Electron-positive pair production by a transverse electromagnetic field in a nonstationary medium

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The probability of e^+e^- pair production by an electromagnetic wave in a nonstationary medium is found on the basis of the Dirac model to first order perturbation in the wave field. It is shown that this is a threshold-free process and, in contrast to the stationary case, it can occur in any medium (whose permittivity is time-dependent) at any frequency of the wave.

When a traverse monochromatic wave propagates through a medium whose properties (permittivity) are time-dependent, a wave spectrum, whose width is determined by the rate of change of the permittivity, is formed.[1] Since the waves in this spectrum all have identical wave vectors ($\mathbf{k} = \text{const}$ owing to the spatial uniformity of the medium), the dispersion relation for these waves allows conservation of energy and momentum for pair production in an arbitrary nonstationary medium, in contrast to the stationary case when pair production can occur only in a plasma.[2,3] The second fundamental difference is that pair production by a photon field in a nonstationary medium is a threshold-free process, so that in this case the electromagnetic wave can have any frequency.

A medium can be nonstationary, i.e., its permittivity ε can be time-dependent, for different reasons: nonlinear polarization of the atoms, formation of plasma accompanying the interaction of laser radiation with the matter, etc. Physically, it is obvious that the time dependence of the medium will give rise to significant effects if ε changes rapidly in time ($\Delta t \ll 2\pi/\omega$, where ω is the frequency of the wave). Such a change in the permittivity can be produced by a sharp change in the density of the medium (gas pressure).[4] The permittivity ε can change much more rapidly when the medium is converted instantaneously into plasma by powerful ultranarrow laser pulses.[5,6] Finally, plasma with strongly time-dependent properties actually exists in and around astrophysical objects, in particular, in the magnetosphere of pulsars.

Let a transverse monochromatic wave with frequency ω propagate in a uniform isotropic medium, whose permittivity changes abruptly at the time $t = 0$ from the value $\varepsilon_1 (t<0)$ to $\varepsilon_2 (t>0)$. Then the wave which for $t<0$ had the form

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0 \exp[i(\omega t-\mathbf{kr})]+\text{c.c.}, \quad t<0, \tag{1}$$

transforms at $t>0$ into two waves—a transmitted wave and a reflected wave:

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_1 \exp[i(\omega_1 t-\mathbf{kr})] + \mathbf{E}_2 \exp[-i(\omega_1 t+\mathbf{kr})]+\text{c.c.}, \quad t\geqslant 0, \tag{2}$$

where $\mathbf{E}_1$ and $\mathbf{E}_2$ are the amplitudes of the electric fields of the transmitted and reflected waves, where frequencies are ω_1 and $-\omega_1$, respectively. The value of ω_1 is determined from the condition of spatial uniformity of the medium and is equal to $\omega_1 = (\varepsilon_1/\varepsilon_2)^{1/2}\omega$. The amplitudes $\mathbf{E}_1$ and $\mathbf{E}_2$, however, are found from Maxwell's equations, and in the case when the incident wave (1) is linearly polarized they have the form[1]

$$\mathbf{E}_{1,2}=\frac{\varepsilon_1^{1/2}(\varepsilon_1^{1/2}\pm\varepsilon_2^{1/2})}{2\varepsilon_2}\mathbf{E}_0. \tag{3}$$

In order to describe pair production in the field (1)–(3) we shall employ the Dirac model (all negative-energy states of the vacuum are filled with electrons and the external field interacts only with this vacuum). The Dirac equation in the field (1)–(3) has the form (here $\hbar = c = 1$)

$$i\frac{\partial\Psi}{\partial t}=[\boldsymbol{\alpha}(\mathbf{p}-e\mathbf{A})+\beta m]\Psi, \tag{4}$$

where

$$\mathbf{A}(\mathbf{r},t)=\begin{cases} i\dfrac{\mathbf{E}_0}{\omega}\exp[i(\omega t-\mathbf{kr})]+\text{c.c.}, & t<0,\\ i\dfrac{\mathbf{E}_1}{\omega_1}\exp[i(\omega_1 t-\mathbf{kr})]-i\dfrac{\mathbf{E}_2}{\omega_1}\exp[-i(\omega_1 t+\mathbf{kr})] + \text{c.c.}, & t\geqslant 0 \end{cases} \tag{5}$$

is the vector potential of the wave in a medium whose permittivity changes abruptly,

$$\boldsymbol{\alpha}=\begin{pmatrix}0 & \boldsymbol{\sigma}\\ \boldsymbol{\sigma} & 0\end{pmatrix}, \quad \beta=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}$$

are the Dirac matrices, and $\boldsymbol{\sigma}$ are the Pauli matrices.

We solve Eq. (4) by perturbing in the field of the wave. This method is valid if

$$\left[1+\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^{1/2}\right]\xi\ll 1, \quad \xi=\frac{eE_0}{m\omega}. \tag{6}$$

We expand the perturbed first-order wave function $\Psi_1(\mathbf{r},t)$ in a complete system of orthonormalized wave functions of the electrons (positrons) with momenta $\mathbf{p}-\mathbf{k}$ and $\mathbf{p}+\mathbf{k}$:

$$\Psi_1(\mathbf{r},t)=\Psi_1^{(-)}(t)e^{i(\mathbf{p}-\mathbf{k})\mathbf{r}}+\Psi_1^{(+)}(t)e^{i(\mathbf{p}+\mathbf{k})\mathbf{r}},$$
$$\Psi_1^{(-)}(t)=\sum_{l=1}^{4}a_l(t)u_l(\mathbf{p}-\mathbf{k},t), \tag{7}$$
$$\Psi_1^{(+)}(t)=\sum_{j=1}^{4}b_j(t)u_j(\mathbf{p}+\mathbf{k},t).$$

Here $a_l(t)$ and $b_j(t)$ are unknown functions and $u_i(\mathbf{p}',t)$

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are orthonormalized bispinor functions which describe the states of particles with energies $\pm\mathscr{E}'=\pm(p'^2+m^2)^{1/2}$:

$$u_{1,2}(\mathbf{p}',t)=\left(\frac{\mathscr{E}'+m}{2\mathscr{E}'}\right)^{1/2}\begin{pmatrix}\varphi_{1,2}\\ \dfrac{\sigma\mathbf{p}'}{\mathscr{E}'+m}\varphi_{1,2}\end{pmatrix}\exp(-i\mathscr{E}'t),$$

$$u_{3,4}(\mathbf{p}',t)=\left(\frac{\mathscr{E}'+m}{2\mathscr{E}'}\right)^{1/2}\begin{pmatrix}\dfrac{-\sigma\mathbf{p}'}{\mathscr{E}'+m}\chi_{3,4}\\ \chi_{3,4}\end{pmatrix}\exp(i\mathscr{E}'t). \qquad (8)$$

The latter functions are normalized to one particle per unit volume: $u_i^+u_j=\delta_{ij}$; the constant spinors are

$$\varphi_1=\chi_3=\begin{pmatrix}1\\0\end{pmatrix},\quad \varphi_2=\chi_4=\begin{pmatrix}0\\1\end{pmatrix}.$$

Under the transformations (5)–(8) the Dirac equation for the perturbed wave function $\Psi=\Psi_0+\Psi_1+...$, $|\Psi_1|\ll|\Psi_0|$

$$\left(i\frac{\partial}{\partial t}-\alpha\hat{\mathbf{p}}-\beta m\right)\Psi_1=-e\alpha\mathbf{A}\Psi_0, \qquad (9)$$

transforms into a system of 16 equations for the unknown functions $a_l(t)$ and $b_j(t)$

$$\left(i\frac{\partial}{\partial t}-\alpha\hat{\mathbf{p}}-\beta m\right)\left[\sum_{l=1}^{4}a_l(t)u_l(\mathbf{p}-\mathbf{k},t)e^{i(\mathbf{p}-\mathbf{k})\mathbf{r}}+\sum_{j=1}^{4}b_j(t)u_j(\mathbf{p}+\mathbf{k},t)e^{i(\mathbf{p}+\mathbf{k})\mathbf{r}}\right]$$
$$=-e\alpha[\mathbf{A}_{(-)}(t)e^{-i\mathbf{kr}}+\mathbf{A}_{(+)}(t)e^{i\mathbf{kr}}]u_s(\mathbf{p},t)e^{i\mathbf{pr}}, \qquad (10)$$

where $s=3$, 4, and

$$\mathbf{A}_{(-)}(t)=\begin{cases} i\dfrac{\mathbf{E}_0}{\omega}e^{i\omega t}, & t<0\\ i\dfrac{\mathbf{E}_1}{\omega_1}e^{i\omega_1 t}-i\dfrac{\mathbf{E}_2}{\omega_1}e^{-i\omega_1 t}, & t\geqslant 0\end{cases},\qquad \mathbf{A}_{(+)}(t)=\mathbf{A}_{(-)}^{*}(t). \qquad (11)$$

The bispinor functions $u_s(\mathbf{p},t)$ appearing in Eq. (10) correspond to the unperturbed states of the Dirac vacuum (they are determined by the expressions (8) with $s=3$ and 4, where $\mathbf{p}'=\mathbf{p}$ and $\mathscr{E}'=\mathscr{E}$ are the momenta and energies of the free vacuum electrons).

According to this model, a pair is produced because of the interaction of the external field with the Dirac vacuum. In first order perturbation theory in the field this leads to electron states in the region of positive energies with the values

$$\mathscr{E}_{(-)}=[(\mathbf{p}-\mathbf{k})^2+m^2]^{1/2},\quad \mathscr{E}_{(+)}=[(\mathbf{p}+\mathbf{k})^2+m^2]^{1/2}.$$

The probabilities of these transitions are determined by the amplitudes $a_{1,2}$ and $b_{1,2}$, respectively (the indices 1 and 2 correspond to two different spin states). Therefore the problem reduces to determining the functions $a_{1,2}(t)$ and $b_{1,2}(t)$ by integrating the system of equations (10). From the latter system we can obtain the following system of equations:

$$\sum_{l=1}^{4}i\frac{da_l}{dt}u_l(\mathbf{p}-\mathbf{k},t)=-e\alpha\mathbf{A}_{(-)}(t)u_s(\mathbf{p},t), \qquad (12)$$

$$\sum_{j=1}^{4}i\frac{db_j}{dt}u_j(\mathbf{p}+\mathbf{k},t)=-e\alpha\mathbf{A}_{(+)}(t)u_s(\mathbf{p},t).$$

Multiplying the first equation in Eq. (12) on the left by $u_l^+(\mathbf{p}-\mathbf{k},t)$ and the second equation by $u_j^+(\mathbf{p}+\mathbf{k},t)$ and taking into account the fact that the bispinors are orthonormal ($u_l^+u_m=\delta_{lm}$), we obtain eight equations for the transitions amplitudes $a_l(t)$ and $b_j(t)$ for a given spinor state s of a vacuum electron ($s=3$ or $s=4$):

$$\frac{da_l(t)}{dt}=ieu_l^+(\mathbf{p}-\mathbf{k},t)\alpha\mathbf{A}_{(-)}(t)u_s(\mathbf{p},t),\quad l=1,\dots 4,$$
$$\frac{db_j(t)}{dt}=ieu_j^+(\mathbf{p}+\mathbf{k},t)\alpha\mathbf{A}_{(+)}(t)u_s(\mathbf{p},t),\quad j=1,\dots 4. \qquad (13)$$

Orienting the z axis parallel to the electric field $\mathbf{E}_0$ of the wave and the x axis parallel to $\mathbf{k}$, we obtain for the amplitudes $a_{1,2}$ and $b_{1,2}$

$$a_{1,2}(t)=ie[u_{1,2}^{+}(\mathbf{p}-\mathbf{k})\alpha_z u_s(\mathbf{p})]\int_{-\infty}^{t}A_{(-)}(t')\times\exp[i(\mathscr{E}+\mathscr{E}_{(-)})t']dt',$$

$$b_{1,2}(t)=ie[u_{1,2}^{+}(\mathbf{p}+\mathbf{k})\alpha_z u_s(\mathbf{p})]\int_{-\infty}^{t}A_{(+)}(t')\times\exp[i(\mathscr{E}+\mathscr{E}_{(+)})t']dt', \qquad (14)$$

where $u_{1,2}^{+}(\mathbf{p}\mp\mathbf{k})$ and $u_s(\mathbf{p})$ are constant bispinors, determined by the expressions (8) [preexponential factors in Eq. (8)].

The probability of electron production from a definite vacuum state $\mathbf{p}$, s is determined by the quantity $|a_1(t)|^2+|a_2(t)|^2|b_1(t)|^2+|b_2(t)|^2$ (the probability of the production of a positron with momentum $-\mathbf{p}$ in a definite spinor state $\mathbf{s}$). The differential probability of pair production, summed over the initial spin states of the Dirac vacuum, in an element of the phase volume $d^3\mathbf{p}/(2\pi)^3$ (the spatial normalizing volume is $V=1$) is

$$dW=2[|a_1(t)|^2+|a_2(t)|^2+|b_1(t)|^2+|b_2(t)|^2]|_{t\to+\infty}\frac{d^3\mathbf{p}}{(2\pi)^3}. \qquad (15)$$

Integrating Eqs. (14) over time, substituting Eq. (11) and making the assumption that the field is switched on and off adiabatically, $E_0(t=-\infty)=E_1(t=+\infty)=E_2(t=+\infty)=0$ (the amplitudes of the incident, transmitted, and reflected waves are assumed to be slowly varying functions of time), we obtain the following expressions for the amplitudes $a_{1,2}$ and $b_{1,2}$ after the wave interacts with the Dirac vacuum:

$$a_{1,2}(t=+\infty)=ie[u_{1,2}^{+}(\mathbf{p}-\mathbf{k})\alpha_z u_s(\mathbf{p})]\times\frac{E_0(\varepsilon_1-\varepsilon_2)(\mathscr{E}+\mathscr{E}_{(-)})}{\varepsilon_2(\mathscr{E}+\mathscr{E}_{(-)}+\omega)[(\mathscr{E}+\mathscr{E}_{(-)})^2-\omega^2\varepsilon_1/\varepsilon_2]}, \qquad (16)$$

$$b_{1,2}(t=+\infty)=ie[u_{1,2}^{+}(\mathbf{p}+\mathbf{k})\alpha_z u_s(\mathbf{p})]$$

$$\times\frac{E_0(\varepsilon_1-\varepsilon_2)(\mathscr{E}+\mathscr{E}_{(+)})}{\varepsilon_2(\mathscr{E}+\mathscr{E}_{(+)}-\omega)[(\mathscr{E}+\mathscr{E}_{(+)})^2-\omega^2\varepsilon_1/\varepsilon_2]}. \quad (17)$$

Calculating the transition matrix elements appearing in Eqs. (16) and (17), we find with the help of Eq. (15) the differential probability of pair production by an electromagnetic wave in a nonstationary medium:

$$dW=\frac{e^2}{(2\pi)^3}\frac{E_0^2(\varepsilon_1/\varepsilon_2-1)^2}{\mathscr{E}}$$

$$\times\left\{\frac{(\mathscr{E}+\mathscr{E}_{(-)})^2[\mathscr{E}\mathscr{E}_{(-)}+m^2+p_x(p_x-k)+p_y^2-p_z^2]}{\mathscr{E}_{(-)}(\mathscr{E}+\mathscr{E}_{(-)}+\omega)^2[(\mathscr{E}+\mathscr{E}_{(-)})^2-\omega^2\varepsilon_1/\varepsilon_2]^2}\right.$$

$$\left.+\frac{(\mathscr{E}+\mathscr{E}_{(+)})^2[\mathscr{E}\mathscr{E}_{(+)}+m^2+p_x(p_x+k)+p_y^2-p_z^2]}{\mathscr{E}_{(+)}(\mathscr{E}+\mathscr{E}_{(+)}-\omega)^2[(\mathscr{E}+\mathscr{E}_{(+)})^2-\omega^2\varepsilon_1/\varepsilon_2]^2}\right\}d^3\mathbf{p}. \quad (18)$$

As one can see from Eq. (18), the process exhibits azimuthal asymmetry with respect to the direction of propagation of the wave. Orienting the polar axis in this direction ($d^3\mathbf{p}=p\mathscr{E}d\mathscr{E}\sin\theta d\theta d\varphi$, where θ is the angle between the vectors $\mathbf{p}$ and $\mathbf{k}$ and φ is the azimuthal angle relative to the direction of polarization of the wave) and integrating over the energy, we obtain the angular distribution of the produced electrons (positrons). We note that since the case of physical interest is an electromagnetic wave having the frequency $\omega\ll m$, the expression (18) simplifies greatly and assumes the form

$$dW=\frac{e^2E_0^2}{2\pi^3}\left(\frac{\varepsilon_1}{\varepsilon_2}-1\right)^2\frac{(\mathscr{E}^2-m^2)^{1/2}}{\mathscr{E}}$$

$$\times\frac{m^2\sin^2\theta\cos^2\varphi+\mathscr{E}^2(1-\sin^2\theta\cos^2\varphi)}{(4\mathscr{E}^2-\omega^2\varepsilon_1/\varepsilon_2)^2}\sin\theta\, d\theta\, d\varphi\, d\mathscr{E}. \quad (19)$$

Integrating Eq. (19) over energy we find the number of pairs produced in the element of solid angle $do=\sin\theta d\theta d\varphi$:

$$dW(\theta,\varphi)=\frac{e^2E_0^2}{128\pi^2m}\left(\frac{\varepsilon_1}{\varepsilon_2}-1\right)^2\left[F\left(2,\frac{1}{2};2;\frac{\omega^2\varepsilon_1}{4m^2\varepsilon_2}\right)\right.$$

$$\times(1-\sin^2\theta\cos^2\varphi)$$

$$\left.+\frac{1}{4}F\left(2,\frac{3}{2};3;\frac{\omega^2\varepsilon_1}{4m^2\varepsilon_2}\right)\sin^2\theta\cos^2\varphi\right]do, \quad (20)$$

where $F(\nu,\mu;\lambda;z)$ is the hypergeometric function.

For the energy distribution of the resulting electrons (positrons) we have

$$dW(\mathscr{E})=\frac{2e^2E_0^2}{3\pi^2}\left(\frac{\varepsilon_1}{\varepsilon_2}-1\right)^2\frac{(\mathscr{E}^2-m^2)^{1/2}(2\mathscr{E}^2+m^2)}{(4\mathscr{E}^2-\omega^2\varepsilon_1/\varepsilon_2)^2}d\mathscr{E}. \quad (21)$$

Integrating the expression (20) over the angles θ and φ [or the expression (21) over energy] we find the total number of electron-positron pairs produced when an electromagnetic wave passes through a nonstationary medium:

$$W=\frac{e^2E_0^2}{48\pi m}\left(\frac{\varepsilon_1}{\varepsilon_2}-1\right)^2\left[F\left(2,\frac{1}{2};2;\frac{\omega^2\varepsilon_1}{4m^2\varepsilon_2}\right)\right.$$

$$\left.+\frac{1}{8}F\left(2,\frac{3}{2};3;\frac{\omega^2\varepsilon_1}{4m^2\varepsilon_2}\right)\right]. \quad (22)$$

We note that in the expressions (19) and (21) the denominators vanish for $\omega(\varepsilon_1/\varepsilon_2)^{1/2}=2\mathscr{E}$. This is the conservation law for single-photon pair production by a wave having the frequency $\omega_1=\omega(\varepsilon_1/\varepsilon_2)^{1/2}$ (by the transmitted and reflected waves) in a medium with index of refraction $n_2=\varepsilon_2^{1/2}<1$.[1)] It is obvious from Eq. (22) that the total probability of the process diverges when $\omega^2\varepsilon_1/4m^2\varepsilon_2=1$. The divergence of the probabilities is associated with the fact that they were determined for an infinitely long interaction time. In perturbation theory probabilities are proportional to the interaction time (under stationary conditions) and diverge as $t\to\infty$. Thus this divergence is not associated with the process studied here, which is governed by the time dependence of the medium, and it can be eliminated by assuming $\omega^2\varepsilon_1/\varepsilon_2<4m^2$. Moreover, for laser frequencies and permittivities realizable in practice $\omega(\varepsilon_1/\varepsilon_2)^{1/2}\ll2\mathscr{E}$ and from Eq. (22) we obtain the following expression for the total number of pairs produced in the volume V due only to the time dependence of the medium:[2)]

$$W=\frac{3e^2E_0^2V}{128\pi m\hbar c^3}\left(1-\frac{\varepsilon_1}{\varepsilon_2}\right)^2. \quad (23)$$

In the general case, for arbitrary frequency of the electromagnetic wave and $\varepsilon_1/\varepsilon_2$, an exact formula for the probability distribution of pairs over the total energy $\mathscr{E}_{\text{total}}=\mathscr{E}_{\text{electron}}+\mathscr{E}_{\text{positron}}$ of the produced particles can be derived from Eq. (18):

$$\frac{dW}{d\mathscr{E}_{\text{total}}}=\frac{e^2E_0^2}{6\pi^2}\left(1-\frac{\varepsilon_1}{\varepsilon_2}\right)^2\left(1-\frac{4m^2}{\mathscr{E}_{\text{п}}^2-k^2}\right)^{1/2}$$

$$\times\frac{\mathscr{E}_{\text{total}}^2(\mathscr{E}_{\text{total}}^2+\omega^2)(\mathscr{E}_{\text{total}}^2+2m^2-k^2)}{(\mathscr{E}_{\text{total}}^2-\omega^2)^2(\mathscr{E}_{\text{total}}^2-\omega^2\varepsilon_1/\varepsilon_2)^2}. \quad (24)$$

[1)] Since the expressions (19)–(22) correspond to the case $\omega\ll m$, the pole in Eq. (19) can be reached, i.e., the laws of conservation of energy and momentum for the process $\gamma\to e^++e^-$ can be satisfied only if $\varepsilon_1/\varepsilon_2\gg1$. This is possible in reality for $\varepsilon_2\ll1$, in agreement with the fact that pair production by a photon field requires that the medium be a plasma.[1,2]

[2)] The quantities $\hbar$ and c are restored in the formula (23).

[1] G. K. Avetisyan, A. K. Avetisyan, and R. G. Petrosyan, Zh. Eksp. Teor. Fiz. **75**, 382 (1978) [Sov. Phys. JETP **48**(2), 192 (1978)].

[2] G. K. Avetisyan, A. K. Avetisyan, and Kh. V. Sedrakyan, Zh. Eksp. Teor. Fiz. **94**, 21 (1988) [Sov. Phys. JETP **67**(4), 660 (1988)].

[3] G. K. Avetisyan, A. K. Avetisyan, and Kh. V. Sedrakyan, Zh. Eksp. Teor. Fiz. **99**, 50 (1991) [Sov. Phys. JETP **72**(1), 26 (1991)].

[4] V. L. Ginzburg, *Theoretical Physics and Astrophysics*, Pergamon Press, New York (1979).

[5] L. V. Keldysh. Zh. Eksp. Teor. Fiz. **47**, 1945 (1964) [Sov. Phys. JETP **20**(5), 1307 (1965).

[6] F. V. Bunkin and A. E. Kazakov, Zh. Eksp. Teor. Fiz. **59**, 2233 (1970) [Sov. Phys. JETP **32**(6), 1208 (1971)].

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