Effect of Langmuir-plasmon diffusion on echos in a weakly turbulent plasma

V. P. Pavlenko and S. M. Revenchuk

Institute of Nuclear Research, Academy of Sciences of the Ukrainian SSR
(Submitted 23 October 1990)
Zh. Eksp. Teor. Fiz. **100**, 815–824 (September 1991)

The attenuation of spatial and spatial-temporal echoes in a weakly turbulent plasma as a result of nonlinear processes is analyzed. The nonlinear processes considered are decay interactions of plasma waves and ion acoustic waves. These interactions are of a diffusive nature. It is possible to determine the plasmon diffusion coefficient in wave-number space either from the Fourier spectrum of the envelope of the spatial-echo signal or from the way the amplitude of the spatial-temporal echo at a fixed point depends on the distance and on the time interval between the pulses of external perturbations which excite the echo.

1. INTRODUCTION

The existence of an echo in a weakly turbulent plasma was first demonstrated by Nemtsov and Éĭdman[1] in the particular case of Langmuir turbulence. Underlying this effect is the retention of a memory of external perturbations in the form of undamped oscillations of the spectral density of Langmuir plasmons. These oscillations are of the same nature as Van Kampen modes in a quiescent plasma. They consist of a set of modulated electron streams. External perturbations, given as low-frequency ion acoustic waves, undergo resonant damping in the gas of Langmuir plasmons. Damping of this sort, which was described by Vedenov and Rudakov,[2] stems from the coincidence of the phase velocity of the sound waves with the group velocity of the plasmons. Like Landau damping, it is not accompanied by irreversible dissipation of the energy of the ion acoustic wave.

It was suggested in Ref. 1 and in some later studies[3–7] of echo effects in weakly turbulent plasmas that the perturbation of the plasmon spectral density is undamped. Mathematically, this assumption corresponds to a zero right side of the kinetic equation for the waves. We know from the theory of echos in a quiescent plasma[8–10] that this phenomenon is exceedingly sensitive to any effect which disrupts the phase memory. Falling in the latter category, in particular, are Coulomb collisions which are accompanied by deflections of the charged particles through small angles. Microturbulence would also fall in this category. This circumstance was utilized in Refs. 11–13 for an experimental determination of the electron mean free path or the diffusion coefficient in velocity space. This quantity appears in the Fokker–Planck collision integral.

In the case of a weakly turbulent plasma, effects which disrupt the phase memory are nonlinear interactions of waves and particles, which are described by the right-hand side of the wave kinetic equation. In the present paper we examine the effect of decay interactions of Langmuir waves (or plasma waves) and ion acoustic waves on the spatial and spatial-temporal echos in weakly turbulent plasmas. Since the ratio of the frequencies of the acoustic and plasma waves is small, these decay interactions are of a diffusive nature,[14] like Coulomb collisions in a quiescent plasma. We show below that in this case echo effects can be utilized to determine the diffusion coefficient for Langmuir plasmons in wave-number space. The results derived on the spatial-echo effect were reported briefly in Ref. 15.

2. FORMULATION OF THE PROBLEM; LINEAR APPROXIMATION

We consider a homogeneous, weakly turbulent plasma in the particular case of a Langmuir turbulence. We write the kinetic equation for plasma waves in the form

$$\frac{dN_{\mathbf{k}}}{dt}=\frac{\partial}{\partial k_i}D_{ij}\frac{\partial N_{\mathbf{k}}}{\partial k_j}. \tag{1}$$

The left side of (1) is the Liouville operator d/dt, which acts on the spectral-density function of the Langmuir plasmons, $N_{\mathbf{k}}$, in the phase space of coordinates and wave vectors. The right side of this equation, which describes nonlinear interactions of waves and particles which lead to a relaxation of the perturbations of $N_{\mathbf{k}}$, is written in the form of a plasmon diffusion operator in wave-vector space.[14] This representation is justified for a homogeneous and isotropic plasma, in which the most important linear processes are decay interactions of plasma waves and ion acoustic waves. For them the diffusion tensor D_{ij} becomes

$$D_{ij}(\mathbf{k})=\int\frac{d\mathbf{k}'}{(2\pi)^3}k_i'k_j'n_{\mathbf{k}'}W_{\mathbf{k},\mathbf{k}',\mathbf{k}-\mathbf{k}'},$$

where $n_{\mathbf{k}}$ is a Fourier component of a low-frequency perturbation of the plasma density caused by ion acoustic waves, and $W_{\mathbf{k},\mathbf{k}',\mathbf{k}-\mathbf{k}'}$ is the matrix element representing the interaction of the waves $\mathbf{k}$, $\mathbf{k}'$, and $\mathbf{k}-\mathbf{k}'$.

Under the assumption that the ion acoustic waves are excited by plane grids (Refs. 16 and 17, for example), we specify the external perturbations to be monochromatic waves with frequencies ω_1 and ω_2, which are applied to the plasma in the $z=0$ and $z=l$ planes, respectively:

$$\rho_{\text{ext}}(z,t)=\rho_1\delta\left(\frac{z}{z_0}\right)\exp(i\omega_1 t)+\rho_2\delta\left(\frac{z-l}{z_0}\right)\exp(-i\omega_2 t). \tag{2}$$

Here ρ_1 and ρ_2 are the amplitudes of the external perturbations, and the constant z_0 has the dimensionality of a length.

Choosing the external perturbations in the form (2) corresponds to an analysis of the spatial echo. This choice allows us to restrict the discussion to a spatially one-dimensional problem, by virtue of the presence of the preferred direction $\hat{\mathbf{z}}$. In this case the Liouville operator d/dt becomes

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\frac{\partial\omega_p}{\partial k_z}\frac{\partial}{\partial z}-\frac{\partial\omega_p}{\partial z}\frac{\partial}{\partial k_z}, \tag{3}$$

and we are left with only a single term on the right side of

 0038-5646/91/090451-05$03.00 © 1991 American Institute of Physics

(1). That term contains D_{zz}. In (3) we have used the notation

$$\omega_p^2(z)=\omega_0^2(1+n/n_0), \quad \omega_0^2=4\pi e^2 n_0/m,$$

where n is the perturbation of the equilibrium plasma density n_0 by the ion acoustic waves. The latter waves are described by the equation for the density perturbation:

$$\frac{\partial^2 n}{\partial t^2}-c_s^2\frac{\partial^2 n}{\partial z^2}=\frac{\omega_0}{2M}\frac{\partial^2}{\partial z^2}\int\frac{d\mathbf{k}}{(2\pi)^3}N_{\mathbf{k}}+\rho_{\text{ext}}, \tag{4}$$

where c_s is the isothermal sound velocity, and m and M are the masses of an electron and an ion, respectively.

We solve Eqs. (1) and (4) by successive approximations, writing the perturbation of the plasma density and the spectral density of the plasmons in series:

$$n=n_0+n^{(1)}+n^{(2)}+\ldots, \quad N_{\mathbf{k}}=N_{\mathbf{k}}^{(0)}+N_{\mathbf{k}}^{(1)}+N_{\mathbf{k}}^{(2)}+\ldots,$$

where $N_{\mathbf{k}}^{(0)}$ is the unperturbed three-dimensional spectrum of the Langmuir turbulence, and the superscripts 1 and 2 specify the deviations of the corresponding quantities from their equilibrium values which are respectively linear and quadratic in the external perturbations. To determine the effect of the right side of (1) on the perturbation of the spectral density of the plasmons, we use O'Neil's approximate method[10] for solving a kinetic equation with a diffusive collision integral. We start from the assumption that the diffusion coefficient D_{zz} is small. Near the planes in which the external perturbations are applied (for definiteness, we will discuss the $z=0$ plane), we can then ignore the right side of the equation for the linear perturbation of the plasmon spectral density,[3]

$$\frac{\partial N_{\mathbf{k}}^{(1)}}{\partial t}+\frac{\partial\omega_p}{\partial k_z}\frac{\partial N_{\mathbf{k}}^{(1)}}{\partial z}-\frac{\omega_0}{2n_0}\frac{\partial n^{(1)}}{\partial z}\frac{\partial N_{\mathbf{k}}^{(0)}}{\partial k_z}=\frac{\partial}{\partial k_z}\left(D_{zz}\frac{\partial N_{\mathbf{k}}^{(1)}}{\partial k_z}\right) \tag{5}$$

in comparison with the last term on the left side. From (5) and (4) we have the following result for the first of the perturbations in (2), in the linear approximation:

$$N_{\mathbf{k}}^{(1)}(z,\omega)|_{z\to 0}=i\frac{\rho_1 z_0\omega_0\omega}{2n_0 v_{gz}^2}\theta(v_{gz})\frac{2\pi\delta(\omega+\omega_1)}{\varepsilon(\omega/v_{gz},\omega)}\frac{\partial N_{\mathbf{k}}^{(0)}}{\partial k_z}\times\exp\left(i\frac{\omega}{v_{gz}}z\right), \tag{6}$$

where $\theta(x)$ is the unit step function, $v_{gz}\equiv\partial\omega_p/\partial k_z$ is a component of the group velocity of the Langmuir plasmons, and

$$\varepsilon(\varkappa,\omega)=\varkappa^2c_s^2-\omega^2-\frac{\omega_0^2\varkappa^3}{4n_0 M}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\partial N_{\mathbf{k}}^{(0)}/\partial k_z}{\omega-\varkappa v_{gz}+i\cdot 0}. \tag{7}$$

In solving Eqs. (4) and (5) we used Fourier transforms in space and time:

$$A(\varkappa,\omega)=\int dz\exp(-i\varkappa z)\int dt\exp(i\omega t)A(z,t).$$

With distance from the $z=0$ plane, the ion acoustic density perturbation $n^{(1)}$ undergoes resonant damping in the gas of Langmuir plasmons. This damping is analogous to collisionless Landau damping. At distances greater than the length scale for this resonant damping we can ignore the last term on the left side in (5), and we can describe the function $N_{\mathbf{k}}^{(1)}$ by the equation

$$-i\omega N_{\mathbf{k}}^{(1)}(z,\omega)+v_{gz}\frac{\partial}{\partial z}N_{\mathbf{k}}^{(1)}(z,\omega)=\frac{\partial}{\partial k_z}\left[D_{zz}\frac{\partial}{\partial k_z}N_{\mathbf{k}}^{(1)}(z,\omega)\right], \tag{8}$$

supplemented with a boundary condition. This condition states that the solution of (8) becomes (6) at small values of z. Using the assumption (made above) that the diffusion coefficient D_{zz} is small, we can replace the derivatives $\partial/\partial k_z$ on the right side of (8) by their approximations found by differentiating the most rapidly varying factor:

$$\frac{\partial}{\partial k_z}N_{\mathbf{k}}^{(1)}(z,\omega)\approx -i\frac{3\omega v_{Te}^2 z}{\omega_0 v_{gz}^2}N_{\mathbf{k}}^{(1)}(z,\omega).$$

Under the assumption that D_{zz} is independent of the coordinate z in the homogeneous plasma, we then finally find

$$N_{\mathbf{k}}^{(1)}(z,t)=-i\frac{\rho_1 z_0\omega_0\omega_1}{2n_0 v_{gz}^2}\theta(v_{gz})\frac{\partial N_{\mathbf{k}}^{(0)}}{\partial k_z}\varepsilon^{-1}\left(-\frac{\omega_1}{v_{gz}},-\omega_1\right)\times\exp\left(i\omega_1 t-i\frac{\omega_1 z}{v_{gz}}-\frac{3\omega_1^2 v_{Te}^4 D_{zz}z^3}{\omega_0^2 v_{gz}^5}\right). \tag{9}$$

The linear perturbation of the plasma density is

$$n^{(1)}(\varkappa,\omega)=\rho_{\text{ext}}(\varkappa,\omega)/\varepsilon(\varkappa,\omega). \tag{10}$$

In (10) we have ignored the contribution from the right side of Eq. (5) to dispersion function (7) by virtue of our assumption that the plasmon diffusion coefficient in wavenumber space is small. This condition is equivalent to the condition that the length scale for the resonant damping of the sound in the gas of Langmuir plasmons be much smaller than the length scale for diffusive damping.

3. SPATIAL ECHO

Working in second-order perturbation theory, we find the following expression for the function $N_{\mathbf{k}}^{(2)}$, which is quadratic in the external perturbations [this expression is valid near the point $z=l$, where we can ignore the right side of Eq. (1)]:

$$N_{\mathbf{k}}^{(2)}(z,\omega)|_{z\to l}=i\frac{\omega_0\omega}{2n_0 v_{gz}^2}\frac{\partial N_{\mathbf{k}}^{(0)}}{\partial k_z}n^{(2)}\left(\frac{\omega}{v_{gz}},\omega\right)\exp\left(i\frac{\omega}{v_{gz}}z\right)$$
$$+i\frac{3v_{Te}^2\rho_1\rho_2 z_0^2\omega_0\omega_1\omega_2\omega_3 l}{4n_0^2 v_{gz}^6}\frac{\partial N_{\mathbf{k}}^{(0)}}{\partial k_z}\times\frac{2\pi\delta(\omega-\omega_3)\theta(v_{gz})}{\varepsilon(-\omega_1/v_{gz},-\omega_1)\varepsilon(\omega_2/v_{gz},\omega_2)}\times\exp\left(i\frac{\omega(z-l)-\omega_1 l}{v_{gz}}-\frac{3\omega_1^2 v_{Te}^4 D_{zz}l^3}{\omega_0^2 v_{gz}^5}\right). \tag{11}$$

The second term on the right side of (11) describes the nonlinear perturbation of the plasmon spectral density at the frequency $\omega_3=\omega_2-\omega_1$ caused by the sources (2). The evolution of this perturbation results in the excitation of a spatial-echo signal. This echo results from modulation of the linear perturbation of the spectral density (9), from the first source, as a result of a linear perturbation of the density of the form (10), caused by the second source.

We describe the effect of the plasmon diffusion on the function (11) in basically the same way as in the preceding section of this paper for the linear perturbation $N_{\mathbf{k}}^{(1)}$. There is the difference that in the present case the derivatives on the right side of Eq. (1) in the region $z>l$ are given approximately by the following expression, as can be seen from (11):

$$\frac{\partial}{\partial k_z}N_{\mathbf{k}}^{(2)}(z,\omega)\approx -i\frac{\omega(z-l)-\omega_1 l}{v_{gz}^2}\frac{3v_{Te}^2}{\omega_0}N_{\mathbf{k}}^{(2)}(z,\omega).$$

Ignoring the effect of plasmon diffusion on the dispersion properties of the plasma, we find

$$N_{\mathbf{k}}^{(2)}(z,\omega)=i\frac{\omega_0\omega}{2n_0v_{gz}^2}\frac{\partial N_{\mathbf{k}}^{(0)}}{\partial k_z}n^{(2)}\left(\frac{\omega}{v_{gz}},\omega\right)\exp\left(i\frac{\omega}{v_{gz}}z\right)$$
$$+i\frac{3\rho_1\rho_2 v_{Te}^2 z_0^2\omega_0\omega_1\omega_2\omega_3 l}{4n_0^2 v_{gz}^6}\frac{\partial N_{\mathbf{k}}^{(0)}}{\partial k_z}$$
$$\times\frac{\theta(v_{gz})\theta(z-l)2\pi\delta(\omega-\omega_3)}{\varepsilon(-\omega_1/v_{gz},-\omega_1)\varepsilon(\omega_2/v_{gz},\omega_2)}$$
$$\times\exp\left(i\frac{\omega(z-l)-\omega_1 l}{v_{gz}}-\frac{3\omega_1^2 v_{Te}^4 D_{zz}l^3}{\omega_0^2 v_{gz}^5}-\frac{9v_{Te}^4 D_{zz}}{\omega_0^2 v_{gz}^5}\int_l^z[\omega(x-l)-\omega_1 l]^2dx\right). \quad (12)$$

The second term in the exponential factor in the last term in (12) describes diffusive damping of the linear perturbation $N_{\mathbf{k}}^{(1)}$ in the region $0<z<l$. The third term describes damping of the nonlinear perturbation $N_{\mathbf{k}}^{(2)}$ at $z>l$. The step functions $\theta(v_{gz})$ and $\theta(z-l)$ reflect the circumstance that in this formulation of the problem an echo can arise only in the region $z>l$, and the only plasmons which contribute to it are those whose z projections of the group velocity are positive.

Substituting (12) into (4), and taking inverse Fourier transforms in space and time, we find the echo perturbation of the plasma density, which is the nonlinear response to external perturbation (2):

$$n^{(2)}(z,t)=-i\frac{3\rho_1\rho_2 z_0^2 v_{Te}^2\omega_0^2\omega_1\omega_2\omega_3^2 l}{8n_0^2M}\int\frac{d\mathbf{k}}{(2\pi)^3}$$
$$\times\frac{\theta(v_{gz})}{v_{gz}^8}\frac{\partial N_{\mathbf{k}}^{(0)}}{\partial k_z}\frac{\exp[i\omega_3(z-l')v_{gz}^{-1}-i\omega_3 t]}{\varepsilon(-\omega_1/v_{gz},-\omega_1)\varepsilon(\omega_2/v_{gz},\omega_2)\varepsilon(\omega_3/v_{gz},\omega_3)}$$
$$\times\exp\left(-\frac{3\omega_1^2\omega_2 v_{Te}^4 D_{zz}l^3}{\omega_0^2\omega_3 v_{gz}^5}\right), \quad (13)$$

where $l'=l\omega_2/\omega_3$. The integral in (13) is zero except near the point $z=l'$, because of the rapidly oscillating exponential function in the integrand. When the function (12) is substituted into the expression for $n^{(2)}$, the upper limit on the integration over z of the exponential factor describing the diffusive damping of the echo in the region $z>l$ is therefore replaced by the value $z=l'$.

Since there is a preferred direction, the z direction, in this problem, it is convenient to switch to cylindrical coordinates $(k_\perp, k_z, \varphi)$ in wave-vector space, with the new volume element

$$d\mathbf{k}=k_\perp dk_\perp dk_z d\varphi.$$

In the case of an isotropic Langmuir-turbulence spectrum $N_{\mathbf{k}}^{(0)}$, the integrand in (13) is independent of the azimuthal angle φ, and only the function $N_{\mathbf{k}}^{(0)}$ depends on $k_\perp$. In the new variables we have[7]

$$\int_0^\infty dk_\perp k_\perp\,\partial N_{\mathbf{k}}^{(0)}/\partial k_z=-k_z N_{\mathbf{k}}^{(0)}(k_z), \quad (14)$$

and expression (13) becomes

$$n^{(2)}(z,t)=i\frac{\rho_1\rho_2 z_0^2\omega_0\omega_1\omega_2\omega_3^3 l}{8n_0^2M(2\pi)^2}\left(\frac{\omega_0}{3v_{Te}^2}\right)^7\int_0^\infty\frac{dk_z}{k_z^7}N_{\mathbf{k}}^{(0)}(k_z)$$
$$\times\frac{\exp[i\omega_3(z-l')v_{gz}^{-1}-i\omega_3 t-(\omega_1^2\omega_2\omega_0^3D_{zz}l^3/81\omega_3 v_{Te}^6k_z^5)]}{\varepsilon(-\omega_1/v_{gz},-\omega_1)\varepsilon(\omega_2/v_{gz},\omega_2)\varepsilon(\omega_3/v_{gz},\omega_3)}. \quad (15)$$

The integration in (15) can be carried out with the help of Cauchy's theorem, if we go over to the complex k_z plane. The integration contour is closed in the upper or lower half-plane, depending on the sign of the difference $z-l'$. The integral is dominated by the poles of the dispersion functions ε, which are conveniently written in the form[3]

$$\varepsilon(\varkappa,\omega)=[\varkappa c_s+\omega(1+i\gamma)][\varkappa c_s-\omega(1+i\gamma)]. \quad (16)$$

Here γ is the dimensionless damping rate of an ion acoustic wave in the gas of Langmuir plasmons, given by

$$\gamma=(\omega_0^4/288\pi n_0Mc_sv_{Te}^4)N_{\mathbf{k}}^{(0)}(k_0),\quad k_0=c_s\omega_0/3v_{Te}^2.$$

We finally find a result for the spatial-echo signal:

$$n^{(2)}(z,t)=-\frac{3\rho_1\rho_2 z_0^2 v_{Te}^2\omega_3 c_s l}{8n_0\omega_1\omega_2\omega_0^2}$$
$$\times\exp\left[-i\omega_3 t+i\frac{\omega_3}{c_s}(z-l')-\frac{\gamma\omega_3}{c_s}|z-l'|\right]$$
$$\times\exp\left(-\frac{\omega_1^2\omega_2\omega_0^3D_{zz}l^3}{81\omega_3 v_{Te}^6k_0^5}\right)$$
$$\times\left[\frac{1}{\gamma}\theta(l'-z)+\frac{\omega_3(z-l')}{c_s}\theta(z-l')\right]. \quad (17)$$

Comparison of (17) with the results of Refs. 3 and 7, which were derived without the right side of Eq. (1), shows that, under our assumption that the plasmon diffusion coefficient in wave-number space is small, the influence of this coefficient on the spatial echo reduces to an exponential decrease in the amplitude, with an argument proportional to D_{zz} and l^3.

4. SPACE-TIME ECHO

In the study of the spatial echo in a weakly turbulent plasma above, we considered sources of external perturbations which were point sources spatially. Strictly speaking, such external perturbations would contradict the condition for the applicability of the adiabatic approximation—a condition which was used in the derivation of Eqs. (1) and (4). However, it was shown in Ref. 7 that in the case of a spatial echo whose signal is an integral over the entire wave-number spectrum the results for point sources differ in no fundamental way from the results for spatially extended sources. The situation is different in the case of a space-time echo excited by external perturbations which are localized in both space and time. In this case it is a matter of fundamental importance to take the finite spatial size of the sources into account.[7] We accordingly consider the space-time echo excited in a weakly turbulent plasma by external perturbations which are wave packets in space and time:

$$\rho_{ext}(z,t)=\rho_1\exp(i\omega_1 t)[\theta(t)-\theta(t-\tau)]\int\frac{dk}{2\pi}A_1(k)\exp(ikz)$$
$$+\rho_2\exp(-i\omega_2 t)[\theta(t-T)-\theta(t-T-\tau)]$$
$$\times\int\frac{dk}{2\pi}A_2(k)\exp[ik(z-l)]. \quad (18)$$

In time, according to (18), the sources are packets of high-frequency waves with rectangular envelopes with a length $\tau \ll T$, which are excited at the times $t=0$ and $t=T$. In space, the external perturbations are wave packets with spectral functions $A_{1,2}(k)$, which are localized at the points $z=0$ and $z=l$, respectively. We restrict the discussion below to packets of Lorentzian shape:

$$A_{1,2}(k)=2\varkappa_{1,2}/[(k\pm k_{1,2})^2+\varkappa_{1,2}^2]. \quad (19)$$

The linear response of the spectral density of the plasmons to the first of the perturbations in (18) is

$$N_{\mathbf{k}}^{(1)}(z,t)=-i\frac{\rho_1\omega_0\omega_1}{2n_0 v_{gz}^2}\frac{\partial N_{\mathbf{k}}^{(0)}}{\partial k_z}\theta(v_{gz})A_1\left(-\frac{\omega_1}{v_{gz}}\right)$$
$$\times\varepsilon^{-1}\left(-\frac{\omega_1}{v_{gz}},-\omega_1\right)$$
$$\times\exp\left(i\omega_1 t-i\frac{\omega_1}{v_{gz}}z-\frac{3\omega_1^2 v_{Te}^4 D_{zz}z^3}{\omega_0^2 v_{gz}^5}\right)$$
$$\times\left[\theta\left(t-\frac{z}{v_{gz}}\right)-\theta\left(t-\frac{z}{v_{gz}}-\tau\right)\right]. \quad (20)$$

Solving Eqs. (1) and (4) by the same method as was used for the spatial echo, but now for the external perturbations (18), we find the following result for the nonlinear perturbation of the plasma density, in place of (15):

$$n^{(2)}(z,t)=i\frac{\rho_1\rho_2\omega_0^3\omega_1^2\omega_2\omega_3^2 l}{8n_0^2 M(2\pi)^2}\exp(-i\omega_3 t)$$
$$\times\int_0^\infty\frac{dk_z}{v_{gz}^7}N_{\mathbf{k}}^{(0)}(k_z)A_1\left(-\frac{\omega_1}{v_{gz}}\right)$$
$$\times A_2\left(\frac{\omega_2}{v_{gz}}\right)\frac{\exp[i\omega_3(z-l')v_{gz}^{-1}-(\omega_1^2\omega_2\omega_0^3 D_{zz}l^3/81\omega_3 v_{Te}^6 k_z^5)]}{\varepsilon(-\omega_1/v_{gz},-\omega_1)\varepsilon(\omega_2/v_{gz},\omega_2)\varepsilon(\omega_3/v_{gz},\omega_3)}$$
$$\times\left\{\theta\left(t-\frac{z-l}{v_{gz}}-T\right)\left[\theta\left(v_{gz}-\frac{l}{T}\right)-\theta\left(v_{gz}-\frac{l}{T-\tau}\right)\right]\right.$$
$$-\theta\left(t-\frac{z-l}{v_{gz}}-T-\tau\right)$$
$$\left.\times\left[\theta\left(v_{gz}-\frac{l}{T+\tau}\right)-\theta\left(v_{gz}-\frac{l}{T}\right)\right]\right\}, \quad (21)$$

where v_{gz} and k_z are related by $v_{gz}=k_z(3v_{Te}^2/\omega_0)$.

The integration over k_z in (21) can be carried out with the help of the mean value theorem, after we have made use of the circumstance that the integrand contains the difference between step functions with approximately equal arguments. As a result we find the following expression for the echo signal at the point of its spatial maximum, $z=l'$:

$$n^{(2)}(l',t)=i\frac{3\rho_1\rho_2\omega_0^2 v_{Te}^2 k_0^2 v_0}{8n_0^2 M\omega_2 c_s^2(2\pi)^2}N_{\mathbf{k}}^{(0)}\left(\frac{k_0 v_0}{c_s}\right)A_1\left(-\frac{\omega_1}{v_0}\right)$$
$$\times A_2\left(\frac{\omega_2}{v_0}\right)[c_s^2-v_0^2(1+i\gamma)^2]^{-2}[c_s^2-v_0^2(1-i\gamma)^2]^{-1}$$
$$\times\exp\left(-i\omega_3 t-\frac{3\omega_1^2\omega_2 v_{Te}^2 D_{zz}l^3}{\omega_0^2\omega_3 v_0^5}\right)E(t). \quad (22)$$

Here $v_0=l/T$, and for the dispersion functions ε we have used the representation (16). The quantity $E(t)$ is the temporal envelope of the echo signal:

$$E(t)\approx\frac{\omega_3}{\omega_1}\left(t-\frac{\omega_2}{\omega_3}T+\frac{\omega_1}{\omega_3}\tau\right)$$
$$\times\left[\theta\left(t-\frac{\omega_2}{\omega_3}T+\frac{\omega_1}{\omega_3}\tau\right)-\theta\left(t-\frac{\omega_2}{\omega_3}T\right)\right]$$
$$+\tau\left[\theta\left(t-\frac{\omega_2}{\omega_3}T\right)-\theta\left(t-\frac{\omega_2}{\omega_3}T-\tau\right)\right]$$
$$+\left[\tau-\frac{\omega_3}{\omega_1}\left(t-\frac{\omega_2}{\omega_3}T-\tau\right)\right]$$
$$\times\left[\theta\left(t-\frac{\omega_2}{\omega_3}T-\tau\right)-\theta\left(t-\frac{\omega_2}{\omega_3}T-\frac{\omega_2}{\omega_3}\tau\right)\right].$$

We wish to stress that expression (22) is valid under the condition that no poles of the dispersion functions are present in the integration interval in (21); i.e., this expression is valid under the condition $v_0\neq c_s$. The value of the velocity v_0 is determined by the distance and by the time interval between the two pulses of external perturbations which excite the spatial-temporal echo. It can serve as an adjustable parameter. In the case $v_0\approx c_s$ the amplitude of the echo signal has a resonant peak, which stems from the poles of the dispersion functions and which is described by an expression like (17).

5. DETERMINATION OF THE PLASMON DIFFUSION COEFFICIENT

The results found above for the spatial and spatial-temporal echos as a function of the diffusion coefficient for Langmuir plasmons in wave-number space can be utilized for an experimental determination of this diffusion coefficient. Specifically, it follows from (17) that one can use the slope of a log plot of the spatial-echo amplitude versus the cube of the distance l between the sources of the external perturbations to find D_{zz}. That method is acceptable if the diffusion coefficient D_{zz} is constant or a very weak function of v_{gz}, because we are dealing with $D_{zz}(c_s\omega_0/3v_{Te}^2)$ after the integration over the poles of the dispersion functions in (17).

By using the space-time echo effect, we can avoid this limitation. If no poles of the dispersion functions ε or the spectral distributions $A_{1,2}$ of the external perturbations appear in the integration interval in (21), we have the value $D_{zz}(v_0\omega_0/3v_{Te}^2)$ in expression (22), where the velocity v_0 can be varied by experimentally varying l or T. Using (22), we can thus find the functional dependence $D_{zz}(v)$ and also the value of c_s from the characteristic resonant peak on the curve of the echo-signal amplitude versus v_0. The unperturbed plasmon spectral-density function $N_{\mathbf{k}}^{(0)}$ in (22) can be determined either by means of the space-time echo effect or from the Fourier spectrum of the spatial envelope of the spatial-echo signal.[5-7] The distance l between the sources of the external perturbations must be chosen small enough that the attenuation of the echo due to the right side of Eq. (1) can be regarded as negligibly small.

There is also the possibility of determining the functional dependence $D_{zz}(k_z)$ by means of the spatial echo. This possibility is based on a method proposed by Dryakhlushin and Romanov[18] for reconstructing the electron distribution function of a quiescent plasma from the spatial spectrum of an echo signal. For this purpose, one needs to know the experimental profile of the echo signal as a function of the coordinate z. We use this information to find the Fourier spectrum of the echo, $n^{(2)}(\varkappa)$. On the other hand, if we ignore the time dependence we find from (15)

$$n^{(2)}(\varkappa)=\int dz\exp[-i\varkappa(z-l')]n^{(2)}(z)$$

$$=\frac{\rho_1\rho_2 z_0^2\omega_0^2\omega_1\omega_2 l}{96\pi n_0^2 M v_{Te}^2\omega_3^3}\varkappa^5 N_{\mathbf{k}}^{(0)}\left(\frac{\omega_3\omega_0}{3v_{Te}^2\varkappa}\right)$$

$$\times\frac{\exp[-(3\omega_1^2\omega_2 v_{Te}^4 l^3 D_{zz}\varkappa^5/\omega_3^6\omega_0^2)]}{\varepsilon(-\varkappa\omega_1/\omega_3,-\omega_1)\varepsilon(\varkappa\omega_2/\omega_3,\omega_2)\varepsilon(\varkappa,\omega_3)},\quad(24)$$

where $D_{zz}=D_{zz}(\omega_3\omega_0/3v_{Te}^2\varkappa)$. The plasmon diffusion coefficient in wave-number space can be calculated by taking the logarithm of (24), into which we substitute the experimental values of $n^{(2)}(\varkappa)$ and $N_{\mathbf{k}}^{(0)}$, the known plasma parameter values, and the expressions for the dispersion functions.

These echo effects can evidently occur if the damping of external perturbations in a gas of Langmuir plasmons, with a linear damping rate $\gamma_L=\varkappa c_s\gamma$, is faster than the nonlinear damping of plasma waves which destroys the memory in the system and which occurs at a rate $\gamma_{\rm NL}$. We take the latter rate to be the reciprocal of the time scale of the decay interaction of plasma waves and ion acoustic waves:[14]

$$\gamma_{NL}=\frac{\pi}{4}\frac{k_0}{\Delta k}\frac{\omega_0 W}{n_0 m v_{Te}^2},$$

where Δk is the initial width of the spectrum of plasma waves. In the case at hand, of a decay interaction of a diffusion type, this width is far larger than k_0. In addition, W is the energy of the plasma waves, given by

$$W=\int W_{\mathbf{k}}\,d\mathbf{k},\qquad W_{\mathbf{k}}=\omega_0 N_{\mathbf{k}}/(2\pi)^3.$$

Since the condition for the applicability of this theory ($\gamma_{\rm L}>\gamma_{\rm NL}$) must hold for all resonant plasmons, i.e., for all $k\geqslant k_0$, we assume

$$N_{\mathbf{k}}=N_{\mathbf{k}}^{(0)}(k_0)(k/k_0)^{-s},\quad s>3.$$

The condition $\gamma_{\rm L}>\gamma_{\rm NL}$ then becomes

$$\frac{k_0^2}{\varkappa\Delta k}(k_0 r_{De})^2<\frac{s-3}{72}\frac{m}{M}.$$

On the other hand, the gas of Languir plasmons remains homogeneous as long as the wave energy satisfies the condition

$$\frac{1}{(2\pi)^3}N_{\mathbf{k}}<\frac{3}{2}\frac{n_0T_e}{\omega_0}(kr_{De})^2,\qquad k>k_0.$$

This condition means[2] that there is no modulation instability. The condition for the applicability of the adiabatic approximation is $k_0/\varkappa\gg1$.

6. CONCLUSION

We have studied the attenuation of spatial and spatial-temporal echos in a weakly turbulent plasma caused by decay interactions of plasma waves and ion acoustic waves. Since these interactions are of a diffusive nature, the effect of the decay processes can be represented as a diffusion of Langmuir plasmons in wave-number space.

The results of this analysis show that the echo effect which arises in a weakly turbulent plasma because the plasmon spectral density retains a memory of external perturbations is exceedingly sensitive to diffusion of plasmons, which leads to a loss of the phase memory of the system. It follows that it is possible to utilize the echo effect to determine the plasmon diffusion coefficient in wave-number space. Two possible measurement techniques have been discussed. In one of them, which is based on the spatial-echo effect, the velocity-dependent diffusion coefficient $D_{zz}(k_z)$ can be determined from the Fourier spectrum of the spatial envelope of the echo signal with the help of expression (24). In the other technique, the spatial-temporal echo, of a pulsed nature, would be utilized. Since the echo signal is generated in this case by only those plasmons whose group-velocity projections v_{gz} fall in a narrow interval $l\tau/T^2$ near a given value v_0, one can determine $D_{zz}(k_z)$ by simply measuring the v_0 dependence of the amplitude of the echo signal at a certain fixed point, e.g., at the point at which this signal is at a maximum, $z=l'$. Recall that expression (22) for the spatial-temporal echo is not valid at $v_0\approx c_s$, in which case its amplitude is determined by the poles of the dispersion functions. A plot of the echo amplitude versus v_0 near the point $v_0=c_s$ will have a resonant peak. One can work from the position of this peak to determine the nonisothermal-sound velocity. The width of this peak will be determined by the dimensionless damping rate γ for an ion acoustic wave in the gas of Langmuir plasmons. This damping rate is related to the plasmon spectral density function.

[1] B. E. Nemtsov and V. Ya. Éĭdman, Fiz. Plazmy **9**, 812 (1983) [Sov. J. Plasma Phys. **9**, 471 (1983)].
[2] A. A. Vedenov and L. I. Rudakov, Dokl. Akad. Nauk SSSR **159**, 767 (1964) [Sov. Phys. Dokl. **9**, 1073 (1965)].
[3] Yu. Nikander, V. P. Pavlenko, and S. M. Revenchuk, Fiz. Plazmy **12**, 402 (1986) [Sov. J. Plasma Phys. **12**, 231 (1986)].
[4] J. Nycander, V. P. Pavlenko, and S. M. Revenchuk, Plasma Phys. Contr. Fusion **28**, 1659 (1986).
[5] J. Nycander, V. P. Pavlenko, and S. M. Revenchuk, Phys. Scr. **34**, 819 (1986).
[6] J. Nycander, V. P. Pavlenko, and S. M. Revenchuk, in *Intern. Conf. on Plasma Phys.*, Kiev, 1987, Proc. Invited Pap. Vol. I, p. 329.
[7] S. M. Revenchuk, Fiz. Plazmy **15**, 414 (1989) [Sov. J. Plasma Phys. **15**, 240 (1989)].
[8] T. M. O'Neil and R. W. Gould, Phys. Fluids **11**, 134 (1968).
[9] C. H. Su and C. Oberman, Phys. Rev. Lett. **20**, 427 (1968).
[10] T. M. O'Neil, Phys. Fluids **11**, 2420 (1968).
[11] T. H. Jensen, J. H. Malmberg, and T. M. O'Neil, Phys. Fluids **12**, 1728 (1969).
[12] C. Moeller, Phys. Fluids **18**, 89 (1975).
[13] H. D. Leppert, H. Schlüter, and K. Wiesemann, Plasma Phys. **23**, 1057 (1981).
[14] V. N. Tsytovich, *Nonlinear Effects in Plasma*, Nauka, Moscow, 1967 (Plenum, New York, 1970).
[15] V. P. Pavlenko and S. M. Revenchuk, Phys. Lett. A **143**, 323 (1990).
[16] H. Ikezi and N. Takahashi, Phys. Rev. Lett. **20**, 140 (1968); H. Ikezi, N. Takahashi, and K. Nishikawa, Phys. Fluids **12**, 853 (1969).
[17] D. R. Baker, N. R. Ahern, and A. Y. Wong, Phys. Rev. Lett. **20**, 318 (1968); A. Y. Wong and D. R. Baker, Phys. Rev. **188**, 326 (1969).
[18] V. F. Dryakhlushin and Yu. A. Romanov, Fiz. Plazmy **5**, 1169 (1979) [Sov. J. Plasma Phys. **5**, 657 (1979)].

Translated by D. Parsons