

# Theory of nonstationary transfer of resonance radiation under conditions of partial frequency redistribution

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A closed set of equations is derived for nonstationary transfer of resonance radiation under conditions of partial frequency redistribution. A method for numerical solution of the set is developed for cylindrical geometry. Theoretical and experimental results are compared. It is shown that the generalization of the theory of resonance radiation transfer to the nonstationary case proposed in Ref. 16 constitutes an excess of accuracy.

## INTRODUCTION

The transfer of resonance radiation in an optically dense medium is determined by the degree of correlation of the frequencies of the absorbed and emitted photons. If many collisions take place during the lifetime of the excited atom  $\tau_R = 1/A$  (where  $A$  is the probability of spontaneous emission) that change the phase and the frequency of the oscillator ( $\gamma_{el}\tau_R \gg 1$  where  $\gamma_{el}$  is the frequency of the oscillator), the emitted photon "kills" the excitation conditions. Such a regime is known as complete frequency redistribution (CFR). The equations that describe the transfer of resonance radiation under these conditions were formulated in the works of Biberman<sup>1</sup> and Holstein.<sup>2</sup> At sufficiently low pressures in gases, in astrophysical objects,<sup>3</sup> and in a plasma of multiply charged ions<sup>4</sup> another limit applies:  $\gamma_{el}\tau_R \ll 1$ , in which effects of frequency "memory" of the conditions of the excitation of the atom are important for the emitted photon. In this case the transfer of radiation takes place under conditions of partial frequency redistribution (PFR). The equations of resonance radiation transfer in this case were investigated in works of the astrophysicists Unno<sup>5</sup> and Hummer<sup>6</sup> and later in Refs. 3, 4, and 7–10. The theory of resonance radiation transfer in the PFR regime has been verified experimentally under nonstationary conditions.<sup>7,11–13</sup> Comparisons of the theory with the results of these experiments were carried out in Refs. 7 and 12–15. The strongest qualitative difference between the experimental results of Ref. 7 and the theory of PFR<sup>1,2</sup> consists in the presence of a minimum in the fluorescence rate at the 1048 Å line of argon, excited in the near-axis region of the cylindrical volume by a proton beam of nanosecond duration, as a function of the pressure of the argon gas ( $P_{\min} \sim 0.04\text{--}0.09$  torr). Similar results were obtained in Refs. 11–13.

In Ref. 16 the question was posed of the applicability of the equations of resonance radiation transfer that describe the effects of PFR<sup>3,7</sup> under nonstationary conditions. The authors of this paper, criticizing Ref. 7, advance a new version of the theory of nonstationary resonance radiation transfer that contains additional terms in the equations for the spectral density of the excited atoms. In the present paper we investigate the question of the applicability of the theory of nonstationary radiation transfer in the PRF regime, and we derive a system of equations, which generalizes the results of Refs. 3, 7, and 16. A technique for the numerical solution of the obtained system of equations is developed and a comparison is made of the results of the calculation with experiment.<sup>7</sup> It is shown, in particular, that the solution of

the equations of the theory developed in Ref. 7 under the conditions of this experiment gives results which are in good agreement with the systematic theory.

## 1. KINETIC EQUATIONS DESCRIBING RADIATION TRANSFER

The theory of transfer under PFR conditions requires the introduction of the spectral density of the excited atoms  $N(\mathbf{r}, \omega, t)$ ,<sup>3,7–9</sup> which characterizes the number of particles capable of emitting a photon with frequency  $\omega$ . The CFR limit corresponds to the equilibrium relation between the spectral density  $N(\omega)$  and the concentration of excited particles  $N$ :

$$N(\omega) = 2\pi N a(\omega), \quad (1)$$
$$\int N(\omega) \frac{d\omega}{2\pi} = N.$$

Here  $a(\omega)$  is the spectral line shape. In the limit  $\gamma_{el}\tau_R \ll 1$  relations of the form (1) break down since the spectral density  $N(\omega)$  is determined by the conditions of radiative excitation. As a result of the Doppler effect the velocity distribution of the excited atoms is also nonequilibrium. In general, a description of the transfer of resonance radiation requires a knowledge of the total velocity–frequency distribution function  $N(\mathbf{r}, \mathbf{v}, \omega, t)$ . The natural apparatus to use to obtain the kinetic equations of excitation radiative transfer is the method of kinetic Green's functions,<sup>17,18</sup> which was applied to these problems in Refs. 8 and 9.

In the present paper we will restrict the discussion to the simplest case of a nondegenerate two-level system. We will use the index 1 to denote the ground state and the index 2 to denote the resonantly excited state. We seek the projections of the Green's functions on the  $i$ th state ( $i = 1, 2$ ). Thus, the main object of the theory is to find the set of Green's functions  $G_{ii}^{\alpha\alpha'}(x, x')$  (the upper indices  $\alpha$  and  $\alpha'$ , in accordance with the Keldysh technique,<sup>17–19</sup> take the values  $+$  and  $-$ , which characterize the order of the field operators). In the absence of coherent pumping the nondiagonal elements  $G_{ik}$  ( $i \neq k$ ) are equal to zero. By virtue of the assumption of nondegeneracy of state 2 the nondiagonal elements over the projections of the magnetic quantum numbers  $m_2$  and  $m_2'$  also do not figure into our treatment. Assuming that the particle field operators  $\hat{\Psi}$  obey Fermi statistics, we present a semiclassical relation between the kinetic Green's function  $G_{22}^{\bar{-}+}$  and the spectral distribution function of the excited particles<sup>18</sup>:

$$N(\mathbf{r}, \mathbf{v}, \omega, t) = -i \int G_{22}^{-+}(\mathbf{r}_1, t_1, \mathbf{r}_2, t_2) \exp(i\omega\tau - i\mathbf{p}\boldsymbol{\rho}) d\tau d\boldsymbol{\rho}. \quad (2)$$

Here

$$\begin{aligned} \tau &= t_1 - t_2, & \boldsymbol{\rho} &= \mathbf{r}_1 - \mathbf{r}_2, & \mathbf{r} &= (\mathbf{r}_1 + \mathbf{r}_2)/2, \\ t &= (t_1 + t_2)/2. \end{aligned}$$

The wave vector  $\mathbf{p}$  is related with the particle velocity  $\mathbf{v}$  by the relation  $\mathbf{v} = \hbar\mathbf{p}/M$  where  $M$  is the mass of the atom.

The transfer equations for the Green's functions are the Dyson equations in which, first, the deviations of the frequencies from the resonance value  $\omega_0 = \omega_2 - \omega_1$  are assumed to be small:

$$\Delta = \omega - \omega_0 \ll \omega,$$

second, the variations of the quantities figuring in the theory in time and space are assumed to be sufficiently smooth (cf. Refs. 17 and 18) for a theory in the form of kinetic equations to apply, and, third, the theory uses the dipole approximation of the interaction of the resonant atoms with the photons.

In addition to the atomic Green's functions, the photon Green's function  $D_{ij}^{\alpha\alpha'}(x, x')$ , defined in terms of the Heisenberg operators of the electric field intensity of the photons, figures in the system of equations. For example, for  $D_{ij}^{-+}(x, x')$  according to Ref. 18 we have

$$iD_{ij}^{-+}(x, x') = \langle \hat{E}_i(x') \hat{E}_j(x) \rangle. \quad (3)$$

For an ideal photon gas in unbounded space the function  $D_{ij}^{-+}(\omega, \mathbf{k})$  has the form<sup>18</sup>

$$\begin{aligned} iD_{ij}^{-+}(\omega, \mathbf{k}) &= (\delta_{ij} - k_i k_j / k^2) (2\pi)^2 \hbar \omega_k \\ &\times [n_k \delta(\omega - \omega_k) + (1 + n_{-k}) \delta(\omega + \omega_k)]. \end{aligned} \quad (4)$$

Here  $\omega_k = c|\mathbf{k}|$ ,  $c$  is the speed of light,  $n_k$  are the photon occupation numbers, which are related to the spectral density of the radiation intensity  $J(\omega_k, \boldsymbol{\Omega})$  (a function of angle and frequency) by the relation

$$n_k = (\pi\lambda^2 / \hbar\omega_k) J(\omega_k, \boldsymbol{\Omega}). \quad (5)$$

In Eq. (5)  $\lambda$  is the wavelength, and  $\boldsymbol{\Omega}$  is the unit vector in the direction of propagation of the photon:  $\boldsymbol{\Omega} = \mathbf{k}/|\mathbf{k}|$ . In most papers on radiative transfer theory the kinetic equation for the photons is formulated in terms of the quantity  $J(\omega_k, \boldsymbol{\Omega})$ , i.e., it is assumed that the expression for  $D_{ij}^{-+}(\omega, \mathbf{k})$  has the form (4). This restriction is lifted in Ref. 9.

We write the Dyson equation for the function  $G_{22}^{-+}(x_1, x_2)$  in the following form (cf. Refs. 17 and 18):

$$\begin{aligned} &\left[ -i\hbar \frac{\partial}{\partial t} - \frac{\hbar^2}{2M} (\Delta_1 - \Delta_2) \right] G_{22}^{-+}(x_1, x_2) \\ &= - \int d^4x_3 [\Sigma_{22}^{--}(x_1, x_3) G_{22}^{-+}(x_3, x_2) + \Sigma_{22}^{-+}(x_1, x_3) G_{22}^{++}(x_3, x_2) \\ &\quad + G_{22}^{-+}(x_1, x_3) \Sigma_{22}^{++}(x_3, x_2) + G_{22}^{--}(x_1, x_3) \Sigma_{22}^{-+}(x_3, x_2)]. \end{aligned} \quad (6)$$

Here  $\Sigma_{22}^{\alpha\alpha'}$  is the mass operator which describes the interaction of the excited particles with the radiation and with the surrounding particles. Let us consider the interaction with the resonant photons in more detail. The explicit expression for the last term on the right side of Eq. (6), which characterizes the part of the approach term (the collision integral) that is associated with the absorption of photons by atoms in

state 1, has the form

$$St_1^{ph} = - \int d^4x_3 G_{22}^R(x_1, x_3) D_{ij}^{-+}(x_3, x_2) N_1(x_3, x_2) d_i d_j. \quad (7)$$

Here use has been made of the relation<sup>17,18</sup>  $G^{-+} \approx G^R$ , which is valid in the case of low atomic density;  $d_i$  is the  $i$ th projection of the matrix element of the dipole moment operator between the states 2 and 1, where summation over the indices  $i$  and  $j$  is assumed;  $G_{22}^R$  is the retarded Green's function for the excited atoms; and  $N_1(x, x') = -iG_{11}^{-+}(x, x')$  [cf. Eq. (2)]. The retarded Green's function satisfies the Dyson equation with the mass operation  $\Sigma^R$ . To find  $\Sigma^R$  one can use the relation

$$\Sigma^R = 1/2(\Sigma^{--} - \Sigma^{++}) - 1/2(\Sigma^{+-} - \Sigma^{-+}). \quad (8)$$

The first term characterizes the level shift due to the interaction (in the given case—the Lamb shift), and the second term characterizes the width of the state, and we have  $\Sigma^{+-} \gg \Sigma^{-+}$ . Thus,

$$\text{Im } \Sigma \approx -1/2 \Sigma^{+-}.$$

The finite width of the state is due to radiative damping and collisions. Calculating the Fourier component in time difference  $\tau = t_1 - t_2$ , we obtain

$$G_{22}^R(\omega_p, \mathbf{r}, \mathbf{r}') = \frac{1}{\hbar} \sum_k \frac{\psi_k^*(\mathbf{r}) \psi_k(\mathbf{r}')}{\omega_p - \omega_2 - E_k - \mu + i\gamma_2/2}. \quad (9)$$

Here  $\psi_k(\mathbf{r})$  are the eigenfunctions of the Laplace operator, which describe the translational shift of the particles in the bounded volume  $V$ ;  $E_k$  are the corresponding eigenvalues. In unbounded space  $\psi(\mathbf{r})$  are plane waves,  $E_k = k^2/2M$ ,  $\hbar\mu$  is the chemical potential (cf. Ref. 18),  $\omega_2$  is the renormalized frequency taking the shift into account. In the purely radiative regime  $\gamma_2 = A$ .

For the function  $G_{11}^{-+}(\omega_p, \mathbf{r}, \mathbf{r}')$  we use the approximate expression (cf. Ref. 18)

$$G_{11}^{-+}(\omega_p, \mathbf{r}, \mathbf{r}') = 2\pi i \sum_k \psi_k(\mathbf{r}) \psi_k^*(\mathbf{r}') \delta(\omega_p - \omega_1 - E_k + \mu) N_1(k), \quad (10)$$

which means that the ground state is not broadened;  $N_1(k)$  are the occupation numbers of the atoms in the state 1 with respect to the quantum numbers  $k$ . Changing over to the Fourier components in the time differences and the coordinates in Eq. (7) and taking account of the definition

$$\begin{aligned} D_{ij}^{-+}(z_3, z_1, \mathbf{q}_\perp, \mathbf{r}_\perp, \omega, \Omega) &= \int d\tau' dt' d\boldsymbol{\rho}' \exp(-i\omega\tau' \\ &\quad + i\Omega t' + i\mathbf{q}_\perp \boldsymbol{\rho}'_\perp) D_{ij}^{-+}(\mathbf{r}_3, \mathbf{r}_2, \tau', t') \end{aligned} \quad (11)$$

(here  $\tau' = t_3 - t_2$ ,  $t' = (t_3 - t_2)/2$ ),  $\boldsymbol{\rho}' = \mathbf{r}_{3\perp} - \mathbf{r}_{2\perp}$ ,  $\mathbf{r} = (\mathbf{r}_3 + \mathbf{r}_2)/2$ , and the index  $\perp$  denotes the direction perpendicular to the  $z$  axis, we obtain for the half-space

$$\begin{aligned} &\int d\tau d\boldsymbol{\rho}_\perp \exp(-i\omega_p\tau + i\boldsymbol{\kappa}_\perp \boldsymbol{\rho}_\perp) St_1 \\ &= J_1 = - \frac{2\pi d_i d_j}{\hbar} \int_0^\infty dz_3 \int \frac{d\omega d\Omega d\mathbf{q}_\perp}{(2\pi)^2} \\ &\quad \times \frac{D_{ij}^{-+}(z_3, z; \omega, \Omega; \mathbf{q}_\perp, \mathbf{r}_\perp) \exp(-i\Omega t)}{\varepsilon_p + \Omega/2 + i\gamma_2/2} \\ &\quad \times \frac{1}{|v|} N_1(\mathbf{p}, \boldsymbol{\kappa}_\perp) \left\{ \cos\left[\frac{\Delta}{v}(z_3 - z)\right] + \cos\left[\frac{\Delta}{v}(z_3 + z)\right] \right\}. \end{aligned} \quad (12)$$

Here  $\tau = t_1 - t_2$ ,  $\mathbf{p}_1 = \mathbf{r}_{1\perp} - \mathbf{r}_{2\perp}$ ,  $\Delta = \varepsilon_p - \omega + \omega_0 + \mathbf{q}_1 \cdot \mathbf{v}_1$ ,  $\varepsilon_p = \omega_p - \omega_2 + E_k + \mu$ ,  $v = \hbar \mathbf{k} / M$ , and  $\mathbf{v}_\perp = \hbar \boldsymbol{\kappa} / M$ . In the derivation of Eq. (12) account was taken of the smallness of the photon wave vector in comparison with the wave vector of the particles, rapidly oscillating functions of the type  $\exp[i\mathbf{p}(z_1 + z_2)]$  were averaged over, etc.

As follows from Eq. (12), the characteristic scale of the nonlocal behavior of the kernel is  $L \sim v/\Delta$ . Far from the boundaries, at distances many times larger than  $L$ , it is possible to extend the integral over  $z_3$  to the entire axis from  $-\infty$  to  $+\infty$ . In this case, changing over to the Fourier components  $D_{ij}^{-+}(z_3, z)$ , we obtain

$$J_1 = -\frac{2\pi d_i d_j}{\hbar} \int \frac{d\omega d\Omega dq d\mathbf{q}_\perp}{(2\pi)^5} \times \frac{D_{ij}^{-+}(z, \mathbf{r}_\perp, q, \mathbf{q}_\perp, \omega, \Omega)}{\varepsilon_p + \Omega/2 + i\gamma_2/2} N_1(\mathbf{p}, \boldsymbol{\kappa}_\perp) \delta(qv + \Delta) e^{-i\omega t}. \quad (13)$$

Making use of the same assumptions, it is possible in an analogous way to simplify the second term on the right side of Eq. (6). Combining it with Eq. (13), we obtain for the approach term

$$\text{St}_+^{ph} = -\frac{2\pi d_i d_j}{\hbar} \int \frac{d\Omega}{2\pi} \frac{d\omega d^3k}{(2\pi)^4} D_{ij}^{-+}(\mathbf{r}, \mathbf{k}, \omega, \Omega) e^{-i\omega t} \times N_1(\mathbf{p}) \delta(\varepsilon_p - \omega + \omega_0 + \mathbf{k}\mathbf{v}) \times [(\varepsilon_p + \Omega/2 + i\gamma_2/2)^{-1} - (\varepsilon_p - \Omega/2 - i\gamma_2/2)^{-1}]. \quad (14)$$

This approximation contains a description of retardation effects. In contrast with the assertion in Ref. 16 that the process of coherent scattering is instantaneous, it follows from Eq. (14) that the rate of photoexcitation is determined by an integral which depends on the rate of variation of the photon Green's function, which in turn is a function of the total time  $t$ . Expanding the expression in brackets in the frequency  $\Omega$ , we obtain

$$\text{St}_+^{ph} = \frac{2\pi i d_i d_j}{\hbar} \int \frac{d\omega d^3k}{(2\pi)^4} N_1(\mathbf{p}) \delta(\varepsilon_p - \omega + \omega_0 + \mathbf{k}\mathbf{v}) \times \left\{ \frac{\gamma_2}{\varepsilon_p^2 + \gamma_2^2/4} D_{ij}^{-+}(\mathbf{r}, t; \mathbf{k}, \omega) + \frac{\partial}{\partial \varepsilon_p} \frac{\varepsilon_p}{\varepsilon_p^2 + \gamma_2^2/4} \frac{\partial}{\partial t} D_{ij}^{-+}(\mathbf{r}, t; \mathbf{k}, \omega) + \dots \right\}. \quad (15)$$

The ellipsis indicates the subsequent terms of the series in the time derivatives of the Green's function. Usually in transport theory the series is truncated at the first term, which means a slow variation of the time dependence of the intensity in comparison with the characteristic time scale

$$\tau_0 \sim \frac{1}{\gamma_2} \left| \left[ 1 - \left( \frac{2\varepsilon_p}{\gamma_2} \right)^2 \right] / \left[ 1 + \left( \frac{2\varepsilon_p}{\gamma_2} \right)^2 \right] \right|.$$

If we use for the function  $D_{ij}^{-+}$  its vacuum approximation (4), we can display the approach term in the form

$$\text{St}_+^{ph} = 2\pi \hbar a_2(\varepsilon_p) \frac{A\lambda^2}{4} \int d\Omega d\omega_k \frac{J(\omega_k, \boldsymbol{\Omega}, \mathbf{r}, t)}{\hbar \omega_k} \times N_1(\mathbf{v}) \delta(\varepsilon_p - \omega_k + \omega_0 + \mathbf{k}\mathbf{v}). \quad (16)$$

In Eq. (16)  $a_2(\varepsilon)$  is the Lorentzian line shape of the excited state:

$$a_2(\varepsilon_p) = \gamma_2/2\pi (\varepsilon_p^2 + \gamma_2^2/4). \quad (17)$$

The probability of spontaneous emission  $A$  is expressed in terms of  $|\mathbf{d}|^2$  in the well-known way.<sup>19</sup> In the case in which the states 1 and 2 are degenerate it is possible to obtain an expression of the type (16) with the substitution  $A \rightarrow (g_2/g_1)A$ , where  $g_i$  is the multiplicity of the degeneracy of the state  $i$ . Thus, the commonly used form of the approach term (16), which is valid if nonlocality and retardation effects [which are taken into account in Eq. (12)] are ignored, describes the nonequilibrium character of the velocity-frequency distribution of the excited atoms during resonant photoabsorption from the ground state.

With the same accuracy with which Eq. (16) was obtained it is possible to write the collision integral of the collisions with the photons [the first and third terms in Eq. (6)], which characterizes the radiative decay of the excited states,  $\text{St}_-^{ph}$ . Not taking stimulated emission into account, we have

$$\text{St}_-^{ph} = -\hbar A N_k(\omega, \mathbf{r}, t). \quad (18)$$

To close the system of equations that describe the radiative transfer of excitations, we must write down the Dyson equation for the Green's function of the photons  $D_{ij}^{\alpha\alpha'}$  (Refs. 17 and 20). For a spatially bounded medium the polarization properties of the function  $D_{ij}^{\alpha\alpha'}$  are found by joining the solutions of the exterior and interior problems.

In the present case we will limit the discussion to the consideration of problems of the decay of the excitations created in the volume, and for  $D_{ij}^{\alpha\alpha'}(\omega, \mathbf{k}, \mathbf{r}, t)$  we will make use of the simpler equation<sup>3,7,9,14-16,21</sup>:

$$-2i\omega_k c(\boldsymbol{\Omega}, \nabla) D_{ij}^{-+} = 4\pi \hbar \omega^2 (\Pi^{+-} D_{ij}^{-+} - \Pi^{+-} D_{ij}^{+-}). \quad (19)$$

This approximation is valid if the transit time of the photon in the medium is small in comparison with the decay time of the excited particles. In addition, we have neglected corrections of the order of  $\sim N\lambda^3$  in comparison with unity.

In the resonance approximation for the polarization operators figuring in Eq. (19) it is possible to obtain

$$\Pi^{+-}(\mathbf{k}, \omega) = i \frac{d^2}{3} \int \frac{d\omega_p d\mathbf{p}}{(2\pi)^4} G_{22}^{+-}(p+k) G_{11}^{-+}(p), \quad (20)$$

$$\Pi^{-+}(\mathbf{k}, \omega) = i \frac{d^2}{3} \int \frac{d\omega_p d\mathbf{p}}{(2\pi)^4} G_{22}^{-+}(p+k) G_{11}^{+-}(p). \quad (21)$$

In Eqs. (20) and (21) as in the derivation of Eq. (16), we have neglected nonlocality and retardation effects. This single-loop approximation assumes that state 1 is not broadened as a result of collisions. As was noted above, use is commonly made in the theory of radiative transfer of the spectral intensity  $J(\omega_k, \boldsymbol{\Omega}, \mathbf{r}, t)$  obtained from the function  $D_{ij}^{-+}$  by integrating over all positive frequencies  $\omega$ . If we assume that the frequency dependence of the function  $D_{ij}^{-+}$  (for  $\omega > 0$ ) is characterized by the ordinary dispersion  $\sim \delta(\omega - \omega_k)$  (cf. Eq. (4) and Refs. 1-8, 14-16), it is possible to go from Eq. (19) to the kinetic equation for the photons, i.e., the equation of the spectral intensity.

## 2. INFLUENCE OF COLLISIONS ON THE TRANSPORT OF RESONANCE RADIATION

The collision integral in the equation for the spectral density of the excited atoms, taking broadening into account, can be derived starting with Eq. (6), taking for the mass operator  $\Sigma^{-+}$  (see Fig. 1) the following expression,

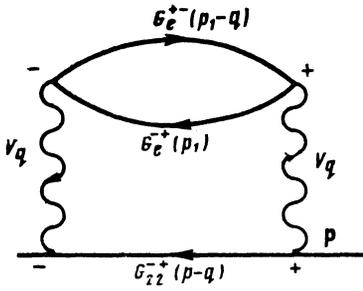


FIG. 1. Feynman diagram of the mass operator  $\Sigma_{22}^{+-}$ , which describes the elastic collision of an electron with an excited atom. The thin lines are the Green's functions of the electrons, the wavy lines are the Fourier components of the interaction potential  $V_q$ , and the straight line is the Green's function of the atom  $G_{22}^{+-}$ .

obtained in the Born approximation:

$$\Sigma^{-+}(p) = - \int \frac{d^4 p_1 d^4 q}{(2\pi)^8} G_{22}^{-+}(p-q) |V_q|^2 \cdot 4\pi^2 n_{p_1} (1-n_{p_1-q}) \times \delta(\omega_{p_1} - \epsilon_{p_1}^e + \mu_e) \delta(\omega_{p_1} - \omega_q - \epsilon_{p_1-q}^e + \mu_e). \quad (22)$$

Here  $V_q$  is the Fourier transform of the potential of the interaction of the excited electron with the other particles (e.g., electrons), characterized by occupation numbers  $n_{p_1}$ , energy  $\epsilon_{p_1}^e$ , and chemical potential  $\hbar\mu_e$ . This interaction leads to a broadening of the excited state. It is assumed that the "broadened" particles satisfy Fermi statistics.

Similarly it is possible to write an expression for  $\Sigma^{+-}(p)$ :

$$\Sigma^{+-}(p) = - \int \frac{d^4 p_1 d^4 q}{(2\pi)^8} G_{22}^{+-}(p-q) |V_q|^2 \cdot 4\pi^2 n_{p_1-q} (1-n_{p_1}) \times \delta(\omega_{p_1} - \epsilon_{p_1}^e + \mu_e) \delta(\omega_{p_1} - \omega_q - \epsilon_{p_1-q}^e + \mu_e). \quad (23)$$

The corresponding right side of the Dyson equation (16) has, according to Refs. 17 and 18, the following form:

$$St_e = \Sigma^{-+}(p) G_{22}^{+-}(p) - \Sigma^{+-}(p) G_{22}^{-+}(p) \quad (24)$$

(here the transition to the Fourier components in the time difference and the coordinates of type (2) has already been made).

Let us consider the case of thermodynamic equilibrium, i.e., let us find the equilibrium form of the Green's function of the excited particles from the equation  $St_e = 0$ . We seek the solution in the form (cf. Ref. 18)

$$G_{22}^{+-}(p) = (G_{22}^R - G_{22}^A) [1 - N(p)], \quad (25)$$

$$G_{22}^{-+}(p) = -(G_{22}^R - G_{22}^A) N(p).$$

Taking into account the relation

$$G_{22}^R - G_{22}^A = -2i\pi a_2(\epsilon_p), \quad (26)$$

where  $\epsilon_p = \omega_p - \omega_2 - E_p + \mu$  and  $a_2(\epsilon_p)$  is defined by Eq. (17), we obtain

$$N(p) = \left\{ \exp \left[ \frac{\hbar(\epsilon_p + \omega_2 + E_p - \mu)}{T} \right] + 1 \right\}^{-1}. \quad (27)$$

Thus, the equilibrium spectral density of the excited particles has the form

$$N(\omega, \mathbf{p}) = 2\pi a_2(\epsilon_p) \left\{ \exp \left[ \frac{\hbar(\epsilon_p + E_p + \omega_2 - \mu)}{T} \right] + 1 \right\}^{-1}. \quad (28)$$

If the line width  $\hbar\gamma_2$  is small in comparison with the temperature,  $\hbar\gamma_2 \ll T$ , then the quantity  $\epsilon_p$  in the argument of the exponent can be neglected, and, taking into account that the particles are not degenerate, we obtain from Eq. (28)

$$N(\omega, \mathbf{p}) \approx 2\pi a_2(\epsilon_p) \exp \left[ - \frac{\hbar(\omega_2 + E_p - \mu)}{T} \right].$$

In this limit carrying out the integral over the momenta in

$$N(\omega) = \int \frac{d^3 p}{(2\pi)^3} N(\omega, \mathbf{p})$$

gives the relation

$$N(\omega) = 2\pi a_2(\epsilon_p) N,$$

where  $N$  is the equilibrium concentration of the excited atoms [cf. Eq. (1)]. For an arbitrary relationship between the line width and the temperature relation (1) is not fulfilled and one must use expression (28).

Under conditions of equilibrium of radiation with matter it is possible to find the equilibrium form of the function  $D_{ij}^{+-}(\omega, \mathbf{k})$  from the condition  $St_i = 0$  [see Eq. (19)]. Using the relation<sup>18</sup>

$$D_{ij}^{+-}(\omega, \mathbf{k}) = D_{ij}^R(\omega, \mathbf{k}) - D_{ij}^A(\omega, \mathbf{k}) + D_{ij}^{-+}(\omega, \mathbf{k})$$

and substituting Eqs. (25) and (26) in the expression for the polarization operators (21) and (20), we obtain

$$\Pi^{-+} = e^{-\hbar\omega/T} \Pi^{+-}, \quad (29)$$

$$D_{ij}^{-+} = \frac{\Pi^{-+}}{\Pi^{+-} - \Pi^{-+}} (D_{ij}^R - D_{ij}^A) = \frac{1}{e^{\hbar\omega/T} - 1} (D_{ij}^R - D_{ij}^A).$$

The retarded Green's function is defined by the expression<sup>22</sup>

$$D_{ij}^R(\omega, \mathbf{k}) = \frac{4\pi\hbar\omega^2}{\omega_k^2 - \omega^2 \epsilon(\omega, \mathbf{k})} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right), \quad D^A = (D^R)^*. \quad (30)$$

Note that if we use a relation of the form (1) in place of Eq. (28) we obtain in Eq. (29) instead of the Planck formula for the occupation number an expression in which the running frequency  $\omega$  is replaced by  $\omega_0$ . Note that transport theory gives in the equilibrium limit for the occupation numbers of the photons the value  $[\exp(\hbar\omega_0/T) - 1]^{-1}$ , i.e., for a broad line the difference from the Planck formula can be substantial (compare the discussion on this matter in Refs. 23-25). In this case in the present transfer theory in the CFR limit on the basis of what has been said it follows that it is necessary to redefine the expressions for the spontaneous source term:

$$\bar{\epsilon} \rightarrow \exp[-\hbar(\omega - \omega_0)/T] \epsilon(\omega) N_2,$$

and the absorption coefficient

$$k_{\omega} \rightarrow \sigma_{\omega} \{N_1 - N_2 \exp[-\hbar(\omega - \omega_0)/T]\}.$$

Since what interests us here is first of all a description of the transport of resonance radiation in the PFR limit, line broadening due to collisions is assumed to be small in comparison with the temperature of the gas. In this case the expression for the collision integral (24) can be simplified by limiting it to the  $\tau$ -approximation:

$$St_e = \epsilon - \hbar [N(\omega, \mathbf{v}) - 2\pi N(\mathbf{v}) a_2(\epsilon_p)] / \tau_{ei}. \quad (31)$$

Here  $N(\mathbf{v})$  is the velocity distribution function of the excited particles,  $\tau_{el}$  is the time between elastic collisions, and  $1/\tau_{el} = \gamma_{el}$ . In an analogous way it is possible to take into account the collisions of the excited particles, which cause the relaxation of their velocity distribution to the equilibrium Maxwell function  $f_M(\mathbf{v})$  after a characteristic time  $\tau_M$  ( $\nu_M = 1/\tau_M$ ). The corresponding collision integral has the form

$$St_v = -\hbar [N(\omega, \mathbf{v}) - N(\omega) f_M(\mathbf{v})] / \tau_M. \quad (32)$$

In the collision integral it is also possible to take account of the decrease of the number of excited particles due to collisions taking place with frequency  $\nu_T$ , and also their formation with rate  $q(\omega, \mathbf{v})$ , also due to collisions. It is assumed that the structure of the corresponding source term has the form

$$q(\omega, \mathbf{v}, \mathbf{r}, t) = 2\pi q(\mathbf{r}, t) a_2(\varepsilon_p) f_M(\mathbf{v}). \quad (33)$$

### 3. SYSTEM OF EQUATIONS OF RESONANCE RADIATION TRANSFER

Within the framework of the above-formulated approximations we can write the basic equations that describe the transport of resonance radiation under nonstationary conditions. The equations for the spectral density of the excited atoms now take the form [cf. Eqs. (16), (18), (31)–(33)]

$$\begin{aligned} \frac{\partial}{\partial t} N(\omega, \mathbf{v}, \mathbf{r}, t) + \mathbf{v} \frac{\partial}{\partial \mathbf{r}} N(\omega, \mathbf{v}, \mathbf{r}, t) \\ = -\gamma_{el} [N(\omega, \mathbf{v}, \mathbf{r}, t) - 2\pi N(\mathbf{v}, \mathbf{r}, t) a_2(\omega)] \\ - \nu_M [N(\omega, \mathbf{v}, \mathbf{r}, t) - N(\omega, \mathbf{r}, t) f_M(\mathbf{v})] \\ - N(\omega, \mathbf{v}, \mathbf{r}, t) (\nu_T + A) + q(\omega, \mathbf{v}, \mathbf{r}, t) + \frac{2\pi A \lambda^2}{4} a_2(\omega) \\ \times \int d\Omega d\omega_k \frac{J(\omega_k, \Omega, \mathbf{r}, t)}{\hbar \omega_k} N_1(\mathbf{v}) \delta(\omega - \omega_k + \omega_0 + \mathbf{k}\mathbf{v}) \end{aligned} \quad (34)$$

(here and below in place of the quantity  $\varepsilon_p$  we use the notation  $\omega$ , where  $\omega$  is reckoned from the transition frequency  $\omega_0$ ). We write the equation for the spectral intensity  $J_k(\omega, \Omega, \mathbf{r}, t)$  in the form [cf. Eqs. (4), (5), (19)–(21)]

$$\begin{aligned} (\Omega \nabla) J = -k_\omega J + \frac{A \hbar \omega_k}{4\pi} \int \frac{d\omega'}{2\pi} d\mathbf{v} N(\omega', \mathbf{v}, \mathbf{r}, t) \\ \times \delta(\omega' - \omega_k + \omega_0 + \mathbf{k}\mathbf{v}). \end{aligned} \quad (35)$$

Here  $k_\omega$  is the absorption coefficient, which, neglecting the corrections for stimulated emission, is given by the expression

$$k_\omega = \frac{1}{4} \lambda^2 A N_1 \varphi(\omega). \quad (36)$$

The quantity  $\varphi(\omega)$  is expressed in terms of the initial line shape  $a_2(\omega)$  by the equation

$$\varphi(\omega) = \int d\mathbf{v} f_M(\mathbf{v}) a_2(\omega_k - \omega_0 - \mathbf{k}\mathbf{v}). \quad (37)$$

As was already noted in the discussion of expression (21), the generalization to the case of degenerate levels consists in multiplying  $A$  in Eq. (36) by the ratio of the statistical weights  $g_2/g_1$ . The line width  $\gamma_2$  is defined in the following way:

$$\gamma_2 = A + \gamma_{el} + \nu_M + \nu_T. \quad (38)$$

As follows from Eq. (35), the spontaneous photon source is determined by  $N_{sp}(\Delta, \Omega, \mathbf{r}, t)$ :

$$N_{sp}(\Delta, \Omega, \mathbf{r}, t) = \int \frac{d\omega'}{2\pi} d\mathbf{v} N(\omega', \mathbf{v}, \mathbf{r}, t) \delta(\omega' - \omega_k + \omega_0 + \mathbf{k}\mathbf{v}). \quad (39)$$

Here  $\Delta = \omega_k - \omega_0$  is the detuning of the photon frequency from resonance. An equation for  $N_{sp}$  can be obtained by multiplying Eq. (34) by the corresponding  $\delta$ -function and integrating over frequencies and velocities as in Eq. (39). As a result we have

$$\begin{aligned} \frac{\partial N_{sp}}{\partial t} + \text{div } \mathbf{j}_{sp} = -\gamma_{el} [N_{sp} - M(\Delta, \Omega, \mathbf{r}, t)] \\ - \nu_M [N_{sp} - P(\Delta, \Omega, \mathbf{r}, t)] - N_{sp} (\nu_T + A) + q(\mathbf{r}, t) \varphi(\Delta) \\ + \frac{A \lambda^2}{4} N_1 \int d\Omega' d\omega_k' \frac{J(\omega_k', \Omega', \mathbf{r}, t)}{\hbar \omega_k'} R_{II}(\omega_k', \omega_k). \end{aligned} \quad (40)$$

In Eq. (40) there arise two new moments  $M$  and  $P$ , defined in the following way:

$$M(\Delta, \Omega, \mathbf{r}, t) = \int d\omega' d\mathbf{v} N(\omega', \mathbf{v}, \mathbf{r}, t) a_2(\omega') \delta(\omega' - \Delta + \mathbf{k}\mathbf{v}), \quad (41)$$

$$P(\Delta, \Omega, \mathbf{r}, t) = \int \frac{d\omega'}{2\pi} d\mathbf{v} N(\omega', \mathbf{v}, \mathbf{r}, t) f_M(\mathbf{v}) \delta(\omega' - \Delta + \mathbf{k}\mathbf{v}). \quad (42)$$

The flux of excited atoms  $\mathbf{j}_{sp}$  has the form

$$\mathbf{j}_{sp} = \int \frac{d\omega'}{2\pi} d\mathbf{v} \mathbf{v} N(\omega', \mathbf{v}, \mathbf{r}, t) \delta(\omega' - \Delta + \mathbf{k}\mathbf{v}). \quad (43)$$

The redistribution function over frequency  $R_{II}(\omega_k', \omega_k)$ , which describes the coherent scattering of the photon  $\mathbf{k} \rightarrow \mathbf{k}'$  in the laboratory frame is defined by the expression (cf. Ref. 3)

$$R_{II}(\omega_k', \omega_k) = \int d\mathbf{v} f_M(\mathbf{v}) a_2(\omega_k - \omega_0 - \mathbf{k}\mathbf{v}) \delta(\omega_k - \omega_k' - \mathbf{k}\mathbf{v} + \mathbf{k}'\mathbf{v}). \quad (44)$$

As follows from the structure of the collision terms (31) and (32), the quantities  $N(\mathbf{r}, t)$ ,  $N(\mathbf{v}, \mathbf{r}, t)$ , and  $N(\omega, \mathbf{r}, t)$  are the corresponding moments of the spectral density  $N(\omega, \mathbf{v}, \mathbf{r}, t)$ . The equations for the moments  $M$  (41) and  $P$  (42) figuring in Eq. (40) have the form

$$\begin{aligned} \frac{\partial M}{\partial t} + \text{div } \mathbf{j}_M = -\nu_M [M - N\varphi(\Delta)] - M(\nu_T + A) + q(\mathbf{r}, t) \varphi(\Delta) \\ + \frac{A \lambda^2}{4} N_1 \int d\Omega' d\omega_k' \frac{J(\omega_k', \Omega', \mathbf{r}, t)}{\hbar \omega_k'} R_{III}(\omega_k', \omega_k), \end{aligned} \quad (45)$$

$$\begin{aligned} \frac{\partial P}{\partial t} + \text{div } \mathbf{j}_P = -\gamma_{el} [P - N\varphi(\Delta)] - P(\nu_T + A) + q(\mathbf{r}, t) \varphi(\Delta) \\ + \frac{A \lambda^2}{4} N_1 \int d\Omega' d\omega_k' \frac{J(\omega_k', \Omega', \mathbf{r}, t)}{\hbar \omega_k'} R_{IV}(\omega_k', \omega_k), \end{aligned} \quad (46)$$

$$\begin{aligned} \frac{\partial N(\mathbf{r}, t)}{\partial t} + \text{div } \mathbf{j} = -N(\nu_T + A) + q(\mathbf{r}, t) \\ + \frac{A \lambda^2}{4} N_1 \int d\Omega d\omega_k \frac{J(\omega_k, \Omega, \mathbf{r}, t)}{\hbar \omega_k} \varphi(\Delta_k). \end{aligned} \quad (47)$$

In Eqs. (45)–(47) three new fluxes arise:

$$\mathbf{j}_M = \int \frac{d\omega' d\mathbf{v}}{2\pi} \mathbf{v} N(\omega', \mathbf{v}, \mathbf{r}, t) a_2(\omega') \delta(\omega' - \Delta + \mathbf{k}\mathbf{v}),$$

$$\mathbf{j}_p = \int \frac{d\omega' d\mathbf{v}}{2\pi} v N(\omega', \mathbf{r}, t) f_M(\mathbf{v}) \delta(\omega' - \Delta + \mathbf{k}\mathbf{v}),$$

$$\mathbf{j} = \int d\mathbf{v} v N(\mathbf{v}, \mathbf{r}, t).$$

Note that contributions from flux terms in problems connected with the decay of excitations are usually small. For example, under the conditions of the experiment in Ref. 7 the ratios of the second term to the first on the left side of Eqs. (40), (45)–(47) are of the order of  $v_T/R\beta \ll 1$ , where  $v_T$  is the characteristic thermal velocity of the particles ( $\sim 10^4$  cm/sec),  $R$  is the radius of the cylindrical volume ( $\sim 1$  cm) and  $\beta$  is the effective decay rate ( $\beta > 10^5 \text{ sec}^{-1}$ ). In addition, in the theory of radiative transfer in optically dense media one typically uses quantities which are averaged over the photon directions (cf. Ref. 3)

$$N(\Delta, \mathbf{r}, t) = \langle N(\Delta, \mathbf{\Omega}, \mathbf{r}, t) \rangle_{\mathbf{\Omega}}.$$

After analogously averaging Eqs. (40), (45), and (46), we can neglect the contribution of the flux terms.

The redistribution functions over frequency

$$R_{III}(\omega_k', \omega_k) = \int d\mathbf{v} f_M(\mathbf{v}) a_2(\omega_k' - \mathbf{k}'\mathbf{v}) a_2(\omega_k - \mathbf{k}\mathbf{v}), \quad (48)$$

which describes incoherent scattering in the center-of-mass system,<sup>3</sup> and

$$R_{IV}(\omega_k', \omega_k) = \int d\omega'' R_I(\omega'', \omega_k') a_2(\omega'') R_I(\omega'', \omega_k), \quad (49)$$

where

$$R_I(\omega', \omega) = \int d\mathbf{v} f_M(\mathbf{v}) \delta(\omega' - \omega + \mathbf{k}\mathbf{v}). \quad (50)$$

figure in the right sides of Eqs. (45) and (46). The function  $R_{IV}$  describes photon scattering under conditions in which many collisions that involve a change in the velocity of the atom but do not lead to a change in the phase of the oscillator take place between the act of absorption and the act of emission.

Averaging the functions  $N_{sp}$ ,  $M$ , and  $P$  over the photon directions, we obtain equations of the type (40), (45), and (46), but without the flux terms. The angle-averaged redistribution functions  $R_{II}$  and  $R_{III}$  (an analogous approximation was made in Ref. 3) figure on the right sides of these equations.

The resulting system of equations (35), (40), (45)–(47) describes the time-dependent transport of resonance radiation under PFR conditions. In the stationary case it is possible to obtain a closed equation for  $N_{sp}(\Delta)$  after eliminating the quantities  $M$ ,  $P$ , and  $N$  from Eqs. (45)–(47). As a result we have

$$v N_{sp}(\Delta) = q\varphi(\Delta)$$

$$+ \frac{v \lambda^2}{A} N_1 \int d\mathbf{\Omega} d\omega_k' \frac{J(\omega_k', \mathbf{\Omega})}{\hbar\omega_k'} R_{II}(\omega_k', \omega_k), \quad (51)$$

where  $v = v_T + A$  is the total rate of decay of the excited particles, and the total redistribution function  $R_{II}(\omega_k', \omega_k)$  is expressed in terms of the angle-averaged functions  $R_{II}$ ,  $R_{III}$ , and  $R_{IV}$  in the following way (cf. Refs. 22, 26, and 27):

$$R_{II}(\omega', \omega) = \frac{A}{\gamma_2} \left\{ \frac{\gamma_{el} v_M}{v} \left[ \frac{1}{v + \gamma_{el}} + \frac{1}{v + v_M} \right] \varphi(\Delta) \varphi(\Delta') \right. \\ \left. + \frac{v_M}{\gamma_{el} + v} R_{IV}(\omega', \omega) + \frac{\gamma_{el}}{v + v_M} R_{III}(\omega', \omega) + R_{II}(\omega', \omega) \right\}. \quad (52)$$

The complete frequency redistribution regime corresponds to the limit  $\gamma_{el} > v_M \gg A, v$ . In this limit we have

$$R_{II}(\omega', \omega) = \frac{A}{v} \varphi(\Delta) \varphi(\Delta'),$$

and Eq. (51) gives [cf. Eq. (1)]

$$N_{sp}(\Delta) = N\varphi(\Delta).$$

The redistribution function obtained in Ref. 26 follows from Eq. (52) by setting  $v_M = 0$ . Note that in the equations for  $N_{sp}(\Delta)$  used in the literature (see Ref. 3), the factor  $v/A$  on the right side of equations of type (51) is commonly omitted.

In nonstationary problems the system of equations (40), (45)–(49) does not reduce to one equation with a redistribution function of the type  $R_{II}(\omega', \omega)$ . However, in Ref. 7 use was made of the equation

$$\frac{\partial N_{sp}}{\partial t} + v N_{sp} \\ = q\varphi + \frac{\lambda^2}{4} N_1 \frac{v}{A} \int d\mathbf{\Omega} d\omega_k' \frac{J(\omega_k', \mathbf{\Omega})}{\hbar\omega_k'} R_{II}(\omega_k', \omega). \quad (53)$$

to describe nonstationary regimes of resonance radiation transfer under PFR conditions. The spectral intensity centering on the right side of Eq. (53) is expressed in Ref. 7 proceeding from Eq. (35) in terms of the corresponding spontaneous source term:

$$\int d\mathbf{\Omega} \frac{J(\omega_k', \mathbf{\Omega})}{\hbar\omega_k'} = A \int \frac{d\mathbf{r}'}{4\pi |\mathbf{r} - \mathbf{r}'|^2} \exp(-k_{\omega'} |\mathbf{r} - \mathbf{r}'|) N_{sp}(\Delta', \mathbf{r}') \\ \equiv A \int \frac{d\mathbf{r}'}{v} G_{\omega_k'}(\mathbf{r} - \mathbf{r}') N_{sp}(\Delta', \mathbf{r}'), \quad (54)$$

Expression (52) with  $v_M$  set to zero was used as the redistribution function in Ref. 7. In addition, for the function  $R_{III}(\omega', \omega)$  there is the widely used approximation (cf. Ref. 3)

$$\langle R_{III}(\omega', \omega) \rangle_{\mathbf{\Omega}} \approx \varphi(\Delta) \varphi(\Delta') \quad (55)$$

and the factor  $v/A$  in Eq. (53) is set equal to unity in Ref. 7. The equations used in Ref. 7 (53) and (54) were subjected to criticism in Ref. 16 because they were not, strictly speaking, derived, but written down heuristically. To derive an equation that describes nonstationary radiative transfer, use was made in Ref. 16 of the relation (see Refs. 28 and 29)

$$N_{sp}(\Delta) = N\varphi(\Delta) + \frac{A\lambda^2}{4} \frac{N_1}{\gamma_{el} + v} \int d\mathbf{\Omega} d\omega_k' \frac{J(\omega_k', \mathbf{\Omega})}{\hbar\omega_k'} \\ \times [R_{II}(\omega', \omega) - \varphi(\Delta) \varphi(\Delta')]. \quad (56)$$

We note that this relation follows from system (40), (45)–(47) if one sets  $v = 0$ , uses the approximation (55), and, extracting Eq. (45) from Eq. (40), neglects the time derivative  $\partial(N_{sp} - M)/\partial t$  in comparison with the quantity  $(\gamma_{el} + v)(N_{sp} - M)$ . In addition, by virtue of Eq. (55) it follows that one should use the relation  $M(\Delta) \approx N\varphi(\Delta)$ . If now we differentiate relation (56) with respect to time, mak-

ing use of the equation for the total concentration of excited atoms (47), and we take into account the time derivative of the spectral intensity, which figures in Eq. (56), expressing it by virtue of relation (54) in terms of the derivative  $\dot{N}_{sp}(\Delta)$ , we then obtain the basic equation of Ref. 16. It differs from Eq. (53) by an additional term containing the derivative  $\dot{N}_{sp}(\Delta)$  under the integral sign in Eq. (54). As follows from the above derivation, taking the additional term into account in Eq. (53), as was proposed in Ref. 16, amounts to an excess of accuracy. The authors of Ref. 16 criticize Ref. 7 for not taking account of the fundamentally instantaneous character of the process of coherent scattering. As was discussed above, in making the transition from Eq. (14) to Eq. (16) it is valid to treat the process as instantaneous if the characteristic time  $\tau$  of variation of the intensity is large in comparison with the time  $\tau_0$ , which depends on the frequency of the scattered radiation. Since in optically thick systems the characteristic decay times of the excitations are large in comparison with the spontaneous time  $\tau_R$ , the corrections which the authors of Ref. 16 propose to take into account are small in comparison with the terms contained in Eq. (53).

#### 4. COMPARISON WITH EXPERIMENT

In order to compare the results obtained from the system of equations (35), (40), (45)–(47) with experiment, we developed a technique of numerical solution of these equations for plane and cylindrical geometries. The initial conditions consisted of a small concentration of excited particles ( $\sim 5 \times 10^{10} \text{ cm}^{-3}$ ) in the layer near the axis with a radius of 3 mm. A study was made of the radial distribution of the excited particles and the spectrum of the radiation emerging from a cylinder of radius  $R = 1.1 \text{ cm}$ . From the time dependence of the spectrally integrated radiation intensity at the resonance line of argon  $^1P_1 - ^1S_0$  ( $\lambda = 1048 \text{ \AA}$ ) we constructed the dependence of the effective decay rate of the excitations  $\beta$  [ $\text{sec}^{-1}$ ] on the argon pressure. The elastic width  $\gamma_{el}$  corresponding to resonant excitation exchange in accordance with Ref. 7 was taken to be of the form  $\gamma_{el}/A = 1.74P$ , where  $P$  is the argon pressure in Torr and  $A = 5.5 \times 10^8 \text{ sec}^{-1}$ . The ratio of the statistical weights is  $g_2/g_1 = 3$  in this case. The transient time of the Maxwell distribution was estimated from the gas-kinetic cross section  $\sigma \approx 5 \times 10^{-15} \text{ cm}^2$ .

Figure 2 shows the time dependence of the intensity of

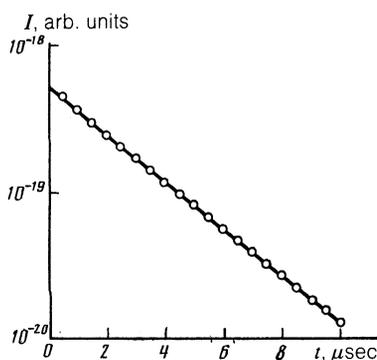


FIG. 2. Time dependence of the intensity of the radiation emerging from the cylinder at the argon line  $^1P_1 - ^1S_0$  ( $\lambda = 1048 \text{ \AA}$ ) at pressure  $P = 0.021 \text{ Torr}$ .

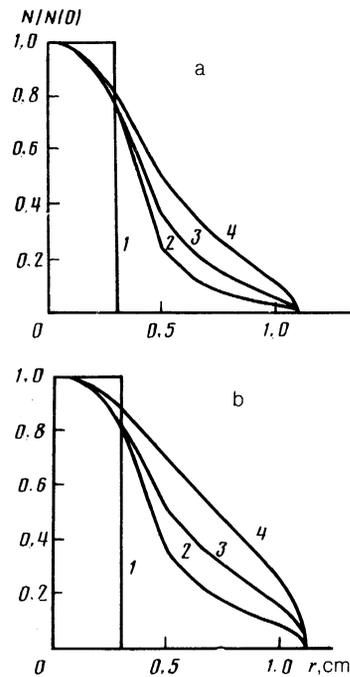


FIG. 3. Radial distribution of the excited particle concentration at various instants of time, normalized to the value of the concentration at the point on the cylindrical axis (pressure  $P = 0.042 \text{ Torr}$ ): a) calculation based on the complete system of equations (the PFR approximation), b) calculation in the CFR approximation; 1)  $0 \mu\text{sec}$ , 2)  $3 \mu\text{sec}$ , 3)  $5 \mu\text{sec}$ , and 4)  $8 \mu\text{sec}$ .

the radiation emerging from the cylinder for argon pressure  $P = 0.021 \text{ Torr}$ . As in Fig. 5 of Ref. 7, this dependence is exponential and is characterized by a decay rate constant  $\beta$  ( $\beta_{\text{theo}} = 3.7 \times 10^5 \text{ sec}^{-1}$ ,  $\beta_{\text{exp}} = 3.8 \times 10^5 \text{ sec}^{-1}$ ). Figure 3 depicts the radial distributions of the concentration of excited particles at various times, normalized to the value of the concentration at the point on the axis of the cylinder. Figure 3a presents the results of a calculation for the complete system of equations for  $P = 0.042 \text{ Torr}$ , and Fig. 3b shows the results obtained for the same conditions in the CFR approximation. Note that at this pressure the quantity  $\beta$ , obtained from the complete system of equations, is  $\sim 3 \times 10^5 \text{ sec}^{-1}$ , while experiment gives  $\beta \approx 3.1 \times 10^5 \text{ sec}^{-1}$ . This is the point at which the difference in the results of the PFR approximation and the CFR theory is greatest. The value of  $\beta$  obtained from the theory<sup>1,2</sup> at this pressure is  $9.83 \times 10^5 \text{ sec}^{-1}$ . For this case we carried out a study of the effects of various approximations on the value of  $\beta$ . If instead of the complete system of equations we use the approximate equation (53), we obtain the same value  $\beta \approx 3 \times 10^5 \text{ sec}^{-1}$  with an accuracy of  $\sim 1\%$ . The effect of the transient time of the Maxwell distribution within the framework of system (40), (45)–(47) on the value of  $\beta$  is insignificant: for  $v_M = 0$  we have  $\beta = 3.006 \times 10^5 \text{ sec}^{-1}$ , and for  $v_M = \infty$  we obtain  $\beta = 3.04 \times 10^5 \text{ sec}^{-1}$ .

The authors of Ref. 14 carried out a comparison of the results of the diffusion approximation of Eq. (53) with experiment<sup>7</sup> (according to the formulas of Ref. 14  $\beta \approx 4.4 \times 10^5 \text{ sec}^{-1}$  for  $P = 0.042 \text{ Torr}$ ). The numerical solution of the complete system of equations in the diffusion approximation that they obtained came within 3% of the exact solution. Figure 4 displays the spectra of the outgoing radiation for a number of pressures as a function of the di-

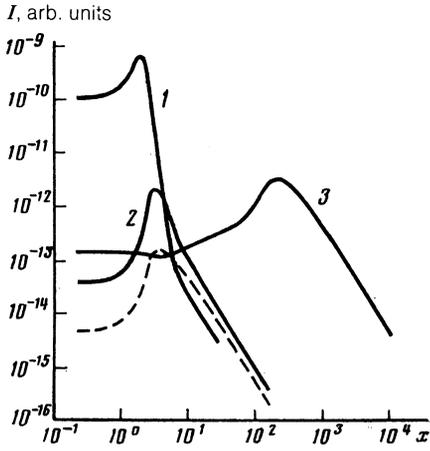


FIG. 4. Spectrum of the outgoing radiation for various values of the pressure  $P$ ;  $x = (\omega - \omega_0)/\omega_D$ . The solid curves correspond to the calculation based on the complete program (the PFR approximation): 1) 0.0015 Torr, 2) 0.042 Torr, 3) 9.35 Torr; the dashed line corresponds to the CFR approximation,  $P = 0.042$  Torr.

dimensionless detuning from the resonance  $x = (\omega - \omega_0)/\omega_D$ , where  $\omega_D$  is the Doppler width. Figure 5 displays the dependence of the decay rate  $\beta$  on the argon pressure.

To estimate the accuracy of the implemented approximation of the effective lifetime of the excited particles (see Refs. 1–3) we also calculated the probability  $\theta$  of emission of a photon, obtained from Eqs. (40), (45)–(47) in the limit  $\nu_M \rightarrow \infty$ . In this approximation we have  $\beta = A\theta$ . In stationary problems we obtain for the quantity  $\theta$

$$\theta(\mathbf{r}) = \int_{-\infty}^{\infty} \frac{a_2(\omega)\theta(\omega, \mathbf{r})d\omega}{1+A\theta(\omega, \mathbf{r})/\gamma_{el}} \left[ \int_{-\infty}^{\infty} \frac{a_2(\omega)d\omega}{1+A\theta(\omega, \mathbf{r})/\gamma_{el}} \right]^{-1}. \quad (57)$$

Here  $\theta(\omega, \mathbf{r})$  is given by the expression

$$\theta(\omega, \mathbf{r}) = \int \frac{d\Omega'}{4\pi} \int d\omega' R_1(\omega', \omega) \exp(-k_\omega \cdot |\mathbf{r} - \mathbf{r}_s|). \quad (58)$$

The function  $R_1(\omega', \omega)$  is defined by Eq. (50). Expression (58) gives the probability of emission of a photon from the point  $\mathbf{r}$ ; the point  $\mathbf{r}_s$  is located on the surface bounding the volume. The integration in Eq. (58) is carried out over all solid angles subtending the surface from the point  $\mathbf{r}$ . In non-stationary problems it is possible to obtain an equation for  $\beta(\mathbf{r})$ :

$$\beta(\mathbf{r}) = A \int_{-\infty}^{\infty} \frac{a_2(\omega)\theta(\omega, \mathbf{r})d\omega}{\gamma_{el} + A\theta(\omega, \mathbf{r}) - \beta(\mathbf{r})} \left[ \int_{-\infty}^{\infty} \frac{a_2(\omega)d\omega}{\gamma_{el} + A\theta(\omega, \mathbf{r}) - \beta(\mathbf{r})} \right]^{-1}, \quad (59)$$

where  $\theta(\omega, \mathbf{r})$  is given by Eq. (58). The difference between expressions (57) and (59) can be discerned only at very low pressures  $P \approx 0.0015$  Torr under experimental conditions. At larger pressures both expressions coincide. Figure 5 presents the results of estimates of the decay rate  $\beta$ , obtained using Eqs. (57) and (58).

The comparison just carried out shows that the system of equations (35), (40), (45)–(47), obtained for problems of resonance radiation transfer in the PFR limit, describes the experiment well under nonstationary conditions. Note also the completely satisfactory accuracy of the heuristic

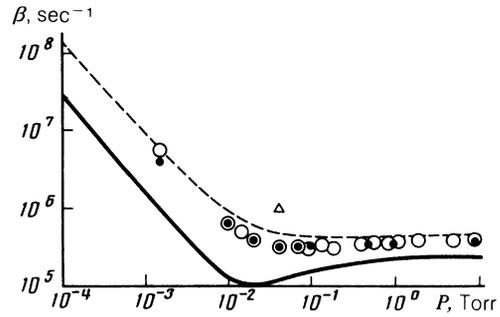


FIG. 5. Dependence of the decay rate of the excited states  $\beta$  on the argon pressure. The solid curve corresponds to the calculation for the stationary problem according to formula (57), the dashed curve—to the calculation in the diffusion approximation,<sup>14</sup>  $\circ$ —experiment,<sup>7</sup>  $\bullet$ —calculated using the complete program (the PFR approximation),  $\Delta$ —calculated in the CFR approximation.

equation (53) for the description of the decay of the excited system in optically thick media. The criticism of this equation in Ref. 16 does not stand. The effective-lifetime approximation gives too low an estimate for the decay rate; however, it qualitatively reproduces the characteristic minimum in the pressure dependence of  $\beta$ . The complete frequency redistribution approximation gives a monotonic trend in this dependence and substantially exceeds the true value of the decay rate in the pressure region  $P \approx 0.04$  Torr.

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