Transient Petschek field-line reconnection in relativistic magnetohydrodynamics

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A transient solution of the Petschek problem of relativistic reconnection of magnetic lines of force is obtained by specifying: a) the initial parameters of a thin layer; b) the electric field along a reconnection line, considered as a function of time. It is shown that in the course of the reconnection process a current layer splits into slow shock waves which accelerate a plasma to the relativisitic Alfven velocity, compress it and heat it strongly, whereas the magnetic field intensity falls steeply. This transforms the magnetic energy into the kinetic and internal energies of the plasma, which is a particularly effective process in the case of a strong magnetic field and a cold plasma. The result is propagation, on either side from a reconnection line, of accelerated relativistic plasma bunches (known as field reversal—FR—regions) bounded by traveling slow shock waves. The FR regions following the path along a current layer transform the whole of the magnetic field in a force tube which is being reconnected and they accelerate all the plasma inside the tube. Different reconnection regimes are possible: quasisteady, similar to the variant considered by Petschek; pulsed, when the FR regions become detached from a reconnection line and move along a current layer as independent objects, etc. The FR regions transport, along a current layer, the magnetic flux being reconnected, as well as the electric field, mass, momentum, and energy. It is therefore possible to regard reconnection as a transport process typical of a highly conducting magnetized medium containing current layers. The process of reconnection may ensure a strong interaction between regions with different magnetic fluxes. The astrophysical applications of the relativistic reconnection process are considered, particularly in the case of solar bursts and flares.

1. INTRODUCTION

The theory of reconnection has arisen from the hypothesis of Giovanelli[1] about the magnetic nature of solar flares. The list of phenomena associated with an explosive-like transformation of the magnetic energy is now much longer: it includes magnetospheric substorms[2,3] and the processes in the magnetopause,[4] in cometary tails,[5] at the fronts of flare fluxes,[6] in thermonuclear devices,[7] in laboratory apparatus,[8,9] etc. There are grounds for assuming that the process of reconnection is also of importance in astrophysics[10] and this requires generalization of the theory to the relativistic case. In fact, the Alfven velocity near current layers on the Sun can reach thousands of kilometers per second,[9,10] which is only two orders of magnitude less than the velocity of light. Therefore, in the case of astrophysical objects with much stronger magnetic fields one should obviously allow for the strongly relativistic effects. We shall generalize the Petschek reconnection model[11,12] to relativistic magnetohydrodynamics. Alternative reconnection mechanisms, such as tearing instability, will not be considered.

2. PETSCHEK APPROACH

Syrovatskiĭ demonstrated[13] that inhomogeneous motion in a magnetized plasma creates electric-current layers which evolve giving rise to an unstable state. This is manifested by the fact that in a small part of a current layer, known as the diffusion region, we can expect a critical value of a plasma parameter (such as the current velocity) followed by the development of wave turbulence and accompanied by a steep fall of the plasma conductivity.

In the Petschek approach[11] the subsequent evolution of the process is dominated by magnetohydrodynamic (MHD) waves (see also Refs. 12 and 14). In the diffusion region there is a normal component of the magnetic field B_n (i.e., the lines of force become "reconnected"), which results in the loss of stability of this part of the current layer where $B_n \neq 0$. An arbitrary discontinuity appears and it begins to decay into a system of MHD waves, including slow shock waves (Fig. 1).

The reason for this decay is the normal component of the magnetic field which is created by dissipation. Decay of the discontinuity also occurs locally in the diffusion region. The newly formed shock waves travel in a plasma in accordance with their own propagation laws. Slow shock waves appear at different moments in time and eventually form a single front bounding a field reversal (FR) region[15] containing an accelerated and heated plasma and the magnetic field being reconnected. Part of the space outside the FR regions will be called the inflow region,[15] from which the plasma enters (flows into) the FR region. In the diffusion region, where the conductivity falls, it is found that (in addition to B_n) an electric field E^* is also generated; from the point where it appears, the field E^* is transferred to a current layer by a surface MHD wave and it performs work there on the currents in the layer and this work is then dissipated in the acceleration and heating of a plasma. The result is an almost the same release of energy as in the case when an anomalous resistance appears instantaneously in the whole current layer. The electric field is generated in one place (diffusion region), acts elsewhere and, moreover, MHD waves ensure

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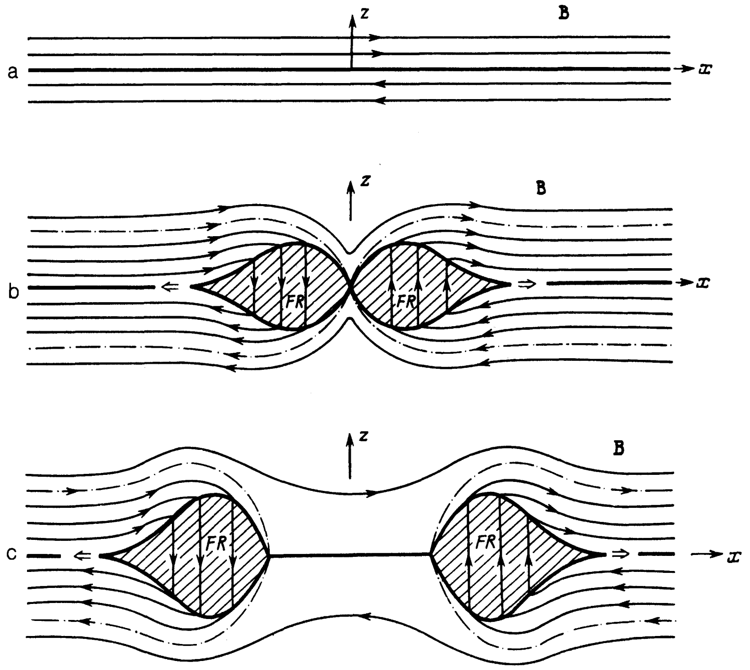

FIG. 1. Transient Petschek reconnection process: a) initial state; b) growth phase; c) expansion phase. The FR regions where an accelerated and heated plasma is concentrated are shown shaded. The dash-dot curves are separatrix lines of force, which are boundaries of a reconnecting magnetic force tube. The scale along the z axis is increased for clarity.

that E^* acts in turn on all the currents in the layer. Physically this appears as the passage of a shock wave along a current layer and work is done by the electric field in the front of this wave. The contribution of the ohmic dissipation proper to the total energy is negligible, because it is approximately equal to the ratio of the size of the diffusion region to the whole length of the current layer. True, in the diffusion region near a neutral line a relatively small fraction of particles is accelerated to exceptionally high energies.[16,17]

The process of magnetic field-line reconnection occurs in two stages. First, the magnetic lines of force are closed in the diffusion region. This alters the structure of the magnetic field, so that uncompensated forces appear and in the second stage of relaxation the system goes over to a new equilibrium position. It should be stressed that relaxation of Maxwellian stresses depends weakly on details of the process of closing the tubes of force in the diffusion region,[14] which provides an opportunity for investigating these stages separately. If the interest lies solely in the large-scale structure of fields and streams, which is the primary need in applications, the diffusion region can be ignored completely and it can be replaced by a boundary condition at the reconnection line. This boundary condition can be conveniently specified in the form of the electric field $E^*(x^0)$ regarded as a function of time, but eventually it is necessary to calculate the profile of the traveling shock waves and the MHD parameters throughout the whole space. Since outside the diffusion region, in what is known as the convection zone, the dissipation is no longer that important, we can use ideal magnetohydrodynamics with an infinite conductivity and this simplifies greatly the solution of the problem.

3. SYSTEM OF EQUATIONS FOR RELATIVISTIC MAGNETOHYDRODYNAMICS AND RELATIONSHIPS AT DISCONTINUITIES

The equations of ideal relativistic magnetohydrodynamics are as follows:[18,19]

$$\nabla_\alpha(\rho u^\alpha)=0, \tag{1}$$

$$\nabla_\alpha(u^\alpha h^\beta-u^\beta h^\alpha)=0, \tag{2}$$

$$\nabla_\alpha T^{\alpha\beta}=0, \tag{3}$$

where ρ is the plasma density; u^α is the four-velocity and h^α is the four-vector of the magnetic field, which are orthogonal to one another; $u^\alpha h_\alpha=0$; $T^{\alpha\beta}$ is the total tensor of the energy–momentum of a plasma and of the magnetic field:

$$T^{\alpha\beta}=\left(\rho w+\frac{1}{4\pi}h^2\right)u^\alpha u^\beta-\left(p+\frac{1}{8\pi}h^2\right)g^{\alpha\beta}-\frac{1}{4\pi}h^\alpha h^\beta, \tag{4}$$

where $g^{\alpha\beta}$ is the metric tensor of the Minkowski space with the signature $(1, -1, -1, -1)$; w is the specific self-enthalpy, given by $w=(p+\varepsilon_{\rm in})/\rho$; p is the gasdynamic pressure in the plasma; $\varepsilon_{\rm in}$ is the internal self-energy per unit volume; $h^2=-h_\alpha h^\alpha$ is the square of the magnetic field intensity.

A shock wave represents a hypersurface in the four-dimensional space and the values of u^α and h^α, as well as the thermodynamic parameters, have a discontinuity on this surface. If n_α is a unit vector along the normal to the shock-wave hypersurface, then the relationships on the shock wave front become[18-20]

$$\{\rho u^\alpha\}n_\alpha=0, \tag{5}$$

$$\{h^\alpha u^\beta-u^\alpha h^\beta\}n_\alpha=0, \tag{6}$$

$$\{T^{\alpha\beta}\}n_\alpha=0, \tag{7}$$

where $\{...\}$ denotes the difference between the quantities on both sides of such a discontinuity.

The following series of scalar relationships for a shock wave follows from Eqs. (5)–(7):

$$\{j\}=\{\rho u_n\}=0, \tag{8}$$

$$\{w\rho u_n h_n\}=0, \tag{9}$$

$$\{H\}=\{h^2u_n^2-h_n^2\}=0, \tag{10}$$

$$\left\{(h^2+H)\left(\frac{wj}{\rho}+H\right)^2\right\}=0, \tag{11}$$

$$\left\{Qu_n^2+p+\frac{1}{8\pi}h^2-\frac{1}{4\pi}h_n^2\right\}=0, \tag{12}$$

$$\left\{Q^2u_n^2-\left(p+\frac{1}{8\pi}h^2\right)^2-2Q\left(p+\frac{1}{8\pi}h^2\right)u_n^2+\frac{1}{2\pi}ph_n^2\right\}=0, \tag{13}$$

where

$$u_n=u^\alpha n_\alpha,\quad h_n=h^\alpha n_\alpha,\quad Q=p+\varepsilon+h^2/4\pi.$$

Fast shock waves correspond to $wj^2/\rho+H>0$, slow shock waves to $wj^2/\rho+H<0$, and Alfven discontinuities to $wj^2/\rho+H=0$.

4. FORMULATION OF THE PROBLEM

As pointed out already, we shall ignore the diffusion region, the effect of which can be replaced by a boundary condition on a reconnection line. This condition can be the electric field $E^*(x^0)$ regarded as a function of time.

Initially there is an electric-current layer near the plane $x'=x$, $x^2=y$, and the parameters of a plasma as well as a magnetic field are specified in the vicinity of this layer: they include the plasma density ρ_0, the pressure p_0, and the magnetic field $h^\alpha=(0,\ \pm B_0,\ 0,\ 0)$ directed along the x axis in the upper half-space, but directed oppositely in the lower half-space.

An ideal plasma can be regarded as a polytropic gas whose specific enthalpy and pressure are related to the density by

$$w=c^2+\frac{\gamma}{\gamma-1}\frac{p}{\rho}, \tag{14}$$

$$p=p_0(\rho/\rho_0)^\gamma, \tag{15}$$

where $\gamma>1$ is the polytropic exponent.

We shall reduce the equations to the dimensionless form relative to the initial values of the density ρ_0, magnetic field B_0, magnetic pressure $p_0=B_0^2/8\pi$, velocity of light in vacuum c, time T_0 for a characteristic change in the electric field on a reconnection line, and distance traversed by light in the time T_0, i.e., cT_0. Then, the dimensionless enthalpy per unit volume $\tilde{\rho}\tilde{w}$ (the tilde means that the quantity is dimensionless) is described by

$$\tilde{\rho}\tilde{w}=\mu\tilde{\rho}+\Gamma\tilde{\rho}, \tag{16}$$

where

$$\mu=8\pi\rho_0c^2/B_0^2,\quad \Gamma=\gamma/(\gamma-1).$$

The initial values of the dimensionless quantities can be rewritten as follows:

$$\tilde{\rho}=1,\quad \tilde{h}^\alpha=(0,\pm1,0,0),\quad \tilde{p}=\beta=8\pi p_0/B_0^2. \tag{17}$$

We shall consider only the weak reconnection case when the electric field along a reconnection line is much less than the Alfven field: $E^*\ll E_A=v_AB_0/c$ (v_A is the Alfven wave velocity). The parameter $\varepsilon=E^*/E_A\ll1$ is small and the expansion is in its terms. We shall show below that in the weak reconnection case the FR region is strongly elongated along the field and compressed in the orthogonal direction, i.e., it represents a boundary layer.

The initial data on the current layer described by Eq. (17) and the boundary condition on the reconnection line (y axis), i.e., the field $E^*(x^0)$, are used to find the solution of the relativistic MHD equations (1)–(3) which satisfy the relationships (5)–(7) for shock waves whose shape is to be determined.

5. ESTIMATES OF VARIOUS QUANTITIES IN A FIELD REVERSAL REGION

We shall begin by estimating the order of magnitude of the various quantities in an FR region. In the weak reconnection case we have $E^*\sim\varepsilon$, so that in the inflow region we can estimate the rate of convection to a reconnection line $v_z\sim\varepsilon$. The velocity component normal to a discontinuity is then also of the same order of magnitude: $u_n\sim\varepsilon$; it follows from Eq. (10) that the normal component of the magnetic field is $h_n\sim\varepsilon$. Therefore, the vertex angle of the discontinuities forming one FR region is of the order of ε, i.e., it follows that $x\sim1$ and $z\sim\varepsilon$, i.e., the FR region is indeed elongated along the field and compressed in the orthogonal direction. A similar analysis based on the conservation laws gives the following estimates:

$$P,\ u_x,\ h_x,\ x\sim1;\quad u_z,\ h_z,\ z\sim\varepsilon,$$

i.e., the quantities tangential to the current layer are of the order of unity, whereas the quantities normal to this layer are of the order of ε. This allows us to find initially the tangential components of the velocity and magnetic field, and then the normal components.

It follows that the FR region is a boundary layer. A well-known property of boundary layers is the constancy of the total pressure in the transverse (z) direction. The proof is exactly the same as in the theory of a viscous Prandtl layer[21] and we need simply estimate the order of the quantities in the equation of motion along the z coordinate. In the weak reconnection approximation we can readily see that the following expression is also valid:

$$P=p+\frac{1}{8\pi}h^2=\mathrm{const}=p_0+\frac{1}{8\pi}B_0^2. \tag{18}$$

Clearly, in the course of the whole reconnection process the total pressure remains constant (in the zeroth approximation).

We shall now make the following comment. When a nonlinear problem with a small parameter is solved, the following situation is usually encountered. In the zeroth approximation the problem still remains nonlinear, but simpler, whereas in higher approximations the problem becomes linear. The main difficulty is to solve the nonlinear problem in the zeroth approximation.

In the reconnection problem this nonlinear zeroth-approximation problem is the Riemann problem of decay of an arbitrary discontinuity or, more exactly, of a relatively simple variant of this problem. In fact, the problem for ρ and p, and for the tangential components of the velocity and magnetic field in the approximation of the zeroth order in ε is self-similar and can be reduced to the Riemann problem. The full Riemann problem is very complex (see Ref. 22), but in the present case some important simplifications are possible. First, the problem is formulated only for the tangential components u^α and h^α. Second, there are no fast shock waves in the decay process, because the total pressure remains constant [see Eq. (18)]. Thirdly and finally, it is natural to assume that in the symmetric case under discussion here the decay process can be regarded as involving two slow switching-off shock waves, as in the original Petschek solution.[11] We shall assume that this is true and show that we can then satisfy all the necessary conditions. The uniqueness of the solution of the Riemann problem then guarantees the uniqueness of the solution of the magnetic reconnection problem. We shall consider only the dimensionless quantities, so that we shall omit the tilde and identify the quantities in the FR region by a bar above them.

6. SOLUTION IN A FIELD REVERSAL REGION

a. *Determination of the pressure.* In the case of a slow switching-off shock wave, which for the sake of brevity we shall call the Petschek wave, the tangential component of the magnetic field vanishes (is "switched off"), so that in an FR region the gas pressure $\bar{p}$ should balance out the total external pressure [Eq. (18)]:

$$\bar{p}=1+\beta. \tag{19}$$

In the weak reconnection approximation the condition (18) is in agreement with the relationships (12) and (13) satisfied in a shock wave.

b. *Determination of the plasma density.* In the case of a Petschek shock wave in front of a discontinuity we have $wj^2/\rho + H = 0$, exactly as in the case of an Alfven discontinuity, whereas beyond the discontinuity we obtain $\bar{h}^2 + \bar{H} \approx \bar{h}^2 - \bar{h}_n^2 \approx 0$, so that the condition (11) is satisfied due to different factors. It follows from the condition $wj^2/\rho + H = 0$, subject to Eq. (8), that

$$(w\rho+2h^2)u_n^2=h_n^2, \tag{20}$$

which gives

$$u_n/h_n=\pm(2/Q)^{1/2}. \tag{21}$$

The sign is deduced from the following considerations. We first select the direction of the outward normal to the FR region. In the first quadrant the line of force of the magnetic field emerges from the FR region (Fig. 1b) and, therefore, we have $h_n > 0$ so that the plasma flux flows into the FR region and we have $u_n < 0$; the sign in Eq. (21) is negative. The signs for the other variants are found in a similar manner.

It follows from Eqs. (8) and (9) that

$$wh_n=\bar{w}\bar{h}_n. \tag{22}$$

Equation (10), in view of the smallness of the field $\bar{h}^2$ and of the normal component of the velocity u_n in the FR region, leads to $u_n^2 - h_n^2 = \bar{h}_n^2$; then, using Eq. (20), we obtain

$$\bar{h}_n/h_n=[(Q-2)/Q]^{1/2}. \tag{23}$$

Substituting in Eq. (16) describing the specific enthalpy, the values of ρ and p on opposite sides of the discontinuity taken from Eqs. (17) and (19), and combining the results with Eq. (23), we obtain

$$\mu+\Gamma\frac{\beta+1}{\bar{\rho}}=(\mu+\Gamma\beta)\left(\frac{Q}{Q-2}\right)^{1/2}. \tag{24}$$

In the inflow region the quantity Q can be calculated from Eq. (16) assuming that $\rho = 1$ and $h^2 = 1$:

$$Q=\mu+\Gamma\beta+2. \tag{25}$$

Substituting Eq. (25) into Eq. (24), we obtain the plasma density in the FR region:

$$\bar{\rho}=\Gamma(\beta+1)/\{[Q(Q-2)]^{1/2}-\mu\}. \tag{26}$$

We can easily see that $\bar{\rho} > 1$, i.e., that the plasma becomes denser, as expected for an evolution shock wave.

c. *Determination of the plasma velocity.* It follows from Eq. (6) that the invariant vector $h_n u^\beta - u_n h^\beta$ is tangent to a shock wave. We shall use $u_t = u^\beta t_\beta$ and $h_t = h^\beta t_\beta$ to denote the projections of the four-velocity and the four-vector of the magnetic field along the tangential direction t_β, which gives

$$\{u_n h_t - u_t h_n\}=0. \tag{27}$$

Writing down this relationship on both sides of the discontinuity and bearing in mind that in the inflow region the quantities u_t and h_n are of the order of ε, whereas $h_t \sim 1$, and in the FR region both $\bar{u}_n$ and $\bar{h}_t$ are of the order of ε, we obtain

$$u_n=\bar{u}_t\bar{h}_n. \tag{28}$$

Equations (28), (21), and (23) allow us to determine the tangential velocity in the FR region:

$$\bar{u}_t=-\frac{u_n}{\bar{h}_n}=\left(\frac{2}{Q-2}\right)^{1/2}. \tag{29}$$

In the weak reconnection approximation all the scales of the quantities along the x axis are considerably greater than the scales along the z axis, as pointed out already, and we can assume that the tangential velocity of the plasma determines its motion in the FR region along the x axis and we have $\bar{u}_t = \bar{u}_x = \bar{u}^1$.

The expression for the x component of the four-velocity $\bar{u}_x = \bar{u}^1$ in terms of the three-dimensional x component of the velocity $\bar{v}_x$ is

$$\bar{u}_x=\bar{v}_x/(1-\bar{v}_x^2)^{1/2}, \tag{30}$$

which then gives $\bar{v}_x$:

$$\bar{v}_x=\frac{\bar{u}_x}{(1+\bar{u}_x^2)^{1/2}}=\left(\frac{2}{Q}\right)^{1/2}=\left(\frac{2}{\mu+\Gamma\beta+2}\right)^{1/2}. \tag{31}$$

The time component $\overline{u^0}$ of the four-velocity is found from

$$\bar{u}^0=\frac{1}{(1-\bar{v}_x^2)^{1/2}}=\left(\frac{Q}{Q-2}\right)^{1/2}. \tag{32}$$

Gathering together the results (29), (30), and (32), we obtain the components of the four-velocity:

$$\bar{u}^{\alpha} = \left(\left(\frac{Q}{Q-2} \right)^{1/2}; \left(\frac{2}{Q-2} \right)^{1/2}; 0; 0 \right). \tag{33}$$

d. *Determination of the z components of the velocity and magnetic field.* We can find the velocity component $\overline{u^3}$ using the equation of continuity, which gives $\partial \bar{u}^3/\partial z = 0$, because $\overline{u^0}$ and $\overline{u^1}$ are constant. Since $\bar{u}^3 = 0$ at $z = 0$ (symmetry condition), it follows that $\bar{u}^3 = 0$ throughout the FR region.

We can determine the field by applying the induction equation (2). Writing down the nonzero components of this equation and bearing in mind that the four-velocity is constant in the FR region [see Eq. (33)], we obtain a homogeneous equation in terms of the first-order partial derivatives:

$$\frac{\partial \bar{h}^3}{\partial x^0} + \frac{\bar{u}^1}{\bar{u}^0} \frac{\partial \bar{h}^3}{\partial x^1} = 0, \tag{34}$$

the general solution of which can be written in terms of an arbitrary function $F(x^0, x)$:

$$\bar{h}^3 = F\left(x^0 - \frac{\bar{u}^0}{\bar{u}^1} x \right). \tag{35}$$

We can find the function $F(x^0, x)$ employing the expression for the magnetic-field four-vector h^{α} in terms of the three-dimensional magnetic-field vector **B** and the three-dimensional velocity **v**, whose spatial components can be represented as follows:[18]

$$h^i = \frac{B^i}{\bar{u}^0} + \frac{\bar{u}^0}{c^2} v^i (\mathbf{B}, \mathbf{v}). \tag{36}$$

The second term in Eq. (36) vanishes, because $\bar{v}_z = 0$, so that we have

$$\bar{h}^3 (x^0, x) = \frac{1}{\bar{u}^0} \bar{B}_z (x^0, x) = \frac{1}{\bar{u}_0 \bar{v}_x} \bar{E}_y (x^0, x) = \frac{1}{\bar{u}^1} \bar{E}_y (x^0, x). \tag{37}$$

In the derivation of the above equation we used the relationships $E_y = v_x B_z$, and $v_x = \bar{u}^1/\bar{u}^0$. Therefore, the function $F(x^0, x)$ is proportional to the electric field and it can be found from the boundary condition on a reconnection line: $F(x^0) = E^*(x^0)/\bar{u}^1$.

Finally, the four-vector of the magnetic field is

$$\bar{h}^{\alpha} = \left(0;\ 0;\ 0;\ \left(\frac{Q-2}{2} \right)^{1/2} E^* \left(x^0 - \frac{\bar{u}^0}{\bar{u}^1} x \right) \right). \tag{38}$$

e. *Determination of the shock wave profile.* The profile of a Petschek shock wave can be found in the form of the function $z = \varepsilon f(x^0, x)$. The normal to the shock-wave hypersurface can be expressed in terms of derivatives of the function f as follows:

$$n_{\alpha} = (-\varepsilon f_{x^0}, -\varepsilon f_x, 0, 1). \tag{39}$$

We shall replace u_n and h_n in Eq. (10) with $\bar{u}_n$ and $\bar{h}_n$ and then, using Eqs. (8) and (23), we obtain

$$\bar{u}_n = -\left(\frac{2}{Q-2} \right)^{1/2} \frac{1}{\bar{\rho}} \bar{h}_n. \tag{40}$$

The sign of a Petschek wave is selected in the first quadrant. Using Eqs. (33), (38), and (39), we find from the condition (40) that f can be described by the following equation:

$$\bar{u}^0 f_{x^0} + \bar{u}^1 f_x = \frac{1}{\bar{\rho}} E^* \left(x^0 - \frac{\bar{u}^0}{\bar{u}^1} x \right), \tag{41}$$

the general solution of which can be represented in the form

$$f(x^0, x) = g\left(x^0 - \frac{\bar{u}^0}{\bar{u}^1} x \right) + \frac{x}{\bar{u}^1 \bar{\rho}} E^* \left(x^0 - \frac{\bar{u}^0}{\bar{u}^1} x \right). \tag{42}$$

The function g describes the effect of motion of a reconnection line. If the line is immobile, then at $x = 0$ the function in question is $f = 0$ and, consequently, we have $g \equiv 0$. In this case the profile of a Petschek wave is given by the function

$$f(x^0, x) = \frac{1}{\bar{u}^1 \bar{\rho}} x E^* \left(x^0 - \frac{\bar{u}^0}{\bar{u}^1} x \right). \tag{43}$$

The velocity of a wave along a current layer (parallel to the x axis) is then

$$\frac{\bar{u}^1}{\bar{u}^0} = \left(\frac{2}{Q} \right)^{1/2} = \left(\frac{2}{\mu + \Gamma\beta + 2} \right)^{1/2}. \tag{44}$$

We thus found the pressure (19), the density (26), the velocity (33), and the magnetic field (38) in the FR region, as well as the profile of a Petschek shock wave (43).

We assumed so far that the electric field E^* is independent of y and that the length of a reconnection line is infinite. However, the above solution is valid (as is easily demonstrated) in the more general case. The field E^* may depend on y, which occurs in the solution only as a parameter. It is natural to assume that at the ends of a reconnection line we have $E^* = 0$, so that the shock wave is a surface stretched on the reconnection line, on the magnetic-field force lines passing through the ends of this line and through the leading edge of the perturbation (Fig. 2). It follows that the solution is quite informative: by specifying the actual dependence $E^*(x^0, y)$, we can study the various reconnection regimes.

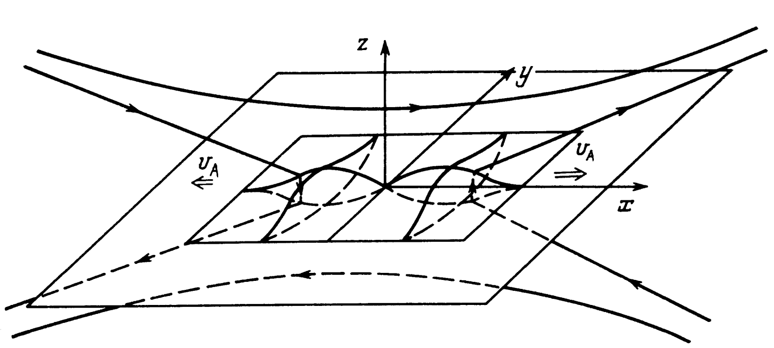

FIG. 2. Appearance of an FR region with a finite reconnection-line length.

f. *Inflow region*. Perturbations of MHD parameters in the inflow region are small and proportional to ε, since the FR region is a thin boundary layer of thickness $\sim\varepsilon$ and the components of the velocity and magnetic field normal to shock waves are also of the order of ε. Therefore, perturbations in the inflow region can be found from a linearized system of relativistic MHD equations. The boundary conditions follow from Eq. (8) for u_n and from Eq. (21) for h_n. On the whole, calculations of perturbations in the inflow region are completely similar to those in the problem of motion of a thin profile (in our case an FR region) in a magnetized plasma. Using the Fourier method or the Green function technique, we can find the solution of this linear hyperbolic problem. However, this is of little help in practice, because the solutions are so cumbersome that nothing obvious can be deduced from them. Even in the nonrelativistic case we can obtain clear results only in the limit either of an incompressible plasma or a cold plasma,[14] whereas in the relativistic case even these simple variants are not obtained. We shall therefore consider the above solution in an FR region on the assumption that in an inflow region the solution is similar to that obtained for the nonrelativistic case.

7. RECONNECTION PHASES

Reconnection characteristics. The most important characteristic of the reconnection process is the magnetic flux $\Phi(x^0)$ which is reconnected. We can calculate it using the theorem on circulation of the electric field. Let us assume that a contour passes through a reconnection line l and then surrounds an FR region. We then find that $E\neq 0$ only on the reconnection line and the circulation theorem gives

$$\Phi(x^0)=\int_0^{x^0}\int_l E^*(\tau,y)\,d\tau. \quad (45)$$

We are allowing here for the fact that the electric field may depend on y.

We shall now calculate the energy inside the FR region:

$$W=\int_{FR} T^{00}dx\,dy\,dz=[\bar{Q}(\bar{u}^0)^2-\bar{p}]\,\frac{\bar{u}^1}{\bar{\rho}(\bar{u}^0)^2}\int_0^{x^0}\Phi(\tau)\,d\tau. \quad (46)$$

Equation (46) is derived after substitution of the variable $\tau=x^0-(\bar{u}^0/\bar{u}^1)x$ and integration by parts. Similarly, the momentum of the FR region is

$$N=\int_{FR} T^{01}\,dx\,dy\,dz=\frac{\bar{Q}(\bar{u}^1)^2}{\bar{\rho}\bar{u}^0}\int_0^{x^0}\Phi(\tau)\,d\tau. \quad (47)$$

The expressions (45)–(47) are valid as long as the FR region moves along a current layer. We can see that the reconnected magnetic flux, energy, and momentum in the FR region are governed entirely by the behavior of the electric field on the reconnection line.

Acceleration. Irrespective of the behavior of the electric field on a reconnection line a plasma is accelerated by the Petschek shock waves which impart exactly the Alfven velocity $cB_0/(4\pi\rho_0c^2+4\pi\Gamma\rho_0+B^2)^{1/2}$ calculated using the unperturbed parameters of the current layer, and the plasma is then compressed and heated [Eqs. (19) and (26)]. On the other hand, the magnetic field is greatly weakened in the FR region, i.e., the magnetic field energy is converted into the plasma energy. The reconnection weakness does not imply that the acceleration is weak but simply that the volume of the FR region with an accelerated and heated plasma is small.

The reconnection process is particularly effective in a strong magnetic field and a low-density plasma ($\mu\ll 1$, $\beta\ll 1$). In this case the plasma is accelerated to relativistic velocities

$$v_x=1-(\mu+\Gamma\beta)/4, \quad (48)$$

is strongly compressed to

$$\bar{\rho}=\Gamma/[2(\mu+\Gamma\beta)]^{1/2}, \quad (49)$$

and is heated:

$$\bar{T}^{00}=2\Gamma/(\mu+\Gamma\beta). \quad (50)$$

It is clear from Eqs. (49) and (50) that $\hat{\rho}$ and $\bar{T}^{00}\to\infty$ for the case when $\mu\to 0$ and $\beta\to 0$ and these quantities are limited mainly only by the condition of validity of the equation of state described by Eqs. (14) and (15). On the other hand, the transverse size of the FR region is $z_{FR}\sim(\mu+\Gamma\beta)/\Gamma\to 0$ and z_{FR} becomes much smaller than the transverse size of a reconnection force tube: $z_{FR}\ll\Phi/B_0l_x$ (l_x is the length of a reconnection or X line). This means that the relativistic plasma is concentrated in a small region.

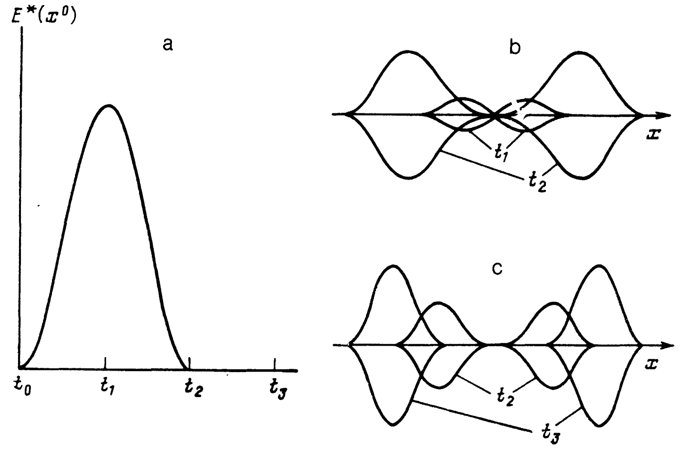

FIG. 3. Behavior of the electric field on a reconnection line (a) and the corresponding profile of Petschek shock waves during the growth (b) and expansion (c) phases.

Growth phase. The solution obtained by us is largely governed by the behavior of the electric field on a reconnection line. Specifying $E^*(x^0)$ in this or another manner, we can model different reconnection regimes (Fig. 3). Let us assume initially that the electric field in the diffusion region is switched on gradually [i.e., $E(0) = 0$] and that after a time we have $E(x^0) > 0$. In this case the leading edge of a Petschek shock wave travels along a current layer at the Alfven velocity (44), whereas the trailing edge is immobile (Fig. 3b).

During this reconnection phase the flux rises, the leading edge of the FR region moves still further and further along the current layer, and all large plasmas are accelerated by the Petschek shock waves, the dimensions of the FR region and its volume increase rapidly with time, and the process is clearly explosive.

Expansion phase. We shall now assume that beginning from a certain moment x_0^0 the process of reconnection stops in the diffusion region $E^*(x_0^0) = 0$. It then follows from Eq. (43) that the FR region becomes detached from the reconnection line, its rear edge travels at the Alfven velocity $x = (\bar{u}^0/\bar{u}^1)(x^0 - x_0^0)$, and it can begin to move along the current layer as an independent object. Behind the expanding FR regions a new current layer is reestablished (Fig. 1b), and the intensity of the magnetic field in this layer is somewhat less than before reconnection. The magnetic energy deficit and the reconnected magnetic flux, which no longer changes during the expansion phase, are carried away by the FR regions.

It is clear from Eq. (43) that during the motion of an FR region along a current layer its transverse size rises linearly with x, i.e., with the distance to the former reconnection line (Fig. 3c). Therefore, the volume of the FR region and the energy enclosed in it increase with time. Physically this is due to the fact that the conversion of the magnetic energy continues for the Petschek waves even after the diffusion region has disappeared. The FR region converts completely the reconnected force tube and traps all the plasma located inside it.

The above solution remains valid up to times of the order of $1/\varepsilon$ when the x and z dimensions of the FR region become comparable and the weak reconnection approximation ceases to be valid.

Limiting case. The above solution considered in the nonrelativistic limit $\mu \gg 1$ reduces to the solution obtained in Refs. 12 and 14 for a transient reconnection process, which during the growth phase in the vicinity of the X line reduces to the Petschek solution.[11]

Electric currents. Alfven criticized strongly the reconnection idea and suggested an interpretation of the plasma phenomena in terms of electric currents.[23] In fact, this approach should reveal new important features of the reconnection process which are not detectable by other points of view.

The electric current decreases in the vicinity of a reconnection line, but the total current in the current layer affected by a perturbation does not change. The rate of energy evolution is not due to the ohmic attenuation of the current, but entirely to the change in the geometry of the current: the current now contracts in the direction away from the reconnection line. A perturbation of the current is usually caused by a surface Alfven wave, as in the nonrelativistic case discussed in Ref. 14. Therefore, the elementary act of reconnection displaces partly the electric current from the diffusion region to the edges of the thin layer, in other words, reconnection gradually destroys the current layer.

8. SOME APPLICATIONS

Transport processes. It is shown above that the FR regions transport (along a current layer) the electric field, reconnected magnetic flux, perturbations of the electric current, mass, momentum, and energy. Hence, we can assume that reconnection is a transport process typical of a highly conducting medium with current layers. Reconnection can result in an effective exchange of mass, momentum, and energy between regions carrying different magnetic fluxes, which should be also of importance in space research.

Laboratory experiment. It is interesting to note that a plasma can be accelerated to relativistic velocities without satisfying any exotic conditions, so that the relativistic reconnection process may occur in principle in a modern laboratory. Generation of a low-pressure plasma characterized by $B_0/c(4\pi\rho)^{1/2} \sim 1$ is not a very difficult task. We shall point out that in the case of a hydrogen plasma with the concentration $n = 10^{13}$ cm^{-3} in a magnetic field of intensity $B = 200$ kG this parameter is $B_0/c(4\pi\rho)^{1/2} \approx 0.5$. Reconnection should then accelerate the plasma only to half the velocity of light. The main difficulty is to create thin current layers separating the regions with magnetic fields so strong that the carriers should also move at the velocities close to that of light. The properties of such relativistic current layers have been investigated insufficiently and the topic requires further study.

Bursts and flares. A typical consequence of field-line reconnection can be a burst or a flare. Depending on the actual situation, there can be an extremely wide range of situations, but we shall consider the specific case of reconnection in a streamer structure, which is quite typical of the solar atmosphere[10] (Fig. 4). At the initial moment in time a current layer stores a magnetic energy (Fig. 4a). In the course of reconnection this energy is transformed into the energy of an accelerated and heated plasma contained in two FR regions. One of them travels to the surface of a star and, falling along a surface parallel to the magnetic lines of force, it creates a radiation burst in the visible, x-ray, and possibly even γ-ray range, depending on the parameters of the current layer (Figs. 4b and 4c). The second FR region travels along the current layer along a direction away from a star: it requires energy, and, therefore, it represents a burst of heated plasma into the surrounding space.

The parameters of the radiation emitted during the growth phase can, in principle, be used to determine the most important characteristics of the reconnection process. In the diffusion region near the X line the particles are accelerated to exceptionally high energies,[16,17] as pointed out already. Therefore, the projection of a reconnection line on the surface of a star should be brighter than the rest of the emitted radiation. Since during the growth phase (Fig. 4b) the reconnected flux rises, the projection A' of the X line A moves in the direction from the projection C' in the current layer C. The area emitting such radiation is $A'C'$ and it is obviously proportional to the reconnected magnetic flux $\Phi(x^0)$, whereas the velocity of a bright edge A' is proportional to the electric field $E^*(x^0)$ along the reconnection line. As shown

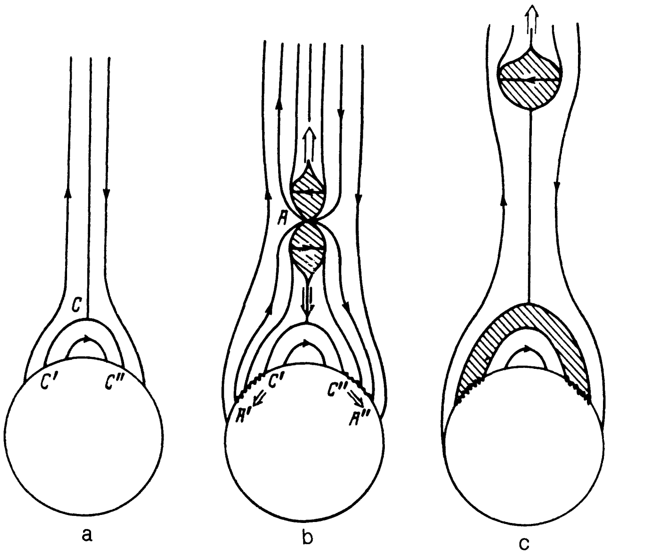

FIG. 4. Formation of flares and bursts due to the reconnection process: a) initial state; b) pulsed phase; c) formation of a burst. Shading identifies the regions with an accelerated and heated plasma.

above [see Eqs. (45)–(47)], $E^*(x^0)$ and $\Phi(x^0)$ essentially determine the whole reconnection process. Therefore, a study of the kinetics of the emission of such radiation and of the magnetic field intensity in this region does indeed allow us to reconstruct the principal characteristics of the reconnection process.

The above scheme clearly accounts for the main features of such phenomena as a solar flare[24] or the magnetospheric storm.[14] We can expect the above scheme to be useful also in the case of astrophysical objects with relativistic current layers.

9. CONCLUSIONS

In astrophysical applications the process of reconnection should obviously occur quite frequently. In the presence of a highly conducting medium any inhomogeneous motion of a plasma stretches and deforms the magnetic force tubes and, consequently, transforms the kinetic energy into the magnetic field energy. The reflection was particularly strong in the vicinity of special (particularly neutral) magnetic field lines, where current layers should form.[13] Gradually a considerable proportion of the energy of the initially inhomogeneous motion is transferred to the magnetic field. During the next stage of the reconnection process the magnetic energy is in turn converted into the kinetic and thermal energy of a plasma; transformation of one type of energy to another may be repeated many times. Therefore, under given boundary and initial conditions a plasma may travel only with the aid of the magnetic field involving the reconnection process. It should be mentioned that in almost all the modern theories (for example, in the models of the magnetospheres of a pulsar[25] or a black hole[26]) the reconnection process is not used at all. This is because the theories are static and the motion of a plasma is simple: it travels only along the magnetic lines of force. An allowance for the reconnection process makes it possible to consider more complex motion of a plasma and to develop dynamic models of magnetospheres, as has been done in the physics of the terrestrial magnetosphere.[2,3,14]

Two topics are the most important in specific applications of the mechanism described above. First of all, how a current layer is formed and is stored. Although there are many ideas on this topic,[7,8,10,13,14,24,26] nevertheless in each case there may be some special features and all theoretical cases have not yet been fully understood. Secondly, we have to consider how the parameters of a plasma in a growing current layer develop and which plasma instability (discontinuity, current, overheating, etc.[10,13,14]) is responsible for the fall of the conductivity in the diffusion region. Unfortunately, this has not yet been investigated sufficiently thoroughly. Therefore, in applications it is preferable to use semiphenomenological models and replace poorly known theoretical predictions with experimental data. For example, measurements of the radiation emitted by a flare (or of some other parameter associated with a flare) make it possible to find with a good time resolution the electric field E^* and, therefore, to determine the whole reconnection process.

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