Nonlinear electromagnetic waves in an antiferromagnetic plate subjected to an external magnetic field

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(Submitted 22 May 1990; resubmitted 10 July 1990)
Zh. Eksp. Teor. Fiz. **98**, 1718–1725 (November 1990)

A theory is proposed for the description of transmission of a nonlinear electromagnetic wave through an easy-axis antiferromagnet subjected to a static external magnetic field directed along the easy magnetization axis of a magnetic crystal. An analysis is made of a wave incident normally on such a plate. The magnetic field of this wave is circularly polarized and it lies in the plane of the plate. A nonlinear interaction between the wave and the spin system of the antiferromagnet gives rise to a dependence of the transmission coefficient of the wave on the power of this wave. It is shown that under certain conditions the wave transmission gives rise to bistable (multistable) states. A dependence of the nonlinear effect on the square of the amplitude of the incident wave is derived. Estimates are obtained for FeF_2 crystals.

1. INTRODUCTION

Two main trends can be distinguished in studies of nonlinear properties of electromagnetic waves traveling in antiferromagnets. The first trend concentrates on surface and bulk polaritons, i.e., on nonlinear electromagnetic waves in easy-axis antiferromagnets.[1,2] The second trend concentrates on nonlinear effects that appear on transmission of an electromagnetic wave across an antiferromagnetic film or plate when the wave frequency is close to the frequency of an antiferromagnetic resonance (AFMR) and the thickness of the sample is comparable with the wavelength of the incident radiation. A theoretical investigation of the transmission of an electromagnetic wave across an antiferromagnet plate reported in Ref. 3 deals with the case when there is no external magnetic field. It is shown there in particular that under certain conditions a wave may become unstable when its power is increased.

We shall report a theoretical investigation of the transmission of an electromagnetic wave by a plate of an easy-axis two-sublattice antiferromagnet subjected to a static external magnetic field when the interaction of the wave with the spin system of the plate is nonlinear and the plate thickness is comparable with the wavelength of the incident radiation. In this most general case the nonlinear wave process is highly complex. The cylindrical symmetry of the spin system of an antiferromagnet, which applies in the linear Faraday effect geometry, now breaks down. The magnetic susceptibility of the antiferromagnet becomes a function of the frequency and power of the incident wave, and also of the external magnetic field. This firstly has the effect that when a wave is transmitted across an antiferromagnetic plate we can expect bistable and, on further increase in the power, multistable states; secondly, it is possible to control nonlinear effects by altering the external magnetic field, which becomes useful in experimental studies of these phenomena.

A linear theory of the transmission of electromagnetic waves by antiferromagnets has been developed sufficiently thoroughly.[4] Nonlinear effects which appear on increase in the wave amplitude are particularly strong when the frequency of an incident electromagnetic wave is close to AFMR frequencies.[3] In the case of an antiferromagnet with the rutile structure (such as FeF_2 or MnF_2) these frequencies lie in the infrared range. If the frequency of the radiation incident on an antiferromagnetic plate lies in this range, the dimensions of this plate cannot be regarded as small compared with the electromagnetic wavelength, as is usually done in descriptions of the interaction of electromagnetic waves with ferromagnets when the wavelength at the resonance frequency is usually considerably greater than the dimensions of the investigated sample.

2. NONLINEAR MAGNETIC SUSCEPTIBILITY OF AN ANTIFERROMAGNET

We shall use the Landau–Lifshitz equation to find the nonlinear susceptibility of an easy-axis antiferromagnet and ignore dissipative processes. This neglect of dissipation is due to the high quality of the antiferromagnetic materials (with AFMR line widths ~ 20 G) used in experiments. We shall consider a two-sublattice model which describes well[4] the properties of real FeF_2 and MnF_2 crystals. We shall assume that a static external magnetic field, directed along the easy magnetization axis z, is of such intensity that the antiferromagnetic sublattices are in the antiparallel state, i.e., that the magnetization $\mathbf{m}_a$ of the sublattice A is directed along the z axis, which is perpendicular to the surface of the antiferromagnetic plate, whereas the magnetization $\mathbf{m}_b$ of the sublattice B is antiparallel to the z axis. We shall assume that m_s is the absolute value of the magnetization in each of the sublattices. We shall introduce the normalized magnetizations of the sublattices:

$$\mathbf{a}=\frac{\mathbf{m}_a}{m_s}, \quad (1a)$$

$$\mathbf{b}=\frac{\mathbf{m}_b}{m_s}. \quad (1b)$$

In the absence of damping the Landau–Lifshitz equations for each sublattice are

$$\partial\mathbf{a}/\partial t=-\gamma\,[\mathbf{a}\mathbf{H}^A], \quad (2a)$$

$$\partial\mathbf{b}/\partial t=-\gamma\,[\mathbf{b}\mathbf{H}^B], \quad (2b)$$

where $\gamma>0$ is the gyromagnetic ratio; $\mathbf{H}^A$ and $\mathbf{H}^B$ are the effective magnetic fields acting on the sublattices A and B, respectively. These fields consist of the external static magnetic field H_0 directed along the z axis, the anisotropy field H_a, the exchange field H_e, and the incident wave field $\mathbf{h}(t)$, and they can be described by[4]

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$$\mathbf{H}^A=\mathbf{z}(H_0+H_Aa_z)-H_E\mathbf{b}+\mathbf{h}(t), \quad (3a)$$

$$\mathbf{H}^B=\mathbf{z}(H_0+H_Ab_z)-H_E\mathbf{a}+\mathbf{h}(t). \quad (3b)$$

The law of conservation of the total magnetic moment in both sublattices can be written as follows:

$$a_x^2+a_y^2+a_z^2=1, \quad (4a)$$

$$b_x^2+b_y^2+b_z^2=1. \quad (4b)$$

If we assume that the variables a_x, a_y, b_x, and b_y are all much smaller than unity, then expanding of a_z and b_z in terms of powers of these variables, we can limit our analysis to the quadratic terms:

$$a_z\approx 1-{}^1/_2(a_x^2+a_y^2), \quad (5a)$$

$$b_z\approx -1+{}^1/_2(b_x^2+b_y^2). \quad (5b)$$

We shall assume that the magnetic field of the incident wave has the circular polarization (wave of the "+" type):

$$\mathbf{h}(t)=\mathbf{h}_+(t)=\mathrm{Re}[(h_x\mathbf{x}+ih_y\mathbf{y})e^{-i\omega t}]. \quad (6)$$

We shall seek the solution of the system (2) in the form of the following Fourier series:

$$\mathbf{a}(t)=\mathrm{Re}[(a_x\mathbf{x}+ia_y\mathbf{y})e^{-i\omega t}+(a_x'\mathbf{x}+ia_y'\mathbf{y})e^{-3i\omega t}+\ldots], \quad (7a)$$

$$\mathbf{b}(t)=\mathrm{Re}[(b_x\mathbf{x}+ib_y\mathbf{y})e^{-i\omega t}+(b_x'\mathbf{x}+ib_y'\mathbf{y})e^{-3i\omega t}+\ldots]. \quad (7b)$$

Solving the system (2) subject to Eqs. (3)–(7), we obtain a system of equations describing the motion of the sublattice magnetizations in the nonlinear case and enabling us to find the susceptibility of an investigated antiferromagnet at the frequency of the incident wave:

$$\mp\omega a_\pm+(\omega_0+\omega_A+\omega_E)a_\pm+\omega_E b_\pm=\gamma\hbar_\pm+A_\pm, \quad (8a)$$

$$\mp\omega b_\pm+(\omega_0-\omega_A-\omega_E)b_\pm-\omega_E a_\pm=-\gamma\hbar_\pm+B_\pm, \quad (8b)$$

where the following notation is used: $\omega_0=\gamma H_0$, $\omega_A=\gamma H_A$, $\omega_E=\gamma H_E$,

$$a_\pm=a_x\pm a_y, \quad (9a)$$

$$b_\pm=b_x\pm b_y, \quad (9b)$$

$$\hbar_\pm=h_x\pm h_y. \quad (9c)$$

The terms which appear in the nonlinear case are

$$A_\pm=-{}^1/_8\gamma\{\hbar_\pm(a_+^2+a_-^2)+\hbar_\mp a_+a_-\}+{}^1/_8\omega_A a_\pm\{a_\pm^2+2a_\mp^2\}$$
$$+{}^1/_8\omega_E\{a_\pm(b_+^2+b_-^2)+b_\pm(a_+^2+a_-^2)+a_\mp b_\mp(a_\pm+b_\pm)\}, \quad (10a)$$

$$B_\pm={}^1/_8\gamma\{\hbar_\pm(b_+^2+b_-^2)+\hbar_\mp b_+b_-\}-{}^1/_8\omega_A b_\pm\{b_\pm^2+2b_\mp^2\}$$
$$-{}^1/_8\omega_E\{a_\pm(b_+^2+b_-^2)+b_\pm(a_+^2+a_-^2)+a_\mp b_\mp(a_\pm+b_\pm)\}. \quad (10b)$$

Substituting $A_\pm=B_\pm=0$, in Eq. (8), we obtain a system of equations describing oscillations of the magnetization in the linear case. The system then has the cylindrical symmetry and, if we use the coordinates of Eq. (9), it splits into two independent subsystems (corresponding to the "+" and "−" signs). We shall consider the reaction of our antiferromagnet to one of the circularly polarized waves (namely the wave with the circular right-handed polarization) in the nonlinear case.

Antiferromagnets of the type under discussion (FeF_2, MnF_2) are characterized by an effective exchange field (~500 kOe) which exceeds greatly the effective anisotropy field (~10 kOe), and also the intensities of the magnetic field of the incident wave attainable in experiments (~10 Oe). The main terms in the system (10) are proportional to ω_E. The nonlinear magnetic susceptibility of an antiferromagnet for a wave with the circular right-handed polarization is found by solving the system (8) by iteration (beginning with the zeroth approximation in the form of the solution for the linear case) and it is given by

$$\chi_+=\chi_+^L+\chi_+^{NL}|\gamma\mathbf{h}_+|^2, \quad (11)$$

where

$$\chi_+^L=(2\omega_A\gamma m_s)/D \quad (11a)$$

is the linear part of the magnetic susceptibility of an antiferromagnet for a right-handed polarized wave, and

$$\chi_+^{NL}=\{\omega_E\omega_A^2\gamma m_s[\omega_A^2-(\omega-\omega_0)^2]\}/(2D^4), \quad (11b)$$

$$D=-(\omega-\omega_0)^2+2\omega_A\omega_E+\omega_A^2. \quad (12)$$

When the polarization of an electromagnetic wave incident on a plate is not circular, the expressions for the magnetic susceptibility of an antiferromagnet depend on the values of the squares of the amplitudes of the waves $h_\pm$ and also on the modulus of the product of the amplitudes h_+ and h_-. We have ignored this case for lack of space.

When the frequency of the incident wave approaches an AFMR frequency $\Omega_+=(2\omega_E\omega_A+\omega_A^2)^{1/2}+\omega_0$, the values of the linear and nonlinear parts of the magnetic susceptibility rise strongly. This is due to the fact that we have ignored dissipation in the system (2). The condition of smallness of the transverse component of the magnetic field compared with the longitudinal component given by Eq. (5) (i.e., the condition of validity of the proposed theory) has the following form on approach to a resonance point:

$$\left(\frac{|\gamma\mathbf{h}_+|}{\Delta\omega}\right)^2\ll 1, \quad (13)$$

where $\Delta\omega=\Omega_+-\omega$. In calculations the quantity on the left-hand side of Eq. (13) does not exceed 10^{-3}.

In this way we can find the nonlinear magnetic susceptibility of an antiferromagnet for an alternating magnetic field and its dependence on an external static magnetic field, and we can determine the range of validity of the theory. We shall now use the results to consider the transmission of a circularly polarized electromagnetic wave (with the right-hand polarization) through an antiferromagnetic plate.

3. TRANSMISSION OF A NONLINEAR ELECTROMAGNETIC WAVE THROUGH AN ANTIFERROMAGNETIC PLATE

We shall consider an antiferromagnetic plate with the rutile structure and of thickness d, and we shall assume that it is subjected to an external static magnetic field H_0 directed along the easy magnetization axis z of the plate at right-angles to its surface. A wave with circular right-hand polarization is incident along the normal to the surface and the magnetic field of this wave has two components h_x and h_y, such that $h_x=h_y$. It is shown in the preceding section that the magnetic susceptibility of an antiferromagnetic plate depends on the square of the amplitude of the incident radiation, i.e., on the wave power. If we assume that the permittivity $\varepsilon_\perp$ of the plate is constant, we find that the Maxwell equations yield a nonlinear equation for the complex amplitude of a right-handed polarized wave

$$d^2h_+/dz^2+\varepsilon_\perp k_0^2(\mu_+{}^L+\mu_+{}^{NL}|\gamma h_+|^2)h_+=0, \qquad (14)$$

where $k_0 = \omega/c$ (ω is the incident-wave frequency and c is the velocity of light in vacuum) and

$$h_+=h_x+ih_y. \qquad (15)$$

The linear and nonlinear parts of the magnetic susceptibility can be expressed in terms of the magnetic susceptibility:

$$\mu_+{}^L=1+4\pi\chi_+{}^L, \qquad (16)$$

$$\mu_+{}^{NL}=4\pi\chi_+{}^{NL}. \qquad (17)$$

It should be noted that Eq. (14) is derived in the one-wave approximation, i.e., assuming that the amplitude of one circularly polarized wave (in this case h_-) is much less than the amplitude of the other wave (h_+). This approximation is valid when the wave incident on an antiferromagnet plate has the right-handed polarization and its coupling to the left-handed wave is only via the coefficient of nonlinearity. We shall seek the solution of Eq. (14) in the form of a product

$$h_+=Hh(z)\exp(i\Phi(z)), \qquad (18)$$

where H is the amplitude factor, $h(z)$ is the normalized amplitude, and $\Phi(z)$ is the eikonal of the circularly polarized wave. Substituting Eq. (18) into Eq. (14), we obtain a pair of equations for the determination of $h(z)$ and $\Phi(z)$:

$$2\frac{dh}{dz}\frac{d\Phi}{dz}+h\frac{d^2\Phi}{dz^2}=0, \qquad (19a)$$

$$\frac{d^2h}{dz^2}-h\left(\frac{d\Phi}{dz}\right)^2+k^2(1+\lambda h^2)h=0, \qquad (19b)$$

where $k^2=k_0^2\varepsilon_\perp\mu_+^L$, $\lambda=k_0^2\varepsilon_\perp\mu_+^{NL}/\mu_+^L$. Integrating the first of these equations, we find that the product of the square of the amplitude of a nonlinear wave and the derivative of the eikonal is a constant:

$$h^2(z)\frac{d\Phi(z)}{dz}=W=\text{const}. \qquad (20)$$

After substituting Eq. (20) into Eq. (19b) and integrating the latter, we obtain the following equation for the amplitude

$$\left(\frac{dh}{dz}\right)^2+\frac{W}{h^2}+k^2h^2+\frac{1}{2}\lambda h^4k^2=A, \qquad (21)$$

where A is the second integration constant. The second integration in Eq. (20) gives

$$\Phi(z)=\Phi(0)+W\int_0^z\frac{1}{h^2(z')}dz', \qquad (22)$$

where $\Phi(0)$ is the eikonal of the wave at the point $z=0$.

We shall now identify the physical meaning of the constant W. The normal component of the time-averaged vector of the energy flux density (Poynting vector) is

$$S_z=\frac{c}{8\pi}\text{Re}[\mathbf{EH}^*]_z, \qquad (23)$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields of the wave. In the case of circularly polarized waves, we obtain

$$S_z=\frac{c}{8\pi}\text{Re}\left[\frac{i}{2}(e_+h_+^*-e_-h_-^*)\right], \qquad (24)$$

where

$$e_\pm=\mp(1/k_0\varepsilon_\perp)(\partial h_\pm/\partial z)$$

are the electric fields of the circularly polarized waves. Substituting Eq. (18) into Eq. (24), we obtain—subject to Eq. (20):

$$S_z=\frac{c}{16\pi}\frac{H^2}{k_0\varepsilon_\perp}W. \qquad (25)$$

Therefore, the constant W governs the energy flux which is carried by a right-handed polarized wave across an antiferromagnetic plate. We can find the unknown parameters W, A, and $\Phi(0)$ by solving the boundary-value problem.

4. SOLUTION OF THE BOUNDARY-VALUE PROBLEM

A wave incident on an antiferromagnetic plate (Fig. 1) can be described by $H_+\exp(ik_0z)$, the wave reflected from the plate is $R_+H_+\exp(-ik_0z)$, that transmitted by the plate is $T_+H_+\exp(ik_0z)$ (here, R_+ and T_+ represent the transmission and reflection coefficients, respectively), and the conditions of continuity of the tangential components of the magnetic and electric fields at the interface between the media yield the following equations for the determination of R_+ and T_+:

$$1+R_+=h(0)e^{i\Phi(0)}, \qquad (26a)$$

$$T_+e^{ik_0d}=h(d)e^{i\Phi(d)}, \qquad (26b)$$

$$1-R_+=\frac{1}{ik_0\varepsilon_\perp}\left\{\frac{dh(0)}{dz}+ih(0)\frac{d\Phi(0)}{dz}\right\}e^{i\Phi(0)}, \qquad (26c)$$

$$T_+e^{ik_0d}=\frac{1}{ik_0\varepsilon_\perp}\left\{\frac{dh(d)}{dz}+ih(d)\frac{d\Phi(d)}{dz}\right\}e^{i\Phi(d)}. \qquad (26d)$$

After some transformations, the system (26) allows us to determine the integration constant W in terms of A:

$$A=W\left(k_0\varepsilon_\perp+\frac{k^2}{k_0\varepsilon_\perp}+\frac{k^2\lambda}{2k_0^2\varepsilon_\perp^2}W\right). \qquad (27)$$

The boundary value of the amplitude of a circularly polarized wave and its derivative at $z=d$ are then given by

$$W=k_0\varepsilon_\perp h^2(d), \qquad (28a)$$

$$\frac{dh(d)}{dz}=0. \qquad (28b)$$

Similar conditions applicable to the wave amplitude and its derivative at the second boundary ($z=0$) are implicit and are of the form

$$4k_0^2\varepsilon_\perp^2-h^2(0)\left(k_0\varepsilon_\perp+\frac{W}{h^2(0)}\right)^2$$
$$=-\frac{W^2}{h^2(0)}-k^2h^2(0)-{}^1/{}_2k^2\lambda h^4(0)+A. \qquad (29)$$

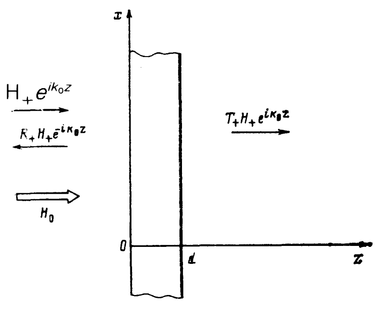

FIG. 1. Geometry of the problem.

The third unknown parameter $\Phi(0)$ is found from the boundary conditions and is given by the expression

$$\Phi(0)=\arcsin\frac{1}{2k_0\varepsilon_\perp}\frac{dh(0)}{dz}. \qquad (30)$$

The transmission coefficient of a circularly polarized wave crossing an antiferromagnetic plate is readily found from Eq. (26b) if we multiply this equation by its complex conjugate which yields

$$|T_+|^2=h^2(d). \qquad (31)$$

The solution of Eq. (21) depends on the sign of the nonlinear coefficient (and this sign depends on the frequency and changes on going through the resonance point). The solution obtained allowing for the boundary conditions can be expressed in terms of the Jacobi elliptic functions[5] and are of the following form if $\lambda>0$:

$$h^2(z)=C_++[C-C_-]\mathrm{cn}^2\left\{\left[-\frac{\lambda}{2}(C-C_-)\right]^{1/2}k(d-z)\,\Big|\,\frac{C-C_+}{C-C_-}\right\}, \qquad (32a)$$

$$C=W/k_0\varepsilon_\perp, \qquad (32b)$$

$$C_\pm=\pm\frac{1}{\lambda}\left\{\left[\left[1+\frac{\lambda W}{2k_0}\right]^2+\frac{2\lambda W}{n^2k_0}\right]^{1/2}\mp\left[1+\frac{\lambda W}{2k_0}\right]\right\}. \qquad (32c)$$

If $\lambda<0$, we obtain

$$h^2(z)=F_1(z)/F_2(z), \qquad (33a)$$

$$F_1(z)=(D_+-C)D_-+(C-D_-)D_+\,\mathrm{cn}^2\left\{\left[\frac{|\lambda|}{2}(D_+-D_-)\right]^{1/2}\times k(d-z)\,\Big|\,\frac{C-D_-}{D_+-D_-}\right\}, \qquad (33b)$$

$$F_2(z)=(D_+-C)+(C-D_-)\mathrm{cn}^2\left\{\left[\frac{|\lambda|}{2}(D_+-D_-)\right]^{1/2}\times k(d-z)\,\Big|\,\frac{C-D_-}{D_+-D_-}\right\}, \qquad (33c)$$

$$D_\pm=\frac{1}{|\lambda|}\left\{\left[1-\frac{|\lambda|W}{2k_0}\right]\pm\left[\left[1-\frac{|\lambda|W}{2k_0}\right]^2-\frac{2|\lambda|W}{n^2k_0}\right]^{1/2}\right\}. \qquad (33d)$$

where $\lambda\neq0$, $n^2=\varepsilon_\perp\mu_+^L$.

The next stages of the solution to the problem have to be carried out numerically.

5. RESULTS AND DISCUSSION

We shall now consider the order in which the problem can be solved numerically. If we specify a definite value of the parameter W, which [see Eq. (28a)] governs the amplitude of the wave at the boundary $z=d$, we can find from Eq. (32) [or Eq. (33), depending on the sign of the nonlinear coefficient λ] the solutions for the amplitude at $z=0$. We can then analyze all possible values of the parameter W and identify those which satisfy the second boundary condition of Eq. (29). The solutions which satisfy both boundary conditions are the solutions of the problem. The range of possible values of W is governed by the law of conservation of energy, in accordance with which the coefficient representing transmission of a wave across an antiferromagnet plate cannot exceed unity. Simple calculations show that $0<W<k_0\varepsilon_\perp$. In calculation of the Jacobi elliptic functions we used the standard subprogram JELF.

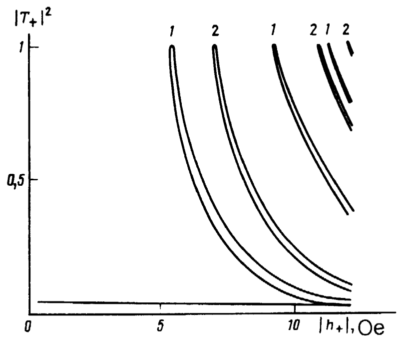

FIG. 2. Dependence of the transmission coefficient of an antiferromagnetic plate on the amplitude of the magnetic field of the incident wave. $H_0=0.5$ kOe, $H_e=515$ kOe, $H_a=8.4$ kOe, $d=100$ μm, $\varepsilon_\perp=4$, $m_s=0.56$ kG, $\Delta\omega/\Omega_+=-1\times10^{-5}$ (1), and -1.2×10^{-5} (2).

Figure 2 gives the results of a numerical calculation of the transmission coefficient of an antiferromagnetic plate as a function of the amplitude of a magnetic field of the wave incident on this plate. The parameters of the problem were selected for FeF_2 crystals. The frequency of a wave incident on such a plate was assumed to be less than the AFMR frequency by an amount $\Delta\omega=\Omega_+\cdot10^{-5}$. Then, $\mu_+^L>0$, $\lambda<0$. We can see that when a certain amplitude of the incident wave is reached, the transmission coefficient of an antiferromagnetic plate becomes a bistable quantity. For the above parameters the value in question is 7 Oe. An increase in the amplitude increases the number of the possible values which the transmission coefficient of the plate has to assume, i.e., the system goes over to a multistable state. The behavior of an antiferromagnetic plate in the field of an incident wave is in many respects similar to the behavior of a Fabry–Perot resonator filled with a medium whose refractive index depends on the power of the radiation propagating in it. As the wave amplitude is increased, the effective refractive index of an antiferromagnetic plate changes and so does the wavelength in the antiferromagnetic medium. When the number of half-wavelengths which can be fitted within the thickness of the plate is an integer, the transmission coefficient tends to unity. This is the reason for the appearance of new peaks on increase in the wave amplitude (Fig. 2). When the incident wave frequency approaches the AFMR frequency, the absolute value of the nonlinear coefficient λ increases. A reduction in the detuning induces bistable operation and we can see from Fig. 1 that this occurs at lower values of the incident-wave amplitude.

6. CONCLUSIONS

We considered the transmission of a nonlinear electromagnetic wave across an antiferromagnetic plate subjected to an external magnetic field. First of all, we found the nonlinear magnetic susceptibility of the antiferromagnetic plate in such a field. We determined the nonlinear corrections to the matrix elements, which depended on the square of the modulus of the amplitude of the electromagnetic wave field.

Secondly, we studied the reflection and transmission of a nonlinear electromagnetic wave across such an antiferromagnetic plate. We found that under certain conditions the transmission coefficient of the wave can be a bistable quantity and, on increase in the wave power, it can become multistable.

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Translated by A. Tybulewicz