab})^{-1}.$$

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