Magnetic force reversal and instability in a plasma with advanced magnetohydrodynamic turbulence

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The effect of fully developed MHD turbulence on the dynamics of a large-scale magnetic field is investigated. It is shown that generation of small-scale magnetic pulsations lowers the total (hydrodynamic plus magnetic) turbulent pressure. This can lead to reversal of the sign of the large-scale magnetic force and to instability of the magnetic field on account of the energy of the turbulent pulsations. The case $\beta \gg 1$ is investigated, where β is the ratio of the plasma pressure to that of the large-scale magnetic field.

1. INTRODUCTION

Investigation of fully developed MHD turbulence is of substantial interest when it comes to explaining the nature of the magnetic fields of the sun, stars, galaxies, and planets.[1-3] The analysis of the interaction of a large-scale magnetic field with fully developed MHD turbulence is usually confined to effects linear in the field. Such an analysis is known to yield the turbulence viscosity, the α effect, and the turbulent diamagnetism (see, e.g., Refs. 4–6). This approach seems natural at first glance, since the energy of the turbulent pulsations exceeds greatly the energy of the large-scale magnetic field.

On the other hand, calculation of fluctuation fields in a plasma with a large-scale magnetic fields **B** and small magnetic Reynolds numbers Rm shows that the change of the large-scale magnetic pressure in the presence of fluctuations is of the order of $\mathrm{Rm}\cdot B^2/8\pi$ (Ref. 7). The change of the magnetic force in a highly conducing plasma with fully developed MHD turbulence (for $\mathrm{Rm} \gg 1$) should therefore be expected to be substantial.

We show in the present paper that in a plasma with fully developed MHD turbulence the elasticity of the large-scale magnetic field is noticeably lowered, and the effective magnetic force can change sign under certain conditions. This effect excites an instability that leads to formation of inhomogeneities of the large-scale magnetic field. The instability energy source are small-scale turbulent pulsations. The modification of the magnetic forces is nonlinear in the field **B**.

The onset of instabilities that are due to sign reversal of force of various types has been studied many times (see Refs. 8–10 and the citations therein). No studies, however were made of the instability of a large-scale magnetic field in plasma with fully developed MHD turbulence for $\beta \gg 1$ (β is the ratio of the plasma pressure to that of the large-scale magnetic field).

The magnetic fields of the sun, stars, galaxies, and planets are highly nonuniform and take the form of magnetic braids and tubes, formed by a mechanism that remains unexplained. An attempt can be made at least to relate the initial phase of formation of magnetic braids with the development of the instability generated by negative magnetic pressure.

2. EFFECT OF NEGATIVE MAGNETIC PRESSURE (QUALITATIVE CONSIDERATION)

We consider fully developed MHD turbulence in which the characteristic scale of hydrodynamic motions is l_0 and the magnetic-pulsation scale is l_m. The minimum scale of the problem (thickness of the filaments of the pulsating magnetic field) is of the order of $l_m \approx l_0 \mathrm{Rm}^{-1/2}$ (Refs. 11, 12), where $\mathrm{Rm} = u_0 l_0/\nu_m$ is the magnetic Reynolds number, u_0 is the characteristic turbulent velocity, and ν_m is the magnetic-diffusion coefficient. Generation of turbulent magnetic fields at the expense of the hydrodynamic pulsations lowers the total (hydrodynamic plus magnetic) turbulent pressure P_T. We explain the essence of the effect qualitatively using isotropic turbulence as the example. Recall that in this case

$$\langle u_i(\mathbf{r}) u_j(\mathbf{r})\rangle = \langle u^2\rangle \delta_{ij}/3,$$
$$\langle h_i(\mathbf{r}) h_j(\mathbf{r})\rangle = \langle h^2\rangle \delta_{ij}/3,$$

where **u** and **h** are random pulsations of the hydrodynamic and magnetic fields, δ_{ij} is the Kronecker delta, and the angle brackets denote averaging over the ensemble. For isotropic turbulence the pressure is given by the relation $P_T = W_m/3 + 2W_k/3$, where $W_m = \langle h^2\rangle/8\pi$ is the energy density of the magnetic pulsations, $W_k = \langle \rho u^2\rangle/2$ is the energy density of the turbulent hydrodynamic motion, and ρ is the plasma density (see, e.g., Refs. 13 and 14). Assume that the turbulence is maintained by an "inexhaustible" energy reservoir. The total energy of the turbulence is then conserved (the dissipation is compensated for by a supply of energy), i.e., $W_k + V_m = \mathrm{const}$. A proof of this statement is given in the Appendix. Allowance for this circumstance permits the change of turbulent pressure in a statistically homogeneous unbounded medium to be expressed in terms of the change ΔW_m of the magnetic energy:

$$P_T = P_T^{(0)} - \Delta W_m/3, \tag{2.1}$$

where $P_T^{(0)}$ is the initial (without allowance for the newly producted field) turbulent pressure. It follows hence that the turbulent pressure is lowered when turbulent magnetic fields are generated (when $\Delta W_m > 0$ holds). This fact is general and is manifested, for example, in generation of the fine-structure magnetic fields predicted by the dynamo theory.[15]

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The total turbulent pressure is lowered also by the "tangling" of the large-scale magnetic field by the turbulent pulsations. In fact, let us apply to the small-scale isotropic MHD turbulence a large-scale magnetic field (with characteristic dimension $L_B \gg l_0$). Such a field is generated, for example, in a turbulent plasma with nonzero average helicity.[4,5] The large-scale magnetic field, "entangled" with the hydrodynamic turbulence, generates supplementary small-scale pulsations.[16,1] It can be assumed that in this case the energy density W_m of the turbulent magnetic pulsations depends principally on W_k and W_B, where $W_B = B^2/8\pi$ is the energy density of the large-scale magnetic field **B**. For weak magnetic fields ($W_B \ll W_k$), expanding the function W_m in a series in W_B, we obtain

$$W_m = W_m^{(0)} + a_T(W_k)B^2/8\pi + \dots,$$

where $W_m^{(0)}$ is the energy density of the magnetic pulsations in the absence of a large-scale magnetic field. Combining this expression with (2.1), we express the turbulent pressure P_T in the form

$$P_T = P_T^{(0)} - a_T B^2/24\pi. \tag{2.2}$$

An important role is played for large-scale processes by the total pressure $P \equiv P_k + P_T + P_B$, were P_k is the usual gasdynamic pressure of the plasma and $P_B = B^2/8\pi$ is the magnetic pressure of the large-scale field. With allowance for (2.2), the total pressure is

$$P = P_k + P_T^{(0)} + (1-q_p)B^2/8\pi, \tag{2.3}$$

where $q_p = a_T/3$. The sign of q_p, as seen from the analysis, is determined by the direction of energy transfer, being positive when magnetic pulsations are generated and negative when they are damped. It follows that in the presence of fully developed MHD turbulence it is possible to reverse the sign of the effective magnetic pressure $P_m = (1 - q_p)B^2/8\pi$ for $q_p > 1$. We note, to be sure, that application of a large-scale field **B** upsets the isotropy of the turbulence. But Eq. (2.3) remains in force, and only the relation between q_p and a_T is changed. The corresponding expression for q_p will be obtained in Sec. 4.

This effect must not be confused with the lowering of the magnetic pressure by turbulent diamagnetism.[5,17] Recall that the nature of turbulent diamagnetism also differs significantly from diamagnetism in classical electrodynamics. The former is a kinematic effect of removing the magnetic field from a region with more intense turbulent pulsations. The total magnetic energy does not depend here explicitly on the magnetic permeability. It follows that turbulence diamagnetism, in contrast to "classical," does not modify the Ampere force. When a magnetic field is taken out of a turbulent region its strength is lowered, and with it the magnetic pressure $P_m = B^2/8\pi$ in this region. The sign of the magnetic pressure, however, remains positive here. In contrast to turbulent diamagnetism, reversal of the sign of magnetic pressure is a dynamic effect, for in this case the structure of the Ampere force is explicitly altered [see expression (4.10) below]. Note also that this effect can amplify the magnetic field in a turbulent region.

3. FUNDAMENTAL EQUATIONS

We consider in Secs. 3 and 4 a quantitative description of the effect of reversing the sign of the magnetic force in the presence of advanced small-scale MHD turbulence.

We represent the velocity $\mathbf{v}(\mathbf{r},t)$ and the magnetic field $\mathbf{H}(\mathbf{r},t)$ in the turbulent medium in the form $\mathbf{v} = \mathbf{V} + \mathbf{u}$ and $\mathbf{H} = \mathbf{B} + \mathbf{h}$, where $\mathbf{V} = \langle \mathbf{v} \rangle$ and $\mathbf{B} = \langle \mathbf{H} \rangle$. We neglect the weak density pulsations. This is valid, in particular for $\beta \equiv 8\pi P_k/B^2 \gg 1$ and over times substantially longer than acoustic. The equation of motion and the induction equation for the average fields **V** and **B** are[1]

$$\rho d\mathbf{V}/dt = -\nabla P_k + \operatorname{div}\hat{\sigma} + \mathbf{F} + \mathbf{F}_{\nu T}, \tag{3.1}$$

$$\partial \mathbf{B}/\partial t = \operatorname{rot}([\mathbf{VB}] - \nu_m \operatorname{rot}\mathbf{B} - c\boldsymbol{\varepsilon}). \tag{3.2}$$

Here c is the speed of light, **F** and $\mathbf{F}_{\nu T}$ are respectively the external force and the force describing the turbulent and kinematic viscosities $\varepsilon = \langle [\mathbf{uh}] \rangle / c$ is the turbulent emf, and $\hat{\sigma}$ is the generalized Maxwell-stress tensor including the Reynolds turbulent-stress tensor:

$$\sigma_{ij} = -\frac{B^2 + \langle h^2 \rangle}{8\pi}\delta_{ij} + \frac{B_i B_j + \langle h_i h_j \rangle}{4\pi} - \rho\langle u_i u_j \rangle. \tag{3.3}$$

To obtain a closed system of equations it is important to find the dependence of the tensors $\langle h_i h_j \rangle$, $\langle u_i u_j \rangle$, and $\langle h_i u_j \rangle$ on the large-scale fields **V** and **B**. To this end we consider the equations for turbulent hydrodynamic and magnetic fields. Changing to a locally comoving (relative to the large-scale flows) coordinate system, we obtain[1]

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{\nabla P_*}{\rho} - (\mathbf{u}\nabla)\mathbf{V} - \frac{[\mathbf{h}\operatorname{rot}\mathbf{B}] + [\mathbf{B}\operatorname{rot}\mathbf{h}]}{4\pi\rho} + \mathbf{T} + \frac{(\mathbf{F}_\nu + \mathbf{F}_r)}{\rho}, \tag{3.4}$$

$$\partial \mathbf{h}/\partial t = \operatorname{rot}([\mathbf{Vh}] + [\mathbf{uB}]) + \mathbf{G} + \nu_m \Delta \mathbf{h}, \tag{3.5}$$

where P_* is the pulsation of the hydrodynamic pressure $\mathbf{F}_\nu$ is a force that takes the kinematic viscosity ν into account, **T** and **G** are terms nonlinear in the pulsations and describe the energy transport through the spectrum of the MHD turbulence:

$$\mathbf{T} = \langle (\mathbf{u}\nabla)\mathbf{u} \rangle - (\mathbf{u}\nabla)\mathbf{u}(-\{[\mathbf{h}\operatorname{rot}\mathbf{h}] - \langle [\mathbf{h}\operatorname{rot}\mathbf{h}] \rangle\}/4\pi\rho,$$
$$G = \operatorname{rot}([\mathbf{uh}] - \langle [\mathbf{uh}] \rangle),$$

and $\mathbf{F}_r$ is a random external field.

The system (3.4), (3.5) of MHD equations for turbulent fields at $\mathrm{Rm} \gg 1$ was heretofore investigated only in an approximation linear in the large-scale field (see, e.g., Refs. 5 and 6). This treatment leads to well known effects (the α effect, turbulent viscosity, turbulent diamagnetism) which vanish in the case of homogeneous and isotropic turbulence with a uniform field **B**. In the present paper we investigate an effect which is *nonlinear* in the large-scale magnetic field when the field **B** and the turbulence are uniform. Turning on the inhomogeneity does not eliminate this effect, and merely modifies it somewhat.

The pulsations are concentrated in small scales in which the derivatives of the large-scale fields are small. With the aid of the MHD equations (3.4) and (3.5), written in the Fourier representation we derive equations that describe the evolution of the second moments:

$$\frac{df_{ij}}{dt}=\frac{i(\mathbf{kB})\phi_{ij}}{4\pi\rho}+Q_{ij}-2\nu k^2 f_{ij}+F_{ij}, \qquad (3.6)$$

$$\frac{dh_{ij}}{dt}=-i(\mathbf{kB})\phi_{ij}+R_{ij}-2\nu_m k^2 h_{ij}, \qquad (3.7)$$

$$\frac{d\chi_{ij}}{dt}=i(\mathbf{kB})\left(f_{ij}-\frac{h_{ij}}{4\pi\rho}\right)+M_{ij}-(\nu+\nu_m)k^2\chi_{ij}. \qquad (3.8)$$

Here

$$f_{ij}(\mathbf{k},t)=\langle u_i(\mathbf{k},t)u_j(-\mathbf{k},t)\rangle,$$
$$h_{ij}(\mathbf{k},t)=\langle h_i(\mathbf{k},t)h_j(-\mathbf{k},t)\rangle,$$
$$\chi_{ij}(\mathbf{k},t)=\langle h_i(\mathbf{k},t)u_j(-\mathbf{k},t)\rangle,$$
$$\phi_{ij}(\mathbf{k},t)=\chi_{ij}(\mathbf{k},t)-\chi_{ji}(-\mathbf{k},t),$$
$$F_{ij}(\mathbf{k},t)=\langle F_i^*(\mathbf{k},t)u_j(-\mathbf{k},t)\rangle+\langle u_i(\mathbf{k},t)F_j^*(-\mathbf{k},t)\rangle,$$
$$\mathbf{F}^*(\mathbf{k},t)=[\mathbf{k}[\mathbf{kF}_r(\mathbf{k},t)]]/k^2\rho,$$

and $\mathbf{k}$ is the wave vector. Recall that here $k\gg l_0^{-1}$, so that $\mathbf{k}\cdot\mathbf{B}$ is not equal to zero. We express the third moments in the form

$$Q_{ij}(\mathbf{k},t)=\langle T_i^*(\mathbf{k},t)u_j(-\mathbf{k},t)\rangle+\langle u_i(\mathbf{k},t)T_j^*(-\mathbf{k},t)\rangle.$$

The expressions for the remaining correlators R_{ij} and M_{ij} are similar, and $\mathbf{T}^*=\mathbf{k}\times[\mathbf{k}\times\mathbf{T}]/k^2$.

Equations of this type raise, as usual, the question of closing the equations for the higher moments. Various approximate methods have been proposed for the solution of problems of this type (see, e.g., Refs. 6, 18, and 19). The simplest closure procedure is the τ approximation, which is widely used in the theory of kinetic equations. As applied to MHD-turbulence problems, this approximation was developed in Refs. 6, 20, and 21. In the simplest variant, it allows us to express the third moments, which determine the energy transport over the spectrum, in terms of the second ones:

$$Q_{ij}-Q_{ij}^{(0)}=-(f_{ij}-f_{ij}^{(0)})/\tau$$

and similarly for the other correlators.[6] The superscript (0) corresponds here to the background MHD turbulence (without the field $\mathbf{B}$), and $\tau(k)$ is the characteristic relaxation time of the statistical moments.

Assume that $\nu k^2\ll\nu_m k^2\ll\tau^{-1}$ for the greater part of the spectrum (this is typical of astrophysical conditions). It is also natural to assume that the characteristic time of variation of the large-scale magnetic field $\mathbf{B}$ is substantially longer than the correlation time $\tau(k)$ for all turbulence scales. The stationary solution of the resulting system of equations then takes the form

$$f_{ij}=f_{ij}^{(0)}+\frac{\psi}{2(1+\psi)}\left(f_{ij}^{(0)}-\frac{h_{ij}^{(0)}}{4\pi\rho}\right), \qquad (3.9)$$

$$\frac{h_{ij}-h_{ij}^{(0)}}{4\pi\rho}=-(f_{ij}-f_{ij}^{(0)}), \qquad (3.10)$$

$$\chi_{ij}=i(\mathbf{kB})\tau\left(f_{ij}^{(0)}-\frac{h_{ij}^{(0)}}{4\pi\rho}\right), \qquad (3.11)$$

where $\psi(\mathbf{k},\mathbf{B})=(\tau(k)\mathbf{kB})^2/\pi\rho$. Here we have taken into account the relations $f_{ij}(\mathbf{k},t)=f_{ji}(-\mathbf{k},t)$ and $h_{ij}(\mathbf{k},t)=h_{ji}(-\mathbf{k},t)$ which hold for a quasihomogeneous background turbulence. We have also assumed $\chi_{ij}^{(0)}(\mathbf{k},0)=0$. In this case it follows from (3.11) that the tensor $\chi_{ij}^{(0)}$ vanishes. The latter is not surprising, since $\chi_{ij}^{(0)}=0$ holds for the background turbulence, and the cross-helicity $[\mathbf{h}\times\mathbf{h}]$ is an integral of the motion.[1]

It is seen from (3.9)–(3.11) that the equipartition state in which $\rho\langle u_iu_j\rangle^{(0)}/2=\langle h_ih_j\rangle^{(0)}/8\pi$ is special. In this case there is no shift from the background turbulence level for any uniform field $\mathbf{B}$.[1)] Getting ahead of ourselves, we note that negative magnetic pressure occurs only when a deviation from equipartition takes place.

Thus, changing to coordinate space and specifying the background turbulence spectrum [including the $\tau(k)$ dependence], we can obtain an expression for the generalized Maxwellian stress tensor σ_{ij} and hence also an expression for the effective magnetic force.

4. EFFECTIVE MAGNETIC FORCE IN A MEDIUM WITH FULLY DEVELOPED MHD TURBULENCE

The spectral propagators $f_{ij}^{(0)}(\mathbf{k})$ and $h_{ij}^{(0)}(\mathbf{k})$ determine the state of the background MHD turbulence (in the absence of a large-scale field), which is assumed to be isotropic. The following relations are valid then in the weak-compressibility approximation

$$f_{ij}^{(0)}(\mathbf{k})=n_0(\mathbf{k})\Delta_{ij}/4\pi k^2,\qquad h_{ij}^{(0)}=m_0(\mathbf{k})\Delta_{ij}/4\pi k^2 \qquad (4.1)$$

(see, e.g., Ref. 1). Here $\Delta_{ij}=\delta_{ij}-k_ik_j/k^2$, while $n_0(k)$ and $m_0(k)$ have the meaning of spectral densities of the corresponding random quantities:

$$\langle u^2\rangle^{(0)}=\int_0^\infty n_0(k)\,dk,\qquad \langle h^2\rangle^{(0)}=\int_0^\infty m_0(k)\,dk.$$

The functions $n_0(k)$ and $m_0(k)$ describe the spectrum of the background MHD turbulence. The wave vector k is normalized to the value $k_0=l_0^{-1}$ determined by the main turbulence scale l_0.

We choose the spectrum of the background MHD turbulence in the form

$$n_0(k)=a_0k^{-5/3},\quad m_0(k)=0\ \text{ for }\ 1\leqslant k<\mathrm{Rm}^{1/4},$$
$$n_0(k)=a_0k^{-5/3},\quad m_0(k)=\varphi(k)\ \text{ for }\ \mathrm{Rm}^{1/4}\leqslant k<A\,\mathrm{Rm}^{1/2},$$
$$n_0(k)=m_0(k)/4\pi\rho=a_0(A\,\mathrm{Rm}^{1/2})^{-1/6}k^{-3/2}\ \text{ for }\ A\,\mathrm{Rm}^{1/2}\leqslant k<\mathrm{Rm}^{2/3},$$
$$n_0(k)=m_0(k)=0\ \text{ for }\ k\geqslant\mathrm{Rm}^{2/3}. \qquad (4.2)$$

Here we have $a_0=2u_0^2/3$, $u_0=[\langle u^2\rangle^{(0)}]^{1/2}$, $A=0.1$–0.5, and $\varphi(k)=k^\lambda\sin[2\pi\ln k/\ln(\mathrm{Rm})]$, $\lambda=0.5$–1.5 (Refs. 11, 12). The background MHD turbulence spectrum is chosen on the basis of the results, confirmed by analytic estimates, of the exact MHD equations,[22] at zero average helicity. Recall that zero average helicity corresponds to the state of background MHD turbulence (without the field $\mathbf{B}$). The characteristic time $\tau(k)$ of energy transport in each of the scales is

$$\tau(k)=2\tau_0k^{-2/3}, \qquad (4.3)$$

where τ_0 is the energy transport time averaged over the energy spectrum.

The transition to the coordinate space is effected with the aid of the relation

$$\langle h_i(\mathbf{x})h_j(\mathbf{x})\rangle=\int h_{ij}(\mathbf{k},\mathbf{x})\,d\mathbf{k} \qquad (4.4)$$

and similar other correlators, Substituting in (3.9) and (3.10) the chosen MHD-turbulence spectrum (4.2), and carrying out the integration in (4.4) over k-space with account taken of relations (4.1) and (4.3), we obtain an expression for the second moments in coordinate space:

$$\langle h_i(\mathbf{x})h_j(\mathbf{x})\rangle=\langle h_ih_j\rangle^{(0)}+(q_p-{}^1/_2q_s)B^2\delta_{ij}-{}^1/_2q_sB_iB_j, \qquad (4.5)$$

$$\langle u_i(\mathbf{x})u_j(\mathbf{x})\rangle=\langle u_iu_j\rangle^{(0)}-[(q_p-{}^1/_2q_s)B^2\delta_{ij}-{}^1/_2q_sB_iB_j]/4\pi\rho, \qquad (4.6)$$

where

$$q_p=\frac{4}{15}\ln(\mathrm{Rm}^*)+\frac{4}{5}\ln\frac{1+\varepsilon_0}{1+\varepsilon_0\,\mathrm{Rm}^{*1/3}}+\frac{8}{15}\varepsilon_0^{-1}a_1-\frac{8}{5}\varepsilon_0^{-2}(a_2+a_3\varepsilon_0^{-1/2}), \qquad (4.7)$$

$$q_s=\frac{8}{45}\ln(\mathrm{Rm}^*)+\frac{8}{15}\ln\frac{1+\varepsilon_0}{1+\varepsilon_0\,\mathrm{Rm}^{*1/3}}-\frac{8}{15}\varepsilon_0^{-1}a_1-\frac{12}{5}\varepsilon_0^{-2}(a_2-a_3\varepsilon_0^{-1/2})+\frac{4}{3}\varepsilon_0^{-3/2}a_4. \qquad (4.8)$$

Here

$$a_1=1-(\mathrm{Rm}^*)^{-1/3},\quad a_2=1-(\mathrm{Rm}^*)^{-2/3},$$
$$a_3=\mathrm{arctg}\,\varepsilon_0^{1/2}-(\mathrm{Rm}^*)^{-5/6}\,\mathrm{arctg}(\varepsilon_0\,\mathrm{Rm}^{*1/3})^{1/2},$$

$$a_4=\mathrm{arctg}\,\varepsilon_0^{1/2}-(\mathrm{Rm}^*)^{-1/2}\,\mathrm{arctg}(\varepsilon_0\,\mathrm{Rm}^{*1/3})^{1/2},$$
$$\mathrm{Rm}^*=A^2\,\mathrm{Rm},\quad \varepsilon_0=4v_A{}^2/u_0{}^2,\quad v_A=B/(4\pi\rho)^{1/2}$$

is the Alfven velocity. The dependences of q_p on q_s on ε_0 for various Rm* are given in the Fig. 1. Estimates show that the contribution of $\varphi(k)$ to the integral (4.4) is quite small, so we confine ourselves for simplicity to the case $\varphi(k)=0$. In the derivation of (4.5) and (4.6) we took into account the natural connection between the parameters u_0, $k_0=l_0^{-1}$ and $\tau(k_0)$, i.e., the relation $\tau(k_0)=l_0/u_0$ where $\tau(k_0)=2\tau_0$ [see expression (4.3) rewritten in dimensional variables]. Expressions (4.7) and (4.8) simplify in two limiting cases:

1) For $\varepsilon_0\ll 1$, $(\mathrm{Rm}^*)^{-1/3}$

$$q_p\approx\frac{4}{15}\ln(\mathrm{Rm}^*)-\frac{4}{7}(\mathrm{Rm}^{*1/3}-1)\varepsilon_0,$$
$$q_s\approx\frac{8}{45}\ln(\mathrm{Rm}^*)-\frac{16}{35}(\mathrm{Rm}^{*1/3}-1)\varepsilon_0.$$

2) For $(\mathrm{Rm}^*)^{-1/3}\ll\varepsilon_0\ll 1$

$$q_p\approx\frac{8}{25}\left(1+\frac{5}{2}\ln\varepsilon_0^{-1}\right)+\frac{4}{7}\varepsilon_0-\frac{4}{3}(\varepsilon_0\,\mathrm{Rm}^{*1/3})^{-1},$$
$$q_s=\frac{8}{225}(1+15\ln\varepsilon_0^{-1})+\frac{16}{35}\varepsilon_0-\frac{2\pi}{3}(\varepsilon_0\,\mathrm{Rm}^{*1/3})^{-3/2}.$$

Substituting (4.5) and (4.6) in (3.3) we obtain a final expression for the generalized Maxwell stress tensor:

$$\sigma_{ij}=-(P_T^{(0)}+(1-q_p)B^2/8\pi)\delta_{ij}+(1-q_s)B_iB_j/4\pi, \qquad (4.9)$$

with $P_T^{(0)}=\rho u_0^2/3+h_0^2/24\pi$, $h_0=(\langle h^2\rangle^{(0)})^{1/2}$.

From (4.9) we get an expression for the effective magnetic force:

$$\mathbf{F}_m=-\nabla(1-q_p)B^2/8\pi+(\mathbf{B}\nabla)(1-q_s)\mathbf{B}/4\pi. \qquad (4.10)$$

We point out that besides the possibility of the magnetic pressure being negative (for $q_p>1$), expression (4.10) contains the possibility that the magnetic tension force can change sign (at $q_z>1$). Relations (4.7) and (4.8) permit an estimate of those magnetic Reynolds number (for a specified ε_0) at which the magnetic forces reverse sign.

For a sufficiently weak large-scale magnetic field [$\varepsilon_0\ll(\mathrm{Rm}^*)^{-1/3}$] the expressions for the second moments are

$$\langle h_ih_j\rangle={}^1/_3h_0{}^2\delta_{ij}+q_s(B^2\delta_{ij}-{}^1/_2B_iB_j), \qquad (4.11)$$

$$\langle u_iu_j\rangle={}^1/_3u_0{}^2\delta_{ij}-q_s(B^2\delta_{ij}-{}^1/_2B_iB_j)/4\pi\rho, \qquad (4.12)$$

where $q_s=8\ln(\mathrm{Rm}^*)/45$. Expression (4.11) is similar in form to the relation obtained in Ref. 7 for the case $\mathrm{Rm}\ll 1$. However, q_s in Ref. 7 is of the same order as Rm, i.e., $q_s\ll 1$. Equations (4.11) and (4.12) correspond to the case when the reversal of the sign of the large-scale magnetic force is

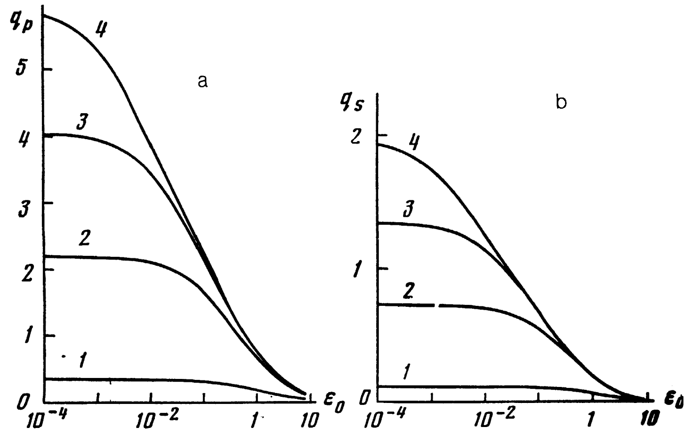

FIG. 1. Dependence of q_p (a) and q_s (b) on $\varepsilon_0=4v_A{}^2/u_0{}^2$ at different values of Rm*: 1—3.7, 2—3.7·10^3, 3—3.7·10^6, 4—3.7·10^9.

quadratic in the field **B**. In strong fields this effect is essentially nonlinear.

Thus, in the presence of fully developed small-scale MHD turbulence the elasticity of the large-scale magnetic field is noticeably lowered. This phenomenon may be the cause of excitation of large-scale MHD perturbations.

5. MHD INSTABILITY GENERATED BY NEGATIVE MAGNETIC PRESSURE (QUALITATIVE TREATMENT)

Let us investigate large-scale effects that evolve on a scale L, where $l_0 \ll L \ll L_B$ holds we describe the influence of small-scale turbulence on these processes by the parameters q_p, q_s, and ν_T, where $\nu_T \approx l_0 u_0/6$ is the turbulent-viscosity coefficient. Note that although q_p, q_s, and ν_T are determined by one and the same turbulence, these parameters are indicative of different effects. The negative turbulence condition to the effective magnetic force which is nonlinear in the large-scale magnetic field, is connected with the first two. The parameter ν_T determines the turbulent diffusion of the large-scale magnetic moment (an effect linear in the field).

Let us investigate the properties of the magnetic buoyancy in the presence of small-scale turbulence. We direct the x axis of a Cartesian system of coordinates opposite to the free-fall acceleration g, and the z axis along the large-scale magnetic field $\mathbf{B}(x)$, i.e., we consider for simplicity a horizontal magnetic field. We separate a magnetic tube located at a level 1 relative to the x axis, with density ρ_1 and magnetic field B_1. We slowly displace upwards the magnetic tube as a whole from position 1 to position 2 (with appropriate parameters ρ_2 and B_2 of the medium around the tube). If, after equalization of the pressures inside and outside the magnetic tube, the density ρ_2^* inside the tube (in position 2) is lower than the density ρ_2 of the surrounding plasma, the tube will continue to float upwards by the action of the buoyant force. The density excess $\Delta\rho = \rho_2^* - \rho_2$ in the absence of dissipative processes and of high thermal conductivity (high thermal conductivity ensures rapid equilization of the temperature inside and outside the rising tube) is obtained, as usual, from the laws of conservation of the mass and magnetic flux inside the tube. We only note that in the presence of small-scale turbulence the condition that the total pressures be equal inside and outside the magnetic tube (in position 2) has the following form:

$$C_s^2\rho_2 + \frac{K_0 B_2^2}{8\pi} = C_s^2\rho_2^* + \frac{K_0 B_2^{*2}}{8\pi},$$

where $K_0 = 1 - q_p$, C_s is the speed of sound, and B_2^* is the magnetic field inside the tube at the point x_2. Assuming the displacement $\xi_* = x_2 - x_1$ to be small, we represent the density ρ_2 and the magnetic field B_2 in the form

$$\rho_2 = \rho_1(1 - \xi/\Lambda_\rho), \quad B_2 = B_1(1 - \xi/\Lambda_B),$$

where $\Lambda_\rho = -\rho dx/d\rho$ and $\Lambda_B = -B dx/dB$ are the corresponding height scales of the density and of the magnetic field. As a result we obtain

$$\Delta\rho = \frac{B_1^2 K_0(\Lambda_B - \Lambda_\rho)}{4\pi C_s^2 \Lambda_\rho \Lambda_B}\xi.$$

The tube can float up (i.e., instability can develop) if we have

$$K_0(\Lambda_B - \Lambda_\rho)/\Lambda_B\Lambda_\rho < 0. \qquad (5.1)$$

In the case of weak turbulence and a relatively small magnetic Reynolds number, with $K_0 \approx 1$, the small-scale turbulence does not influence the large-scale processes. It follows then from (5.1) the instability criterion imposed by the magnetic buoyancy is of the form $\Lambda_B < \Lambda_\rho$. This means that instability develops only when the scale on which the initial magnetic field changer is less than the density height scale.[3]

In a medium with fully developed small-scale MHD turbulence, however, the situation is radically changed. Thus, for $q_p > 1$ the effective magnetic pressure of the plasma becomes negative ($K_0 < 0$) and the usual magnetic buoyancy is missing in a strongly inhomogeneous magnetic field [see (5.1)]. On the other hand, for $\Lambda_B > \Lambda_\rho$ and $K_0 < 0$, instability develops in a large-scale magnetic field. It is seen from (5.1) that the instability will develop even in an initially quasihomogeneous large-scale magnetic field.

Let us estimate the instability growth rate. Neglecting for simplicity the dissipative processes, we retain in the equation of motion of the magnetic tube only the buoyant force

$$\frac{d^2\xi}{dt^2} = -\left(\frac{v_A^*}{C_s}\right)^2 \frac{gK_0(\Lambda_B - \Lambda_\rho)}{\Lambda_B\Lambda_\rho}\xi, \qquad (5.2)$$

where $v_A^* = B_1/(4\pi\rho_1)^{1/2}$ is the Alfven velocity. We seek the solution of (5.2) in the form $\xi = \xi_0 \exp(\gamma_{\rm inst} t)$. The instability growth rate is then

$$\gamma_{inst} = \frac{v_A^*}{\Lambda_\rho}\left[-K_0\left(1 - \frac{\Lambda_\rho}{\Lambda_B}\right)\right]^{1/2}, \qquad (5.3)$$

where it is taken into account that $\Lambda_\rho \approx C_s^2/g$.

The energy for this instability comes from the small-scale turbulent pulsations. This circumstance distinguishes in principle the instability considered in the present paper from the Parker instability.[3] The latter develops as a result of the work done by the force of gravity in a very inhomogeneous magnetic field ($\Lambda_B < \Lambda_\rho$). In this sense the Parker instability is similar to the families Rayleigh–Taylor instability.

As to the role of turbulent viscosity, it leads either to weakening or to complete elimination of the instability. Note that under astrophysical conditions the turbulent viscosity is frequently considerably longer than the magnetic and kinematic viscosity. We are interested only in this case. The damping rate in this case is of the order of ν_T/L^2. This means that the instability we are studying has a threshold. It is seen from (5.3) that the instability growth rate is proportional to the large-scale magnetic field. It follows that the instability occurs only in sufficiently strong magnetic fields $B > B_{\rm cr}(\nu_T, \rho, {\rm Rm})$, where $B_{\rm cr}$ is the instability threshold for the large-scale magnetic field, and is determined from the equation

$$\varepsilon_0(B_{cr})\,[1 - q_p(\varepsilon_0, {\rm Rm})] + (l_0\Lambda_\rho/3L^2)^2 = 0.$$

We have thus revealed one more channel for energy conversion from small-scale turbulence to a large-scale magnetic field. As a result, the instability can lead to formation of inhomogeneities of the large scale magnetic field, in the form of striations or braids

6. CONCLUSION

We have considered the effects, nonlinear in the large-scale magnetic field, of modifying the magnetic force of an advanced small-scale MHD instability. For $\mathrm{Rm} \gg 1$, the sign of large-scale magnetic force can reverse. This effect excites an instability in the large-scale magnetic field due to energy transfer from the turbulent pulsations. In the course of development of the instability, the large-scale magnetic field becomes inhomogeneously distributed. The case $\beta \gg 1$ is investigated.

The instability mechanism consists of the following. An isolated tube of magnetic-field lines, moving upwards, turns out to be lighter than the surrounding plasma, since the fall of the magnetic field in it, due to expansion of the tube, is accompanied by an increase of the magnetic pressure inside the tube. This increase, due to the fact that the effective magnetic pressure is negative, leads to a decrease of the density inside the tube and to the appearance of a buoyant force. Thus, the upwards floating of the tube, i.e., the development of stability, is at the expense of the energy of small-scale turbulent pulsations.

An example of a medium in which the processes described in this article can develop is the turbulent convective zone located under the visible surface of the sun. In this region, convective cells (granules) of dimension $l_0 \simeq (5-10) \cdot 10^7$ cm are created and annihilated, a large-scale magnetic field ($L_B \gg l_0$) is generated, and fine-structure magnetic pulsations are excited.

At a depth $\sim 10^9$ cm (from the sun's surface) the plasma has the following parameters (see, e.g., Ref. 23): $\mathrm{Rm} \approx 3 \cdot 10^7$, $u_0 \approx 10^4$ cm/sec, $\rho_0 \approx 5 \cdot 10^{-4}$ g/cm^3, and $B_0 \approx 10^2$ G. We then have $K_0 \approx -2$ and the effective magnetic pressure is negative. The instability that develops in this case apparently determines the formation of the magnetic braids in the convective zone. They float up from under the sun's surface leading to the onset of the observed sunspots.

APPENDIX: DERIVATION OF THE LAW OF CONSERVATION OF THE TOTAL ENERGY OF HOMOGENEOUS TURBULENCE WITH A UNIFORM LARGE-SCALE MAGNETIC FIELD

Let us multiply Eqs. (3.6) and (3.7) by $\rho/2$ and $(8\pi)^{-1}$, respectively, and add them. The equation for the trace of the resultant tensor is

$$\frac{d}{dt}\left(\frac{\rho\langle u^2\rangle}{2}+\frac{\langle h^2\rangle}{8\pi}\right)_{\mathbf{k}} = \Pi(\mathbf{k})+I(\mathbf{k})-D(\mathbf{k}), \qquad \text{(A1)}$$

where $I(\mathbf{k}) = \rho F_{ii}/2$ is the spectral density of the power of the external source maintaining the turbulence

$$D(\mathbf{k})=2k^2\left(\nu\rho\frac{\langle u^2\rangle}{2}+\nu_m\frac{\langle h^2\rangle}{8\pi}\right)_{\mathbf{k}}, \quad \Pi(\mathbf{k})=\frac{\rho Q_{ii}}{2}+\frac{R_{ii}}{8\pi}.$$

Note that the terms containing the magnetic field $\mathbf{B}$ are eliminated from Eq. (A1). This reflects the fact that the uniform large-scale magnetic field performs no work on the turbulence.

We change over to coordinate space in Eq. (A1). By calculations similar to those described in Ref. 24 it is easy to show that

$$\int \Pi(\mathbf{k})\, d^3k = -\operatorname{div}\langle \mathbf{\Phi}\rangle = 0,$$

where $\langle \Phi \rangle$ is the flux of magnetomechanical energy of the homogeneous turbulence. It follows that in coordinate space Eq. (A1) takes the form

$$\partial W_T/\partial t = I - D, \qquad \text{(A2)}$$

where $W_T = W_k + W_m$. Calculations show that $D = W_T/\tau_0$ for fully developed background turbulence with a power-law spectrum. It is easy to verify that this fact indeed follows from the condition that the energy spectrum be constant over the spectrum. In this case the solution of Eq. (A2) with $t \gg \tau_0$ reaches a stationary value $W_T =$ const that does not depend on the field $\mathbf{B}$.

[1)] A small shift from the background state can appear in the case of strong inhomogeneity of the magnetic field **B**. It is determined by the spatial derivatives of the large-scale field.

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