Magnetic dislocations in a stripe domain structure

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We conduct a theoretical and experimental investigation of the properties of stripe domain structures with magnetic dislocations. We obtain the equations of motion for a solitary magnetic dislocation, and show that its velocity is determined by the combined action of a braking force, a surface tension force, and forces caused by external magnetic and demagnetization fields. We also discuss how magnetic dislocations differ from ordinary dislocations in crystals.

In analyzing the properties of completely ordered stripe domain structures (DS) in magnetic films, the following question arises: how does the period d of this type of distribution of magnetization vector $\mathbf{M}$ specifically change as we vary the magnetic field $\mathbf{H}$, the temperature T, etc.? Despite the apparent simplicity of this question, an answer to it is far from evident. Actually, in films with unbounded transverse dimensions subjected to uniform external perturbations, a smooth change in the period is forbidden by symmetry considerations; for finite-size films it is forbidden because of the existence of an energy barrier between states with differing numbers of domains. Therefore, if for certain values of $T = T_0$ and $\mathbf{H} = \mathbf{H}_0$ an equilibrium DS exists in the film with period $d_{00} = d_0(T_0, \mathbf{H}_0)$, any change in T or $\mathbf{H}$ will make this DS metastable. If $d_{00} > d_0(T, \mathbf{H})$, then the DS will be found in a state of tension and, depending on the value of the difference $\Delta d_0 = d_{00} - d_0$, the following situations are possible: either a kink instability in the domain wall system against a sinusoidal distortion of the profile, a translational (modulation) instability of the position of the domain wall, or (for quasi-uniaxial films) a first-order phase transition to a hexagonal lattice of cylindrical magnetic domains. For $d_{00} < d_0(T, \mathbf{H})$ the stripe DS will be under compression, and for a certain critical value of Δd_0 it is possible to have either translational (modulation) instability of the domain wall or a transition to a cylindrical magnetic domain lattice.[1-7]

The situation changes if we take into account the possibility that defects of magnetic dislocation (MD) type can exist in the stripe DS. Note that the creation of MD, which disrupts the translational order in the DS and thereby increases the entropy of the system, becomes favorable in the neighborhood of a second-order phase-transition line (or a first-order line that is close to second-order), at which the corresponding modulus of rigidity of the DS reduces to zero.[8,9] In this case, for $T \neq 0$ relaxation of the metastable stripe DS to equilibrium can take place by way of creation, annihilation, fusion, and motion of MD.

Point and line defects in the crystal structures of real films (including the transverse sample boundaries) facilitate the birth and fusion of dislocations. In addition, these defects exert coercive forces on the cores of the MD, stabilizing the positions of the latter. Therefore, in films with distributed coercive forces the average period of the DS can vary within wide limits due to motion of the MD, while the instabilities that are inherent in an ideally-ordered DS will in general not arise in this case (such mechanisms are discussed in Refs. 7 and 10–12).

In this paper we investigate the properties of stripe DS with MD both theoretically and experimentally. We show that the velocity of a MD is determined by the combined action of a braking force and a surface-tension force, as well as forces which arise from external magnetic fields and demagnetization fields. In contrast to normal dislocations (i.e., in crystals), which can translate parallel to glide planes (when the continuous media is not disrupted), MD acted on by uniform external forces move in such a way that the velocity of the core always coincides with the direction of the domain wall of the stripe DS, i.e., perpendicular to the glide planes. This is connected with the fact that the climb of dislocations in crystals takes place because of mass transfer, while the motion of MD is mediated by changes in the direction or magnitude of $\mathbf{M}$. If the external influence (e.g., $\mathbf{H}$) is uniform, motion of the MD can occur in the glide planes as well; in this case the motion has a threshold character, and begins if the "elastic" stress in the stripe DS exceeds some start-up value determined by the Peierls force.

1. THEORY

1.1. Derivation of basic equations

An analytic theory that describes the behavior of an MD in a stripe DS can be constructed fairly simply for uniaxial ferromagnetic films in the neighborhood of a second-order spontaneous or orientational phase-transition line (or a first-order line that is close to second-order), for which the distribution of magnetization can be accurately approximated by using a Fourier series containing a small number of terms. For films with strong "perpendicular" anisotropies [$\beta_u \gg 4\pi$, where β_u is the uniaxial anisotropy constant; the axis of easy magnetization is parallel to the normal $\mathbf{n}$ to the surface], this type of phase transition involving the participation of a DS was investigated in Refs. 8 and 9. The external parameters that can induce a spontaneous phase transition are the temperature T (below the Curie temperature T_c), and, for an orientational phase transition, a magnetic field $\mathbf{H}$ roughly parallel to the surface of the film (i.e., $H_\perp \gg H_\parallel$, where $H_\perp$ and $H_\parallel$ are the projections of $\mathbf{H}$ on the film surface and on the normal $\mathbf{n}$ to the surface, respectively).

It has been established that a phase transition with the participation of a DS is described by a three-component order parameter, one of whose components is defined as the value of the magnitude of the magnetization vector $\mathbf{M}$ or

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some one of its projections M_i, while the two other components characterize the degrees of translational and orientational order of the domain wall.

The analysis carried out in Refs. 8 and 9 showed that, depending on the parameters T and $\mathbf{H}$, inhomogeneous magnetic states in the film can be realized in the following modifications: a "crystalline" phase (i.e., a fully-ordered DS), the so-called BKT (Berezinskii-Kosterlitz-Thouless) phase[13–15] (i.e., a DS with bound MD), a "liquid-crystal" phase (i.e., a DS with free MD), and a "liquid" phase (i.e., a DS with free magnetic disclinations). In what follows we will investigate films with quasi-ordered DS corresponding to the BKT or liquid-crystal phases.

We limit the discussion to the case of a thick magnetically uniaxial ferromagnetic film ($l_z \gg \alpha^{1/2}$, where l_z is the film thickness and α is the inhomogeneous exchange interaction constant) with its easy magnetic axis parallel to the surface normal, $\mathbf{n}\|\mathbf{e}_z$, in a magnetic field $\mathbf{H} = M(h_\perp \mathbf{e}_y + h_\| \mathbf{e}_z)$ near an orientational phase transition (i.e., for $h_\| \ll h_\perp$; $|\xi| \ll \max\{4\pi, \beta_u\}$, where $\xi = \beta_u - h_\perp$). If we parametrize the direction of the vector $\mathbf{M}$ in a spherical system of coordinates by the angles $\theta = \arcsin(M_z/M)$ and $\psi = \arctan(M_x/M_y)$, we can write the free energy of the system in the form

$$\mathcal{F} = {}^1/_2 M^2 \int d\mathbf{r} \{\alpha[(\nabla\theta)^2 + \cos^2\theta(\nabla\psi)^2] + \beta_u \cos^2\theta - 2h_\perp \cos\theta\cos\psi - 2h_\| \sin\theta - \mathbf{m}\mathbf{h}_D\}, \tag{1}$$

where $\mathbf{m} = \mathbf{M}/M$ and $\mathbf{h}_D = \mathbf{H}_D/M$ is the normalized magnetostatic field. Since $\theta \ll 1$ and $\psi \ll 1$ hold near a phase-transition line, it follows from the Landau-Lifshits equation and the equations of magnetostatics[7,16,17] that

$$(\mu\nabla_x^2 + \nabla_y^2)[-\alpha\nabla^2\theta + (h_\perp - \tilde{\beta})\theta + {}^1/_2\beta_u\theta^3 + \omega_0^{-1}(\lambda_r - \lambda_e\nabla^2)\dot{\theta} + (h_\perp\mu\omega_0^2)^{-1}\ddot{\theta}] + 4\pi\nabla_z^2\theta = 0, \tag{2a}$$

$$\nabla_x[(\omega_0 h_\perp \mu)\psi + (\lambda_r - \lambda_e\nabla^2)\dot{\psi}] - \nabla_z\dot{\psi} + 4\pi\omega_0\nabla_z\theta + [\nabla_x + (\lambda_r - \lambda_e\nabla^2)\nabla_z]\dot{\theta} = 0, \tag{2b}$$

where $\tilde{\beta} = \beta_u - (3/32\pi^2)\beta_u h_\|^2$, $\mu = 1 + 4\pi h_\perp^{-1}$, λ_r and λ_e are relaxation constants whose origins are relativistic and exchange effects respectively, while $\omega_0 = gM$, where g is the gyromagnetic constant.

In the course of calculating the distribution of magnetization and the spin-wave spectrum in the film, it is customary to write the angle θ in the form of a sum of a static $\theta_0(\mathbf{r})$ and a dynamic $\tilde{\theta}(\mathbf{r}, t)$ component, where $|\theta_0| \gg |\tilde{\theta}|$. For a regular stripe DS with a fixed period d (in a film with unbounded transverse size the period d can be arbitrary; for films with transverse dimension l_x the period equals $d = l_x N_x^{-1}$, where N_x is the number of periods), the solutions to Eqs. (2) have the form[18,19]

$$\theta_0(\mathbf{r}) = \sum_{n=0}^{\infty} \lambda^n A_n(z)\cos(nkx),$$

$$\psi_0(\mathbf{r}) = \sum_{n=0}^{\infty} \lambda^n B_n(z)\sin(nkx), \tag{3}$$

where $\lambda \ll 1$ is the order parameter; $k = 2\pi d^{-1}$; $q = \pi/l_z$; $A_0 = (4\pi)^{-1}h_\|$; and

$$A_1 = a_1\cos(qz) + {}^1/_2\beta_u\lambda^2 a_1^3 a_3\cos(3qz);$$

$$A_2 = {}^1/_2\beta_u A_0 a_1^2[b_0 + b_2\cos(2qz)],$$

$$A_3 = {}^1/_2\beta_u a_1^3[c_1\cos(qz) + c_3\cos(3qz)].$$

Explicit expressions for the coefficients a_n, b_n, and c_n are obtained from Eq. (13) of Ref. 19 if we make the replacement $\delta \to \beta_u/2$.

Substituting the series (3) into (1) and (2) and taking into account the boundary conditions at the film surface:

$$(\mathbf{n}, \nabla\mathbf{M})|_S = 0, \quad (\mathbf{n}, (\mathbf{H}_{Di} + 4\pi\mathbf{M} - \mathbf{H}_{De}))|_S = 0, \quad [\mathbf{n}, (\mathbf{H}_{Di} - \mathbf{H}_{De})]|_S = 0, \tag{4}$$

where $\mathbf{H}_{Di} = M\mathbf{h}_{Di}$ and $\mathbf{H}_{De} = M\mathbf{h}_{De}$ are the demagnetization and scattering fields respectively, we obtain an expansion of the free energy density in the parameter $\lambda a_1 \equiv \lambda a$ for fixed values of k and ξ:

$$V^{-1}\mathcal{F} = V^{-1}\mathcal{F}_0 + \sum_{p=1}^{3} (\lambda a)^{2p} B_{2p}(\xi, k) M^2/2p, \tag{5}$$

and the equation that determines the dependence of the parameter λa on k and ξ is:

$$B_2 + (\lambda a)^2 B_4 + (\lambda a)^4 B_6 = 0, \tag{6}$$

where V is the film volume and $\mathcal{F}_0$ is the free energy of the film in its uniform magnetized state, which equals

$$\mathcal{F}_0 = -(A_0 M)^2(2\pi - \xi/2 + 3\beta_u A_0^2/8);$$

the functions B_{2p} are defined by the expressions

$$B_2 = {}^1/_4\varkappa^2(1 + \zeta_0^4 - \tilde{\xi}\varkappa^{-2}), \tag{7a}$$

$$B_4 = \frac{9\beta_u}{128}\left[1 - \frac{\beta_u A_0^2}{\varkappa^2 - \tilde{\xi}} \frac{3(4 - \tilde{\xi}\varkappa^{-4}) + 2\zeta_0^4}{4 - \tilde{\xi}\varkappa^{-2} + \zeta_0^4}\right], \tag{7b}$$

$$B_6 = 6231\beta_u^2(512\varkappa_{c0})^{-2}. \tag{7c}$$

Here $\tilde{\xi} = \xi - \frac{3}{2}\beta_u A_0^2$; $\varkappa = k\alpha^{1/2}$; $\varkappa_{c0} = k_{c0}\alpha^{1/2} = (4\alpha\pi^3/\mu_\perp l_z^2)^{1/4}$; $\zeta_0 = \varkappa_{c0}\varkappa^{-1} = k_{c0}k^{-1}$.

In Ref. 9 it was shown that in the course of an orientational phase transition the uniform magnetized state that exists in a strong transverse magnetic field, i.e., for $h_\perp \gtrsim \beta_u$, loses its stability against a transition to a stripe DS in thermodynamic equilibrium along the curve

$$h_{\perp c}(h_\|) = \beta_u - 2\varkappa_c^2 - {}^3/_{32}\beta_u h_\|^2\pi^{-2}, \tag{8a}$$

furthermore, the DS, in its turn, becomes unstable with respect to a transition to a uniform state for

$$h_{\perp c}^*(h_\|) = h_{\perp c}(h_\|) + 0{,}388(\varkappa_{c0}\varepsilon_{kc})^2 h_{\|kc}^{-4} w(\varepsilon_{kc}), \tag{8b}$$

while the critical value of the normalized reciprocal period for such a DS along the curves defined by Eqs. (8a) and (8b) equals

$$\varkappa_c(h_\|) = 2\pi\alpha^{1/2}d_c^{-1} = \varkappa_{c0}(1 - 3{,}19\cdot 10^{-5}\beta_u h_\|^2\alpha^{-1/2}l_z), \tag{9a}$$

$$\varkappa_c^*(h_\parallel)=\varkappa_c(h_\parallel)-0{,}17\varkappa_{c0}h_{\parallel kc}^{-2}\varepsilon_{kc}w(\varepsilon_{kc}). \quad (9b)$$

Here d_c is the critical period; $\varkappa_{c0}=\varkappa_c(0)$; $w(\varepsilon_{kc})$ is the Heaviside function; $\varepsilon_{kc}=h_\parallel^2-h_{\parallel kc}^2$;$h_{\parallel kc}=2\sqrt{3}\pi\varkappa_{c0}\beta_p^{-1/2}$ is the ordinate of a tricritical point at which two phase transition lines which separate the stripe DS with equilibrium period d_c from the uniform state join together. Across one of these, the phase transition is first-order, while across the other it is second-order.

Equations (8) and (9) were obtained from the condition that the free energy Eq. (5) be a minimum with respect to the order parameter, i.e.,

$$\partial\mathcal{F}/\partial(\lambda a)=0,\quad \partial^2\mathcal{F}/\partial(\lambda a)^2\geqslant 0, \quad (10)$$

with subsequent minimization with respect to the normalized inverse period of the DS $\varkappa=2\pi d^{-1}\alpha^{1/2}$. If the minimization is not carried out, i.e., the parameter $\varkappa$ is arbitrary but fixed, the system (10) will determine the behavior of a nonequilibrium DS with $\varkappa\neq\varkappa_c$. The first of the conditions (10) is identical to Eq. (6); the sign of the equation in the second condition determines the position of the line along which the DS loses its stability for a given $\varkappa$, that is

$$h_{\perp f}=h_{\perp c}-\varkappa^2(1-\zeta^2)^2, \quad (11a)$$

for the second-order phase transition, where $\zeta=\varkappa_c\varkappa^{-1}$, and

$$h_{\perp f}^*=h_{\perp f}+0{,}388(\varkappa_c\varepsilon_k)^2h_{\parallel k}^{-4}w(\varepsilon_k) \quad (11b)$$

for the first-order phase transition; the amplitude of the z-component of the magnetization of the DS on these curves comes to

$$(\lambda a)_f=8(\varkappa_{c0}/\pi\varkappa)\,|\varepsilon_k w(\varepsilon_k)E_1/(2077E_2)|^{1/2}. \quad (12)$$

Here $\varepsilon_k=h_\parallel^2-h_{\parallel k}^2$, $E_1=9-\zeta_0^4$, $E_2=3-\zeta_0^4$, while the ordinate of the tricritical point $H_{\parallel k}$ which separates the first-order and second-order phase transition lines across which the stripe DS with nonequilibrium period d evolves into the uniformly magnetized state is determined by the expression $h_{\parallel k}=4\pi\varkappa(3\beta_u^{-1}E_1^{-1}E_2)^{1/2}$.

It follows from (11) that the line on which a DS with fixed period d on the plane $(h_\perp, h_\parallel)$ loses stability shifts to the left relative to the corresponding curves for a DS with equilibrium period, and is tangent to the latter (for $d>d_{c0}$, i.e., $\varkappa<\varkappa_{c0}$) for $d=d_c$. We note, however, that a transition of the nonequilibrium DS to the uniformly magnetized state in the regime of quasistatic variation of the magnetic field does not take place, since an instability of another type will develop first (see Ref. 7). The field dependence of λa is determined by the expression

$$(\lambda a)^2=[(B_4^2-4B_2B_6)^{1/2}-B_4](2B_6)^{-1}; \quad (13)$$

in the limiting case $\Delta h_\perp=h_{\perp f}-h_\perp\ll\varkappa^2h_{\parallel k}^{-2}|\varepsilon_k|$

$$(\lambda a)^2={}^{32}/_9\Delta h_\perp h_{\parallel k}^2(\beta_u|\varepsilon_k|)^{-1}, \quad (14a)$$

while in the case $\varkappa^2h_{\parallel k}^{-2}|\varepsilon_k|\ll\Delta h_\perp\ll\varkappa^2$

$$(\lambda a)^2=3{,}24\beta_u^{-1}\varkappa_{c0}(\Delta h_\perp)^{1/2}. \quad (14b)$$

In order to investigate the nonlinear dynamics of distortions in the shape of the DS it is necessary to introduce a field which specifies the displacement of a point with a given value of magnetization from its position in the regular DS, $u\equiv u_x(x,y)$, i.e., to cast the distribution of magnetization in the form[2,7,8]

$$\theta(\mathbf{r},t)=\sum_{n=0}^{\infty}\lambda^nA_n(z)\cos\{nk[x-u(x,y,t)]\}, \quad (15a)$$

$$\psi(\mathbf{r},t)=\sum_{n=0}^{\infty}\lambda^nB_n(z)\sin\{nk[x-u(x,y,t)]\}. \quad (15b)$$

Then by limiting the discussion to long wavelength distortions of the shape of the DS, we can write the "elastic" part of the free energy associated with shifts of the domain boundary in the following form:

$$\mathcal{U}=\mathcal{U}_L+\mathcal{U}_{NL}, \quad (16)$$

where

$$\mathcal{U}_L=\frac{1}{2}\int dx\,dy[C_x(\nabla_xu)^2+C_y(\nabla_yu)^2+k^{-2}C_3(\nabla_y^2u)^2], \quad (17a)$$

$$\mathcal{U}_{NL}={}^1/_2\int dx\,dy[C_{1,2}\nabla_xu(\nabla_yu)^2+C_{0,4}(\nabla_yu)^4]. \quad (17b)$$

The effective moduli of rigidity C_x and C_y of the stripe DS equal

$$C_x=C_0E_3[1-2\varkappa^2E_3^{-1}E_4^2(\Delta h_\perp)^{-1}], \quad (18a)$$

$$C_y=C_0\{1-\zeta_0^4\mu^{-1}+6\Delta h_\perp h_\parallel^2|\varepsilon_k|^{-1}\varkappa^{-2}E_2E_1^{-1}[2E_2^{-2}+{}^1/_9(1-{}^1/_4\zeta_0^4\mu^{-1})]\}, \quad (18b)$$

while the remaining quantities entering into (17) and (18) are defined in the following way:

$$C_0={}^1/_2(\lambda a)^2\varkappa^2M^2l_z,\quad C_3=C_0\zeta_0^4\mu^{-2},\quad C_{1,2}=-4\zeta_0^4C_0,$$
$$C_{0,4}=\zeta_0^4C_0,\quad E_3=1+3\zeta_0^4,\quad E_4=1-\zeta_0^4.$$

In order to derive the equation of motion for the shift of the domain wall we also introduce kinetic energy and dissipation functions:

$$\mathcal{T}_{kin}=\frac{1}{2}\int dx\,dy\rho\dot{u}^2, \quad (19)$$

$$\mathcal{W}_d=-\frac{1}{2}\int dx\,dy[\Gamma\dot{u}^2+\eta_x(\nabla_x\dot{u})^2+\eta_y(\nabla_y\dot{u})^2]. \quad (20)$$

Then the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{u}}+\frac{\partial}{\partial x_i}\frac{\partial\mathcal{L}}{\partial(\nabla_iu)}=\frac{\partial\mathcal{W}_d}{\partial\dot{u}}-\frac{\partial}{\partial x_i}\frac{\partial\mathcal{W}_d}{\partial(\nabla_i\dot{u})}, \quad (21)$$

where $\mathcal{L}=\mathcal{T}_{kin}-\mathcal{U}$ is the Lagrangian that completely determines the nonlinear behavior of the domain wall system, acquires the form

$$\rho\ddot{u}+\Gamma\dot{u}=C_i^*\nabla_i^2u-C_3k^{-2}\nabla_y^4u+\frac{\partial}{\partial x_i}\frac{\partial\mathcal{U}_{NL}}{\partial(\nabla_i\dot{u})}, \quad (22)$$

where $\rho=(\lambda a)^2k^2(2\mu\beta_ug^2)^{-1}l_z$ is the effective density of the stripe DS, and $C_i^*(\omega) = C_i - i\omega\eta_i$ is the effective complex modulus of rigidity ($i = x,y$);

$$\eta_x=\lambda_e C_0(\alpha\omega_0)^{-1}(1-4\varkappa^2\Delta h_\perp^{-1}E_4),\quad \eta_y=\lambda_e C_0(\alpha\omega_0)^{-1},$$
$$\Gamma=\rho\mu\beta_u\omega_0(\lambda_r+\lambda_e k^2).$$

Rather than proceed to an analysis of the motion of the MD, we will pause to make the following remark: magnetic dislocations exist in two dual modifications, which differ by the direction of **M** at the center of the dislocation core. For $H_\parallel = 0$ the behaviors of both MD modifications are identical; however, an arbitrarily small field $H_\parallel$ makes them inequivalent. In particular, for the same values of the external parameters a change in the type of MD may involve a change in the direction of motion. The inequivalence of the behavior of MD for which the vector **M** has different directions at dislocation cores arises because the properties of "different-polarity" domains are not identical for $H_\parallel \neq 0$. The period of an asymmetric DS in the general case is given by the expression

$$d=d_1+d_2={}^1/_2 d(1+\varepsilon_d)+{}^1/_2 d(1-\varepsilon_d), \tag{23}$$

where $\varepsilon_d(\lambda a) \approx \arcsin(h_\parallel/2\pi^2\lambda a)$ for $|h_\parallel| \leqslant 8\lambda a$. The equilibrium period d_0 of the DS for $|h_\parallel| \ll h_{\parallel kc}$ depends on the field **H** in the following way:

$$d_0=d_{10}+d_{20}={}^1/_2 d_0(1+\tilde\varepsilon_d)+{}^1/_2 d_0(1-\tilde\varepsilon_d)$$
$$=d_{c0}\{1+\Delta h_\parallel[{}^7/_8 h_\parallel^2|\varepsilon_{kc}|^{-1}+\alpha^{1/4}l_z^{-1/2}(4\pi\mu)^{-3/4}](4\varkappa_{c0}^2)^{-1}\}, \tag{24}$$

where

$$\tilde\varepsilon_d=\varepsilon_d(\lambda a)_0,\qquad (\lambda a)_0={}^{32}/_9\beta_u^{-1}h_{\parallel kc}^2|\varepsilon_{kc}|^{-1}(h_{\perp c}-h_\perp).$$

1.2. Motion of dislocations in a quasi-ordered stripe DS

The static distribution of **M** around a solitary MD with Burgers vector $\mathbf{B}\parallel\mathbf{e}_x$, where $|\mathbf{B}| \approx d$, is given[20] by Eqs. (15):

$$u=(B/2\pi)\,\mathrm{arctg}\,[(y/x)(C_x/C_y)^{1/2}]\quad \text{for } x\gg x_c,\ \ y\gg y_c, \tag{25a}$$

$$u=\frac{1}{4}B\Phi\left(\frac{y}{y_c}\left(\frac{x_c}{x}\right)^{1/2}\right)\operatorname{sign} x\quad \text{for } B\ll x\ll x_c,\ B\ll y\ll y_c, \tag{25b}$$

where

$$x_c=(C_3C_x)^{1/2}(kC_y)^{-1},\quad y_c=\left(\frac{C_3}{C_y}\right)^{1/2}k^{-1},$$

$$\Phi(x)=\frac{2}{\pi^{1/2}}\int_0^x e^{-t^2}dt.$$

The equation of motion of the MD can be written in the form[21]

$$m_{ik}W_k=F_i+F_i^{\mathrm{int}}+F_i^{\mathrm{br}}. \tag{26}$$

Here m_{ik} is the effective mass tensor of the MD, whose components are equal in order of magnitude to $[\rho B^2/4\pi]\ln(l_x/r_0)$, where $r_0 \sim d$; **W** is the acceleration of the MD; $\mathbf{F}^{\mathrm{br}}$ is the braking force on the MD; **F** is the force that results from the action of the external magnetic field, the magnetization fields, and the surface tension of the warped domain wall, on the MD; and $\mathbf{F}^{\mathrm{int}}$ is the force exerted on the MD by other dislocations. In particular, for a pair of MD with Burgers vectors $\mathbf{B}_1$ and $\mathbf{B}_2$, for $|\mathbf{r}_{12}| \ll |\mathbf{r}_{1c} - \mathbf{r}_{2c}|$ we have

$$F_x^{\mathrm{int}}=\frac{B_1B_2}{2\pi}\frac{x(C_xC_y)^{1/2}}{x^2+y^2C_xC_y^{-1}},\qquad F_y^{\mathrm{int}}=\frac{B_1B_2}{2\pi}\frac{yC_x^{3/2}C_y^{-1/2}}{x^2+y^2C_xC_y^{-1}}; \tag{27}$$

here x and y are components of the two-dimensional vector $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$, where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the radius vectors to the MD cores. In a metastable DS (i.e., with $d \neq d_0$) the force **F** can be written as a sum $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$. The force $\mathbf{F}_1$ which is caused by the "stress"

$$\sigma_x=\delta\mathcal{U}/\delta(\nabla_x u)=C_x\nabla_x u=C_x(d-d_0)/d=-p,$$

is calculated by using the Peach-Koehler formula

$$F_{1y}=pB.$$

This force, which is directed along the domain wall of the stripe DS, expels the MD from the sample for $d<d_0$ and causes it to move in the opposite direction for $d>d_0$. The force $\mathbf{F}_2$ gives rise to the inequivalence of dislocations with orientations of the vector **M** in the core parallel to the field $\mathbf{H}_\parallel = H_\parallel \mathbf{e}_z$ (type 1) and antiparallel to the latter (type 2). Estimates give

$$F_{2y}^{(1,2)} \approx \int_s f_n^{(1,2)}\cos\alpha_H\, ds=\tilde p^{(1,2)}B, \tag{28}$$

where $f_n^{(1,2)} = \mp 4\lambda a M l_z(H_\parallel - H_{\parallel s})\pi^{-1}$ is the force acting on a unit length of a contour s in the shape of a semicircle with radius $\sim\frac{1}{2}d_{1,2}$ in the plane $z=0$, which bounds the core of the MD and is directed along the external normal to the contour; $\tilde p^{(1,2)} = \mp 4\lambda a M l_z d_{1,2}(\pi B)^{-1}(H_\parallel - H_{\parallel s})$; the angle α_H is formed by the radius vector passing from the center of the MD core to a given point on the contour and the y axis; $\cos\alpha_H = [1 + (dy/dx)^2]^{-1/2}$; and $H_{\parallel s}$ is the field corresponding to the equilibrium period of the DS. When the other conditions are the same the force $F_{2y}^{(1)}$, for example, pulls an MD of type 1 into the sample, while an MD of type 2 is expelled from the sample under the action of the force $F_{2y}^{(2)}$. For $|F_{2y}^{(2)}| > |F_{1y}|$ a change in the MD type entails a change in the direction of motion.

The braking force $\mathbf{F}^{\mathrm{br}}$ is caused by processes in which the MD interacts with magnons and phonons, as well as with defects in the magnetic and crystallographic structure. However, if the velocity of MD motion $\mathbf{V}\parallel\mathbf{e}_y$ is small (i.e., $V_y \ll v_m$ where $v_m = (C_y/\rho)^{1/2}$), then the braking force may be determined phenomenologically without specifying the mechanism which leads to its appearance. The equation for the free ("above-barrier") motion of a dislocation has the form[22]

$$\omega\nabla_i u^{(k)}+k_i v_x^{(k)}=iI_i^{(s)},$$
$$(\rho\omega+i\Gamma)v_x^{(k)}+\sigma_x^{(k)}k_x+\sigma_y^{(k)}k_y=0. \tag{29}$$

Here $u^{(k)}$, $\sigma_x^{(k)}$, $\sigma_y^{(k)}$, and $\mathbf{v}^{(k)}$ are the Fourier components of u, σ_x, σ_y, and **v**, where $\mathbf{v} = \mathbf{v}(\mathbf{r}_\perp, t)$ is the velocity of motion of elements of the medium far from the dislocation, and $I_i^{(s)}$ is the symmetric part of the incommensurateness tensor: $I_i^{(s)} = e_{ik} v_k B \exp(-i\omega t)$; e_{ik} is the totally antisymmetric tensor of second rank; and $\omega = \mathbf{kv}$. Converting the system of Eqs. (29) to the form

$$[\omega(\rho\omega+i\Gamma)-C_x^{*}k_x^2]\sigma_x^{(k)}-C_x^{*}k_xk_y\sigma_y^{(k)}=i(\rho\omega+i\Gamma)C_x^{*}BV,$$

$$(C_y^{*}+C_3k_y^2k^{-2})k_xk_y\sigma_x^{(k)}-[\omega(\rho\omega+i\Gamma)$$

$$-(C_y^{*}+C_3k_y^2k^{-2})k_y^2]\sigma_y^{(k)}=0, \qquad (30)$$

we find the component σ_x of the stress field generated by a moving dislocation which, at time $t=0$ occupies the position $y=0$, $x=0$, and which makes up an additional stripe domain extending the direction $y>0$. As a result we obtain

$$\sigma_x(\mathbf{r}_\perp,t)=\frac{B}{4\pi^2}\int d\mathbf{k}_\perp\frac{\Delta_x(\mathbf{k}_\perp)}{\Delta(\mathbf{k}_\perp)}\exp[i(k_xx+k_y\tilde{y})], \qquad (31)$$

where

$$\Delta_x(\mathbf{k}_\perp)=V[\Gamma C_x^2k_x^2+k_y^2(\eta_x\tilde{C}_y^2k_y^2+\eta_yC_x^2k_x^2)]+iC_x(\tilde{C}_y+C_3k_y^2k^{-2})(C_xk_x^2+\tilde{C}_yk_y^2+C_3k_y^4k^{-2})k_y,$$

$$\Delta(\mathbf{k}_\perp)=(C_xk_x^2+\tilde{C}_yk_y^2+C_3k_y^4k^{-2})^2+V^2k_y^2(\Gamma+\eta_xk_x^2+\eta_yk_y^2)^2,$$

$$\tilde{C}_y=C_y-\rho V^2,\quad \tilde{y}=y-Vt.$$

The force of viscous friction acting on a dislocation equals

$$F_y^{\text{тр}}=-B\sigma_x(0)=-\frac{\Gamma VB^2}{4\pi}\left(\frac{C_x}{2C_y}\right)^{1/2}$$

$$\times\int\limits_{\xi_{min}}^{\xi_{max}}d\xi\frac{1}{[b+(ac)^{1/2}]^{1/2}}\left[\frac{1+\gamma_y\xi^2}{a^{1/2}}+\frac{\gamma_x\xi^4}{c^{1/2}}\right], \qquad (32)$$

where

$$a=1+\gamma\gamma_x^2\xi^2,\quad b=\xi^2+\xi^4+\gamma\gamma_x\xi^2(1+\gamma_y\xi^2),$$

$$c=\xi^4(1+\xi^2)^2+\gamma\xi^2(1+\gamma_y\xi^2)^2,$$

$$\gamma=C_3(\Gamma V)^2k^{-2}C_y^{-3},$$

$$\gamma_x=\eta_x(C_yk)^2(C_3C_x\Gamma)^{-1},\quad \gamma_y=\eta_yC_yk^2(C_3\Gamma)^{-1},$$

$$\xi_{min}={}^1/_2Bl_y^{-1}(C_3/C_y)^{1/2},\quad \xi_{max}={}^1/_2(C_3/C_y)^{1/2}.$$

If the MD is found at a large distance from the boundaries of the film and from other dislocations, then we have $|F_{1y}+F_{2y}|\gg|F_y^{\text{int}}|$ and the MD moves with a velocity $V=V(p+\tilde{p})$ determined from the equation $F_y^{\text{br}}=-(F_{1y}+F_{2y})$. In the general case the function $V=V(p+\tilde{p})$ can be determined by numerical methods. Let us introduce approximate expressions for the velocity of an MD in certain special cases. For $\Gamma\gg\eta_ik_c^2$, i.e., for $\lambda_r+\lambda_ek_c^2\gg\lambda_ek_c^2$, the time dependence of the elastic moduli can be neglected by setting $\eta_i=0$; then both types of MD move with a velocity which for small values of $V_y\ll\pi C_y(l_y\Gamma)^{-1}$ is determined by the expression

$$V_{1,2}=G(p+\tilde{p}_{1,2}-p_c)B, \qquad (33)$$

where

$$G=8\pi\{B^2\Gamma C_x^{1/2}C_y^{-1/2}\ln[2l_yB^{-1}([1+C_3(4C_y)^{-1}]^{1/2}+1)^{-1}]\}^{-1}$$

is the mobility of the MD, and p_c is the coercive magnetic pressure for the motion of the MD. The mobility G depends on the magnetic pressure p because the parameters C_i are functions of p. For $\pi C_y/l_y\Gamma\ll V_y\ll v_m$ the velocity of the MD is found from the approximate equation

$$V_{1,2}=4\pi C_y^{1/2}C_x^{-1/2}\{[1+\ln(2\gamma^{-1/4})]\Gamma B\}^{-1}(p+\tilde{p}_{1,2}-p_c). \qquad (34)$$

In films with large anisotropy ($\beta_u\to\infty$), the expressions for the elastic moduli simplify when $H_\parallel=0$ ($C_y\approx p$, $C_x\approx 4C_0$, $C_3\approx C_0$); neglecting the coercive force, we find from (34) that $V\approx\pi p^{3/2}C_0^{1/2}(\Gamma B)^{-1}$.

The solution to the system of Eqs. (30) allows us to calculate the components of the tensors σ_i and ∇_iu for the stress and strain fields generated by a moving MD. For $x\ll x_c$ and $\tilde{y}\ll y_c$ the components of the magnetic strain tensor ∇_iu are defined as derivatives of the quantities u [see (25b)] with the subsequent replacement $y\to\tilde{y}$. For $x\gg x_c$ or $\tilde{y}\gg y_c$ the term $C_0k_y^4k_c^{-2}$ and the terms proportional to η_i in the denominator of the expressions under the integral sign can be neglected and the expression for the component ∇_yu of the strain tensor acquire the form

$$\nabla_yu=\frac{B\Gamma Vx}{4\pi\tilde{Y}(C_xC_y)^{1/2}}K_1\left(\frac{\Gamma V\tilde{Y}}{2C_y}\right)\exp\left(-\frac{\Gamma V\tilde{Y}}{2C_y}\right), \qquad (35)$$

where $\tilde{Y}=(\tilde{y}^2+x^2C_yC_x^{-1})^{1/2}$; $K_1(x)$ is the modified Bessel function of the second kind.

For $|x|\gg|\tilde{y}|$ and $\tilde{Y}\gg 2C_y(\Gamma V)^{-1}$ we obtain

$$\nabla_yu=\frac{B}{4\pi^{1/2}}\left(\frac{\Gamma V}{C_x}\right)^{1/2}\frac{x}{|\tilde{y}|^{3/2}}\exp\left[-\frac{\Gamma V}{2C_y}(\tilde{y}+|\tilde{y}|)\right], \qquad (36)$$

i.e., the components of the strain tensor $\nabla_iu(\mathbf{r}_\perp,t)$ fall off exponentially at large distances from the dislocation core in the direction of motion of the latter, while in the opposite direction they fall off as a power law. If $\tilde{Y}\ll 2C_y(\Gamma V)^{-1}$, then the function $\nabla_iu(\mathbf{r}_\perp,t)$ equals

$$\nabla_yu=xB[2\pi(x^2+y^2C_xC_y^{-1})]^{-1}, \qquad (37)$$

which is analogous to the case of a nonmoving dislocation, as follows from Eq. (25a). The change in the decay law of the strain generated by a moving MD [see (36)] compared with normal dislocations arises from the specifics of the equation of motion (22).

Near the boundary of the film or near defects of the crystalline and magnetic structure, dislocations move nonuniformly; the energy released in this case is radiated in the form of spin waves (compare with Refs. 21–24). The departure of the dislocation from the lateral surface of the film is also accompanied by radiation of spin waves; this is because we can treat this process as one in which dislocations annihilate with their mirror images, accompanied by the release of an energy

$$\mathcal{U}_a\approx\rho(BV)^2(4\pi)^{-1}\ln(l_yB^{-1}).$$

1.3. Kinetics of the transition of a metastable DS with magnetic dislocations to the equilibrium state

The concentration of dislocation pairs of types 1 and 2 of length y in a metastable DS ($d<d_0$) equals

$$N_{1,2}(y)=(1/S)\exp\left[-\mathscr{F}_{1,2}(y)/T\right], \tag{38}$$

where $\mathscr{F}_{1,2}(y)$ is the thermodynamic potential per unit volume (for a single pair of dislocations) of the strained DS, which equals

$$\mathscr{F}_{1,2}(y)=2\mathscr{F}^{\text{nuc}}(p+\tilde{p}_{1,2})By+\mathscr{F}^{\text{int}}. \tag{39}$$

Here $\mathscr{F}^{\text{nuc}}\sim(C_xC_y)^{1/2}B^2$ is the thermodynamic potential for the formation of a dislocation core; $\mathscr{F}^{\text{int}}(y)=T^*\ln(|y|/B)$ is the energy of interaction between dislocations in a single pair; $S=B^2$; and $T^*=(C_xC_y)^{1/2}B^2/2\pi$.

The dependence of the potential per unit volume of an MD pair on the distance between the constituent MD is shown schematically in Fig. 1. For $y=y_b$ the attractive force F^{int} is balanced by the forces F_{1y} and F_{2y}, which arise from the compressive stress σ, the action of the magnetic field $H_\parallel$ in the nonequilibrium stripe DS, and the force of dislocation repulsion F_{3y} originating from exchange and magnetostatics (both MD of a pair have the same magnetic charge). The quantities y_a and y_b are comparable in magnitude to B; therefore, it is necessary to use numerical calculations to determine them. For $y\gg b$ the forces satisfy $F_{3y}\ll F^{\text{int}}$, $F_{1y}+F_{2y}$, and we need not take them into account. For either a type 1 or a type 2 MD pair, the critical distance between MD for which the pair is stable against dissociation equals

$$y_{\text{cr}_{1,2}}=B^{-1}(p+\tilde{p}_{1,2})^{-1}T^*. \tag{40}$$

For $y>y_{\text{cr}}$ the force $F_{1y}+F_{2y}>F_y^{\text{int}}$ and the distance between dislocations increases; for $y<y_{\text{cr}}$ the potential per unit volume of a dislocation pair $\mathscr{F}(y_{\text{cr}})=\mathscr{F}_{\text{cr}}$ is a maximum. From this we see that the problem of the kinetics of the transition of a metastable DS to the equilibrium configuration is analogous to the corresponding problem in the classical theory of nucleation.[25] A similar situation arises with regard to the motion of kinks during the glide of dislocations[24] and the nucleation of soliton lines in an incommensurate phase.[26]

According to Ref. 24, we can determine the rate of formation of dislocations per unit length of a solitary stripe domain:

$$J=DN_{1,2}(y_{\text{cr}})y_T^{-1}, \tag{41}$$

where D is the coefficient of diffusion of a solitary dislocation, which is connected with its mobility by the Einstein relation $D=GT$, and y_T is the distance (measured from the position y_{cr}) at which the energy of a dislocation pair equals $\mathscr{F}_{\text{cr}}-T$. A pair separated by a distance $y>y_{\text{cr}}+y_T$ can be considered as having surmounted an activation barrier $\mathscr{F}_{\text{cr}}$, while pairs with $y_{\text{cr}}<y<y_{\text{cr}}+y_T$ are partially coupled and remain at the position $y=y_b$ and are partially dissociated. The value y_T is determined from the equation

$$-{}^1/_2(y_T)^2\nabla_y^2\mathscr{F}(y)\,|_{y=y_{\text{cr}}}=T,$$

whose solution gives

$$y_{T_{1,2}}=(2TT^*)^{1/2}/[B(p+\tilde{p}_{1,2})].$$

From the relations $JL\tau'_{1,2}\sim1$ and $V\tau'_{1,2}\sim L$ we can estimate the average lifetime of pairs of type 1 and type 2

$$\tau'_{1,2}\sim(V_{1,2}J_{1,2})^{-1/2}=G^{-1}[(p+\tilde{p}_{1,2})(p+\tilde{p}_{1,2}-p_c)]^{-1/2}$$

$$\times(2T^{-1}T^*)^{1/4}\exp[\mathscr{F}_{\text{cr}}(2T)^{-1}], \tag{42}$$

where L is the mean free path of a solitary MD along the y axis. A dislocation can annihilate with the MD of another pair; it can also link up behind dislocations in neighboring domains or with defects in the crystal lattice. These processes are characterized by mean free paths L_1, L_2, and L_3 respectively; therefore,

$$L\approx\left(\sum_{n=1}^{3}L_i^{-1}\right)^{-1}.$$

The relation (42) is valid for $L\ll l_y$. In the other limiting case $L\gg l_y$, we have $\tau''_{1,2}\sim(J_{1,2}l_y)^{-1}$ and $\tau'''_{1,2}\sim l_yV_{1,2}^{-1}$ determine the average times necessary to create a pair of dislocations of types 1 and 2 and the time necessary to annihilate an MD at the boundaries of the sample, respectively. Therefore, the average time during which a dislocation pair exists is $\tau=\max\{\tau''_{1,2},\tau'''_{1,2}\}$. The estimates obtained here for the lifetime τ are valid if the stress p changes insignificantly in the process of MD motion. However, as new pairs of MD appear and are annihilated the stress p in the system decreases, while the critical size of an MD pair y_{cr} increases. The final stage of the process is analogous to the coalescence stage in the theory of nucleation.[25,26] According to Refs. 25 and 26, in order to determine the average size of a pair $\bar{y}(t)=y_{\text{cr}}(t)$, we obtain a system of equations

$$\frac{dy}{dt}=\frac{2D}{T}[B\Delta(t)-T^*y^{-1}]=\frac{2DT^*}{T}[(y_{\text{cr}})^{-1}-y^{-1}], \tag{43}$$

$$P(t)+\Delta(t)=\text{const}=p,\quad \frac{\partial f}{\partial t}+\frac{\partial(fV_y)}{\partial y}=0, \tag{44}$$

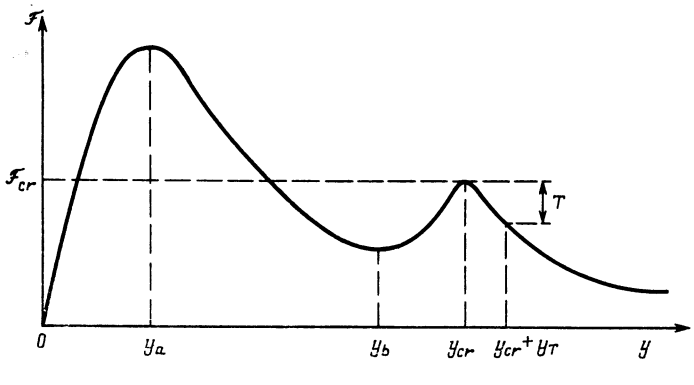

FIG. 1. Dependence of the thermodynamic potential of a bound dislocation pair on the distance between the cores.

where $P = C_x \Sigma_i B y_i$ is the stress relieved within the stripe DS due to the formation of dislocation pairs, which is connected with the dislocation pair distribution function $f(y,t)$ by the relation $P(t) = \int y f(y,t) dy$; y_i is the length of the ith pair of dislocations, $\Delta(t) = [B y_{cr}(t)]^{-1} T^*$ is the remaining stress in the DS arising from the stress p; and $V_y = dy/dt$. We find from Eqs. (43) and (44) that the average size of a pair depends on time according to the law $\bar{y}(t) = y_{cr}(t) \propto t^{1/2}$.

The theory we have developed can also be used to analyze the behavior of DS during phase transitions that are close to the Curie point. For this, in the free energy of the system we must add a term

$$\Delta\mathcal{F} = -{}^1/_2 \xi M^2 + {}^1/_4 \tilde{\delta} M^4 M_0^{-2},$$

where $\xi = \tilde{\delta} \tilde{M}^2(T) M_0^{-2}$; $\tilde{M}(T)$ is the equilibrium value of the magnetization in an unbounded medium and $\tilde{\delta}$ is the uniform exchange interaction constant; in the equations given in paragraphs 1.1–1.3 we make the replacements

$$h_\perp \to T\xi_0', \quad \beta_u \to 2\tilde{\delta}, \quad \mu \to 1, \quad C_0 \to {}^1/_2 \varkappa^2 (\lambda a)^2 l_z M_0^2,$$
$$\mu g^2 \beta_u \to (\tilde{\omega}_0/M_0)^2, \quad \mu \beta_u \omega_0^2 \to \tilde{\omega}_0^2,$$
$$\tilde{C}_y \to C_y, \quad \Gamma \to \rho \tilde{\omega}_0 (\lambda_r + \lambda_e k^2).$$

Here $\tilde{\omega}_0$ is the characteristic frequency in the Landau-Khalatnikov equation

$$\tilde{\omega}_0^{-1} (\lambda_r + \lambda_e \nabla^2) \dot{\mathbf{M}} = -\delta\mathcal{F}/\delta\mathbf{M}. \qquad (45)$$

1.4. Generalization of the theory to the region of magnetic fields $H_\perp \ll \beta_u M$

In order to generalize the theory to the case of biaxial ferromagnetic films with DS located in a magnetic field $|H_\perp| \ll \beta_u M$ and $|H_\parallel| \sim 4\pi M$, we must add a term $\Delta\mathcal{F} = \frac{1}{2}\beta_x M_x^2$ to the free energy density, where β_x is the constant of "rhombic" anisotropy ($0 < \beta_x \ll \beta_u$) and then use the results of Refs. 6 and 27, where the ground state and spin-wave spectra of such films, i.e., with stripe DS, were investigated for $\beta_x = 0$ and $H_\perp = 0$. According to Ref. 27, we can show that the "elastic" part of the free energy $\mathcal{U}$ is given by Eqs. (17a) and (17b), where

$$C_x = 4\pi k l_z^2 M^2 (W_{\nu_1\nu_1} - W^2_{\nu_1\nu_2} W^{-1}_{\nu_2\nu_2}),$$
$$C_y = 4\pi l_z M^2 [W_{\nu_1} + (1 + \beta_x \beta_u^{-1} + \pi H_\perp \beta_u^{-1} M^{-1}) l_w l_z^{-1}],$$
$$C_3 = 12 l_z M^2 \nu_1^{-1} \sum_{n=1}^{\infty} n^{-5} [1 - (1 + \nu_1 n + \nu_1 n^2/3) \times \exp(-\nu_1 n)] \sin^2(n\nu_2/2),$$
$$\Gamma = 2\lambda_r M (1 + \zeta_e)(g\Delta)^{-1}, \quad \rho = \bar{m}, \quad \eta_i = 0,$$
$$p = C_x (1 - d d_0^{-1}), \quad \tilde{p}_{1,2} = \mp 2 M H_z B^{-1} d_{1,2} l_z, \qquad (46)$$
$$W_{\nu_i} = \partial W / \partial \nu_i, \quad W_{\nu_i \nu_k} = \partial^2 W / \partial \nu_i \partial \nu_k,$$

$$W = \nu_2 [H_\parallel (4\pi M)^{-1} - 1] + 2(\pi \nu_1)^{-1} \sum_{n=-\infty}^{\infty} |n|^{-3} [1 - \exp(-\nu_1 |n|)] \sin^2(\nu_2 n/2) - {}^1/_4 H_\parallel / M + \pi/2 + \nu_1 l_w l_z^{-1} [1 + \beta_x \beta_u^{-1} + \pi H_\perp (\beta_u M^{-1})],$$

$$\zeta_e = \lambda_e (3\lambda_r \Delta^2)^{-1};$$

here $l_w = \sigma_0 (4\pi M)^{-1}$ is a characteristic length in the material; $\Delta = \Delta_0 [1 + \beta_x \beta_u^{-1} + \pi H_\perp (2\beta_u M)^{-1}]$; $\bar{m}$, σ_0, and Δ_0 are the effective mass, energy density and width of the DB respectively,[28] and we have written $\nu_1 = k l_z$ and $\nu_2 = k d_2$. The width of a domain d_2, in the center of which $\mathbf{M} = -M\mathbf{e}_z$, is calculated from the equation $W_{\nu_2} = 0$, while the equilibrium period d_0 of the DS is determined from the solution to the equation $W_{\nu_1} = 0$. In deriving Eq. (46) we have assumed that $\Delta \ll l_z$, while σ does not depend on the curvature of the domain wall.

From this we see that the results obtained above, taking into account the transformation specified by Eq. (46) can be used to analyze the behavior of an MD in a stripe DS. The distribution of $\mathbf{M}$ around the core of the MD, the character of its motion and the kinetics of reconstruction of the DS can differ from those discussed above due to the formation of vertical Bloch lines which accompanies the disruption of domains. In a stripe DS with a single-polarity domain wall, in the course of the destruction of a domain the number of vertical Bloch lines that appear in each of the MD is odd, i.e., $n = 2j + 1$, while for a DS with different polarity domain wall this number is even: $n = 2j$. In order to take into account the influence of the vertical Bloch lines, we must add an interaction force F_L for the vertical Bloch lines to the right side of the MD equation of motion (26). For example, in a DS with a single-polarity domain wall, for $n = 1$ a vertical Bloch line in a dislocation pair is attracted with a force $F_L = -8\pi^2 \Delta^2 l_z M^2 y^{-1}$, where y is the spacing between the MD of the pair.[28] In addition, it is necessary to take into account the change in the attenuation constant Γ caused by the vertical Bloch lines in the expression (32) for the forces of viscous friction acting on the MD, and also in Eqs. (33) and (34) for the velocity and mobility of the MD. To do this, in Eq. (46) we must set[17]

$$\Gamma = 2M\lambda_r \{1 + \zeta_e + 2\Delta n [1 + {}^7/_2 \zeta_e] (\pi d Q_u^{1/2})^{-1}\} (g\Delta)^{-1}, \qquad (47)$$

where $Q_u = \beta_u / 4\pi$ is the Q-factor of the material. To the potential $\mathcal{F}(y)$ [see Eq. (39)] it is necessary to add a term $\mathcal{F}_L(y) = 2n\mathcal{F}_L^{(0)} + \mathcal{F}_L^{B3}$, where $\mathcal{F}_L^{(0)} = 4\alpha l_z M^2 Q_u^{-1/2}$ is the energy of an isolated vertical Bloch line, and $\mathcal{F}_L^{\text{int}}$ is the vertical Bloch line interaction energy. For $n = 1$ in a DS with a single-polarity domain wall

$$\mathcal{F}_L^{\text{int}} = 8\pi^2 \Delta^2 l_z M^2 \ln(y\Lambda^{-1}),$$

where $\Lambda = (\alpha/4\pi)^{1/2}$ is the width of a solitary vertical Bloch lines.[28]

In order to estimate the position of the points y_a and y_b in Fig. 1 (for the case where $y_{a,b} \ll d$ and we can neglect long-range interactions by the "elastic" fields around the MB), we will use a model which approximates the dislocation dipole by a rectangular break in a stripe domain of width d_1 or d_2 on the portion of length y. Then the increase in the potential $\mathcal{F}_{1,2}(y)$ due to the formation of a pair for $d \ll l_z$ and $H_\parallel < 4\pi M$ equals[29]

$$\mathcal{F}_{1,2}(y) = 8\pi M^2 d_{1,2} l_z \{ y [1 \pm (4\pi M)^{-1} H_\parallel + l_w (1 - y d_{1,2}^{-1})]$$
$$+ 2\pi^{-1} [y \operatorname{arctg}(y l_z^{-1})$$
$$+ 4 l_z^{-1} y^2 \ln(1 + l_z^2 y^{-2}) - {}^1/_4 l_z \ln(1 + l_z^{-2} y^2)$$

$$-2y(\mathrm{arctg}(d_{1,2}l_z^{-1})$$
$$+{}^1/_4\,d_{1,2}l_z^{-1}\ln(1+l_z^2d_{1,2}^{-2})$$
$$-{}^1/_4\,l_zd_{1,2}^{-1}\ln(1+d_{1,2}^2l_z^{-2}))]\}+\mathcal{F}_L(y).$$

The quantities y_a and y_b are determined by numerical calculation from the equation

$$\partial\mathcal{F}_L(y)/\partial y+8\pi M^2d_{1,2}l_z\{1\pm(4\pi M)^{-1}H_\parallel-l_wd_{1,2}^{-1}$$
$$+2\pi^{-1}[\mathrm{arctg}(yl_z^{-1})$$
$$+{}^1/_2\,l_z^{-1}y\ln(1+l_z^2y^{-2})$$
$$-2\,\mathrm{arctg}(d_{1,2}l_z^{-1})-{}^1/_2d_{1,2}l_z^{-1}\ln(1+l_z^2d_{1,2}^{-2})$$
$$+{}^1/_2\,l_zd_{1,2}^{-1}\ln(1+d_{1,2}^2l_z^{-2})]\}=0.$$

The quantities $d_{1,2}(H_\parallel)$ can be taken to be the same as for the ordered stripe DS; the form of the function $\mathcal{F}_L(y)$ is assumed to be known.

The structure of the core can turn out to have a strong influence on the process of MD motion. Thus, for example, if the "tip" of a stripe domain has the form of a wedge with an angle φ at the vertex, and if during the motion of the domain it expands with a "transverse" velocity $v_\perp$, then the velocity of MD motion will equal $\tilde{V}=V+v_\perp[\sin(\varphi/2)]^{-1}$, where the value V is determined by Eq. (33). If the angle φ is small, then $\tilde{V}$ can considerably exceed V. It is possible that this, too, has been observed by the authors of Ref. 30.

Let us discuss the effect of a field $H_\perp$ on the results we have obtained. Since increasing $H_\perp$ results in a decrease in the quantity d_0 (for $H_\parallel=\mathrm{const}$), the stripe DS which forms for a certain value of $H_\parallel$ with $H_\perp=0$ will become unstable against the appearance of a sinusoidal modulation of the profile of the domain wall as $H_\perp$ increases. If the field $\mathbf{H}_\perp$ is directed along the domain wall, then for certain initial conditions it is possible to have repolarization of the Bloch domain wall if the direction of the vector $\mathbf{M}$ at the center of the domain wall is antiparallel to $\mathbf{H}_\perp$.[28] This repolarization can occur as a result of the birth of vertical Bloch line pairs and subsequent attraction of these pairs towards the edges of the film. An investigation of the kinetics of repolarization of Bloch domain wall can be carried out in the same way as in paragraph 1.3. The velocity of a solitary vertical Bloch line under the action of a field $H_\perp$ is expressed in terms of the vertical Bloch line mobility

$$\mu_L=\pi g\Delta_0Q_u^{1/2}\{2\lambda_r[1+3\nu+{}^5/_3\nu Q_u^{-1}]\}^{-1},$$

where $\nu=\lambda_e(3\Delta_0^2\lambda_r)^{-1}$, in the following way:[17]

$$V_L=\mu_L(H_\perp-H_\perp^{(c)})\equiv G_L(F_L^{(1)}-F_L^{(c)}). \tag{48}$$

Here $F_L^{(1)}=2\pi\Delta_0l_zMH_\perp$ is the force exerted on the vertical Bloch line by the field $H_\perp$; $F_L^{(c)}=2\pi\Delta_0l_zMH_\perp^{(c)}$ is the coercive force; and $H_\perp^{(c)}$ is the coercive field. Using Eq. (48) we find the diffusion coefficient of the vertical Bloch line

$$D_L=G_LT=\mu_LT(2\pi\Delta l_zM)^{-1}.$$

The concentration $N(y)$ of vertical Bloch line pairs of length y and the rate J of formation of vertical Bloch line pairs per unit length of the domain wall will be determined by relations (38) and (41), respectively, where

$$S=d\Lambda,\ \mathcal{F}_L(y)=2\mathcal{F}_L^{(0)}+\mathcal{F}_L^{\mathrm{int}}-\mathcal{F}_L^{(1)}y,$$
$$\mathcal{F}_L^{\mathrm{int}}=8\pi^2\Delta_0^2l_zM^2\ln(y\Lambda^{-1}).$$

The function $\mathcal{F}_L(y)$ is analogous to that shown in Fig. 1; the position of the points y_a and y_b is determined by numerical calculations,[29,31] while y_{cr} and y_T are determined by the expressions

$$y_{cr}=4\pi\Delta_0H_\perp^{-1}M,\ y_T=[4T/l_zH_\perp^2]^{1/2}.$$

After this, from the relations $JL\tau'\sim1$ and $v_L\tau'\sim L$, we can estimate the average lifetime of a vertical Bloch line (for $L\ll l_y$), which comes to

$$\tau'\sim(v_LJ)^{-1/2}\sim D_L^{-1}(F_L^{(1)}-F_L^{(c)})^{1/2}(4l_z^{-1}H_\perp^{-2}d^2\Lambda^2T^3)^{1/4}$$
$$\times\exp[\mathcal{F}_{cr}/2T],$$

where L is the mean distance between the vertical Bloch line (along the domain wall) in a pair until it annihilates with other vertical Bloch lines. For $L\gg l_y$ the average times required to create a vertical Bloch line pair and to annihilate it at the boundaries of the film are determined by the quantities $\tau''\sim(Jl_y)^{-1}$ and $\tau'''\sim l_yv_L^{-1}$, respectively; the average time during which the vertical Bloch line pair exists is $\tau=\max\{\tau'',\tau'''\}$. In contrast to the case where the period of the DS changes because of formation of MD pairs, the force $F_L^{(1)}$ acting on a vertical Bloch line does not change in the course of repolarization of the domain wall.

2. EXPERIMENT AND DISCUSSION OF RESULTS

Our experimental investigation of the behavior of MD in a regular stripe DS was carried out in quasi-uniaxial epitaxial films of the ferrite-garnet compound $(YGdYbBi)_3(FeAl)_5O_{12}$, which was grown on a nonmagnetic substrate made of $Gd_3Ga_5O_{12}$ with (111) orientation. The intrinsic DS of such films usually is labyrinthine; a regular stripe DS is created with the help of the following method. Using a magnetic head, a harmonic oscillation was recorded on a magnetic strip whose spatial wavelength λ_0 was close to the equilibrium period d_0 of the labyrinthine DS of the film at room temperature. Then the strip with its record was placed in close contact with the magnetic film and was pulled through along the entire surface of the film in such a way that the direction of motion of the strip coincided with the direction of the "dashes" of the record. The procedure described allowed us to form an almost ideally regular stripe DS which was preserved after withdrawal of the magnetic strip. By changing the pressure on the strip in the process of forming the induced DS in the film, we were able to vary the density of MD from zero to 10^2 cm^{-2} and larger, obtaining both single MD and dislocation dipoles of various types (Figs. 2 and 3).

If MD were present in the original DS, then the average period $\bar{d}$ of the DS can be changed within certain limits because of the motion of the latter. In a defectless film of finite

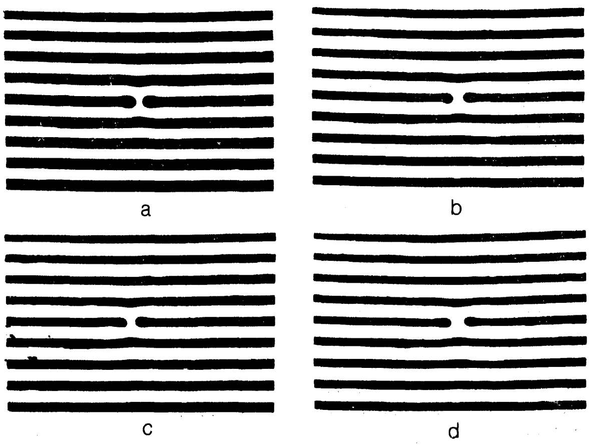

FIG. 2. Behavior of an isolated dislocation dipole in the field $H_\parallel$ for film No. 1 ($d_0 = 11\ \mu$m): a—0 Oe; b—6.5 Oe; c—10.5 Oe; d—13.2 Oe.

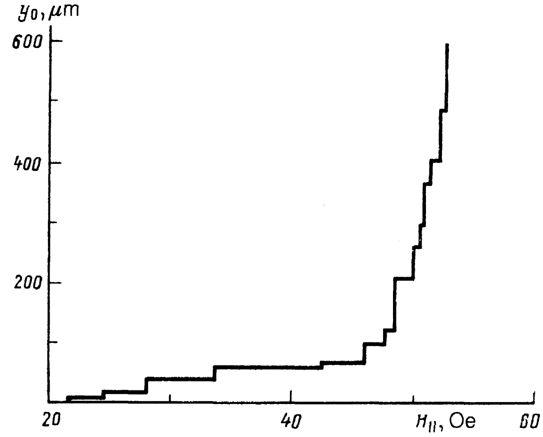

FIG. 4. Dependence of the position of the core of a single dislocation on field $H_\parallel$ for film No. 2 ($d_0 = 6\ \mu$m).

size, if the values of **H** and T are fixed, the MD are spaced in such a way that the average period $\bar{d}$ will equal (if this is possible) the thermodynamic-equilibrium value $d_0(\mathbf{H}, T)$; changing **H** or T causes a shift in the noninteracting MD to new positions without hysteresis.

In actual films, because the cores have a tendency to be pinned at defects, the motion of MD takes place as a succession of jumps. This is illustrated by Fig. 4 where we show how the position of the core of a solitary dislocation y_0 depends on the magnetic field $H_\parallel$ for film No. 2. The function $y_0(H_\parallel)$ resembles $d_0(H_\parallel)$, since for fixed increments $|\Delta H_\parallel|$ the tensile stresses that act on the MD core are proportional to the difference $d_0(H_\parallel + \Delta H_\parallel) - d_0(H_\parallel)$ which increases as $|\Delta H_\parallel|$ increases. It follows from Fig. 4 that the coercive force for cores lies within the limits 1–10 Oe.

Photographs of the various dislocation dipoles and solitary MD in the DS for film No. 1 are shown in Figs. 2 and 3,

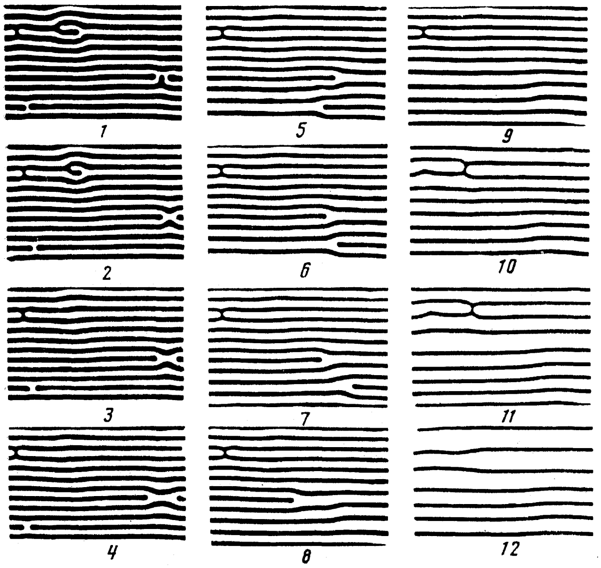

FIG. 3. Behavior of solitary and bound dislocations of various types for film No. 1 in a field $H_\parallel$ (in Oe): 0 *(1)*, 5.5 *(2)*, 8.8 *(3)*, 11.0 *(4)*, 12.7 *(5)*, 13.2 *(6)*, 14.3 *(7)*, 16.1 *(8)*, 16.3 *(9)*, 18.7 *(10)*, 22.0 *(11)*, 25.3 *(12)*.

which correspond to various original MD densities. It is clear that dislocations can be bound in pairs not only when they are located in a single stripe domain [e.g., Fig. 2 for dipole No. 1; the left-hand lower dipole (No. 2) in Fig. 3, *1–4*; the left-hand upper dipole (No. 3) in Fig. 3, *1–11*; the right-hand lower dipole (No. 4) in Fig. 3, *1–4*] but also in cases where the MD are separated from each other by one or several "unbroken" stripe domains [the right-hand lower dipole (No. 5) in Fig. 3, *5–7*]. The behavior of the dipoles in a magnetic field is illustrated in Fig. 5 where we show $H_\parallel$ as a function of the distance y between vertices of the dislocation cores in a pair.

For a pair in whose cores $\mathbf{M}\cdot\mathbf{H} < 0$ holds, a monotonic increase in y is characteristic (if the dipole was formed in a field $H_\parallel = 0$) as $H_\parallel$ increases (curves *1, 2, 4* in Fig. 5; the labels of the curves correspond to enumeration of the dipoles shown in Figs. 2 and 3); for a certain value of $H_\parallel$ dissociation begins. If the dipoles are formed in a field $H_\parallel \neq 0$ (dipole No. 5 in Fig. 3, *5–7*, which was formed during the dissociation of dipoles No. 2 and No. 4 in a field 11 Oe $< H_\parallel <$ 12.7 Oe), then y initially passes through a minimum and then begins to increase rapidly and monotonically (curve *5* in Fig. 5).

For a pair with cores in which $\mathbf{M}\cdot\mathbf{H} > 0$, the distance y initially decreases monotonically as $H_\parallel$ increases, and then, after bulk dissociation and "dispersal" of the dipoles formed by MD with $\mathbf{M}\cdot\mathbf{H} < 0$, the distance y increases slightly due to an increase in the average period of the DS; after this the decrease continues until the pair annihilate (curve *3* in Fig. 5).

From this we see that dislocation dipoles can be divided into two groups, depending on the direction of **M** in the cores relative to the field $\mathbf{H}_\parallel$. Dipoles in the first group are characterized by an increase in y with increasing $H_\parallel$ and dissociation at a certain critical value of $H_\parallel = H_\parallel^{(1)}$; in dipoles of the second group, y decreases with increasing $H_\parallel$ and for $H_\parallel = H_\parallel^{(2)}$ annihilation of a pair begins, where $|H_\parallel^{(2)}| > |H_\parallel^{(1)}|$. As the direction of the field $\mathbf{H}_\parallel$ changes, the dipoles of the first group begin to behave like dipoles of the second group and conversely.

Solitary dislocations and bound pairs of similar MD do not exhaust the set of possible magnetic defects in a stripe DS. Thus, for example, a "finger" of a neighboring stripe

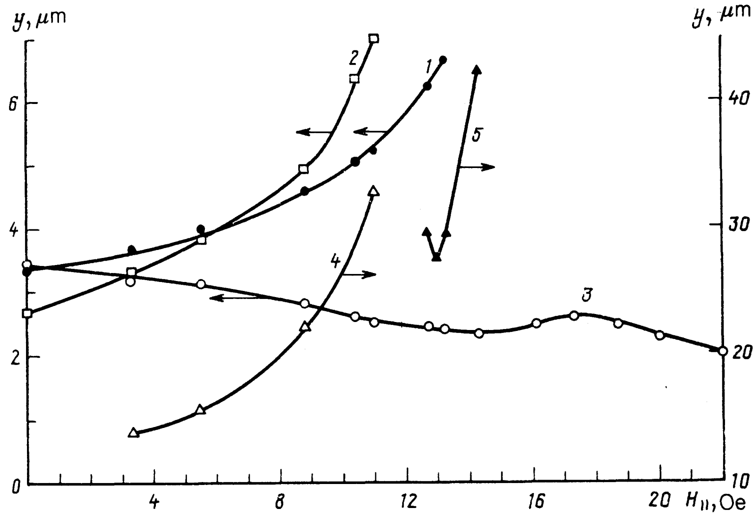

FIG. 5. Dependence of the distance between cores of bound dislocation pairs of various types on the field $H_{\parallel}$ for film No. 1.

domain with the same direction of the vector **M** as in the cores (Fig. 3, *1*) can be "sucked" into the gap between the cores of a dislocation dipole; MD with the vector **M** oriented antiparallel in their cores and located in neighboring stripe domains can bind in pairs (Fig. 3, *1, 2*). Such defects, however, are "self-healing" in relatively weak magnetic fields $H_{\parallel}$.

A comparison of the results of our experiments with the conclusions of theory indicates good qualitative agreement between them. The decisive role of MD in the processes by which a DS reconstructs as the external parameters vary is confirmed. We have observed "nonreciprocal" behavior of dislocation pairs in a field $H_{\parallel}$: for $\mathbf{M}\uparrow\uparrow\mathbf{H}$, where **M** is the magnetization vector in the cores of the dislocations, as the field intensity $H_{\parallel}$ increases we observe pair annihilation; for $\mathbf{M}\uparrow\downarrow\mathbf{H}$, we observe dissociation.

The question of how an MD behaves in a stripe DS was discussed in Ref. 4; however, the theoretical analysis carried out in Ref. 4 does not apply to actual systems, since the authors used the model of an isotropic crystal, which does not admit the existence of regular stripe DS; a stable DS, according to the classification of Ref. 9, corresponds to the "liquid-crystal" phase. The expressions given in this paper for the effective moduli of rigidity of the DS and the functional dependence of the DS period on film thickness and magnetic field are erroneous. The transition from the BKT phase (strictly speaking, it is only for this phase that the theory developed in paragraphs 1.1–1.4 is valid) to the "liquid-crystal" phase is accompanied by a massive multiplication of the MD (see Ref. 9); in this case the static and dynamic characteristics of the MD must change significantly. Equations for the transition line from the BKT phase to the "liquid-crystal" phase for an equilibrium stripe DS were obtained in Ref. 9; for the nonuniform (i.e., "compressed" or "under tension") DS this line on the $H_\perp H_\parallel$ (or $TH_\parallel$) plane shifts in the direction of lower values of $H_\perp$ (or T), since the "elastic" stresses in the region of two-dimensionality decrease the "elastic" moduli C_x and C_y. The equation for the phase transition line discussed here can be obtained from the condition that the energy expended in the formation of a single dislocation, which equals

$$\delta\mathcal{U}=(C_xC_y)^{1/2}(B^2/4\pi)\ln(r_\perp/r_0),$$

be cancelled by the entropic contribution which comes to

$$-T\delta S=-2T\ln(r_\perp/r_0);$$

see also Ref. 9.

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