Finite amplitude fast magnetosonic waves in a low-density plasma

A. V. Danilov

High Temperature Institute, Academy of Sciences of the USSR
(Submitted 21 August 1991)
Zh. Eksp. Teor. Fiz. **101**, 89–98 (January 1992)

We consider exact solutions of the quasi-hydrodynamical equations which describe periodic fast magnetosonic waves propagating in a non-isothermal plasma ($T_e \gg T_i$) at an arbitrary angle (not too close to $\pi/2$) to the external magnetic field. In our considerations we use an effective potential depending on a single parameter. We find the conditions for which the ion dispersion cannot stop the nonlinear steepening of the wave and for which there appears an internal rotational discontinuity in its structure. We compare the results with the observations of low-frequency waves in the region in front of the bow shock wave of planets and comets.

1. INTRODUCTION

Nonlinear waves in low-density laboratory and cosmic plasmas have been well studied.[1] The main efforts have been aimed at collisionless shock waves[2,3] and solitons.[4–7] Far less attention has been paid to finite amplitude periodic waves which are usually considered only as part of the shock wave structure (oscillatory precursor or oscillations behind the front).[8–10]

However, periodic waves in a low-density plasma are of interest by themselves. To a large degree this interest is due to the waves with a frequency of about $0.1\omega_i$ (ω_i is the proton gyrofrequency) observed in the upstream region from the bow shock wave of planets and comets. These low-frequency waves (LFW) have a right-handed polarization in the frame fixed to the solar wind and usually propagate at small angles to the magnetic field. This kind of wave, observed in the Earth's magnetosphere,[11] was studied in detail in Ref. 12. Later, similar waves were observed in the vicinity of other planets, and also of comets. Particularly valuable information was obtained using high-resolution apparatus in studying the Giacobini-Zinner comet.[13]

The measurements have shown that in the neighborhood of the Giacobini-Zinner comet the spectral density of the noise has a maximum in the region of periods of the order of 10^2 s, corresponding to waves of this kind. An extremely high positive correlation between the magnitude of the magnetic field and the plasma density is observed where these waves have a maximum amplitude. This makes it possible to assume that the waves considered are fast magnetosonic waves and that they play an important role in the dynamics of the turbulent plasma near the bow shock wave of the comet.

Earlier studies had already revealed two kinds of low-frequency waves:[5] quasiharmonic waves and a sequence of pulses with a strong nonlinear distortion of their shape. Both kinds of wave are close in frequency and amplitude to the perturbations of the magnetic field [$\Delta B^2/B^2 = O(1)$] but differ greatly in shape. The first ones are almost monochromatic and in the case of a quasilongitudinal propagation the absolute magnitude of the magnetic field and hence the plasma density is practically unchanged in them. For waves of the second kind, the so-called shocklets, a strong nonlinear steepening of the leading front of the pulse and a considerable compression of the plasma [$\Delta\rho/\rho = O(1)$] independent of the propagation direction are characteristic. In the region preceding the front there is as a rule a high-frequency precursor with a frequency of the order of ω_i.

Very similar kinds of LFW have been observed in the vicinity of the Giacobini-Zinner comet.[13] A change from quasimonochromatic waves to shocklets occurred in that case when the apparatus approached the comet. A study of the shocklets showed that there are no jumps in density, like shock waves, in their structure. The sudden change of the magnetic field at the leading front of the shocklet is a rotational discontinuity which is a component of its structure complementing the angle of rotation of $\Delta\mathbf{B}$ in the plane perpendicular to the propagation direction of the wave to 360°.

According to contemporary ideas[14,15] the LFW are generated in the Earth's magnetosphere thanks to cyclotron resonance with ion beams reflected from the bow shock wave. In the vicinity of comets the ion currents generating LFW are formed when gas currents emerging from the comet head are photoionized. The group velocity of the waves are somewhat higher than the Alfvén velocity so that they are carried away by the solar wind. The amplitude of the wave is established due to a balance between the energy influx obtained from the ions and the energy transfer to the region of smaller wavelengths thanks to the nonlinear steepening with subsequent dissipation due to cyclotron absorption.[16] The simplest model of such a wave is a finite amplitude periodic wave propagating in a plasma without ion currents and without dissipation.

We consider in the present paper, in the framework of the single-fluid hydrodynamics of a plasma consisting of cold ions and hot electrons, traveling periodic fast magnetosonic (FMS) waves. We assume that the angles at which the waves propagate relative to the magnetic field are not too close to $\pi/2$ so that the dispersion is determined by the fact that the ion Larmor radius is finite. We carry out our analysis using an effective potential, and the wave amplitude is then determined by a parameter proportional to the energy flux of the wave in a frame fixed to it. We find the conditions under which there appears a rotational discontinuity in the structure of the wave. The results are compared with the observed LFW and also with numerical calculations[15] and with the results of a study of model equations obtained using a modification of the nonlinear Schrödinger equation.[17]

 0038-5646/92/010048-05$05.00

2. BASIC EQUATIONS

We shall start from the quasihydrodynamical equations[18] which describe a collisionless plasma consisting of cold ions and isothermal electrons ($T_i \ll T_e = \text{const}$):

$$d\rho/dt+\rho\nabla\mathbf{V}=0, \tag{1}$$

$$\rho d\mathbf{V}/dt+\nabla p+[\mathbf{B}[\nabla\mathbf{B}]]/4\pi=0, \tag{2}$$

$$\nabla p+Ne(\mathbf{E}+[\mathbf{V}_e\mathbf{B}]/c)=0, \tag{3}$$

$$\nabla\mathbf{B}=0,\quad c[\nabla\mathbf{E}]=\partial\mathbf{B}/\partial t, \tag{4}$$

$$c[\nabla\mathbf{B}]=4\pi Ne(\mathbf{V}-\mathbf{V}_e), \tag{5}$$

where N is the electron and ion density, ρ the plasma density, p the plasma pressure, $\mathbf{E}$ and $\mathbf{B}$ the electric field and magnetic induction field strengths, and $\mathbf{V}$ and $\mathbf{V}_e$ the hydrodynamic velocities of the whole plasma and of the electron component. The electron inertia has been neglected in Eqs. (1) to (5) and we used the assumption that the plasma is quasineutral and also that all characteristic velocities are small compared with the electron thermal velocity and the frequencies are low compared with the ion cyclotron frequency. The equation of state has, in agreement with the assumption that the electrons are isothermal, the form $p/N = \text{const}$.

We shall look for the exact solutions of Eqs. (1) to (5) describing stationary one-dimensional periodic or solitary waves propagating in the laboratory frame along the x axis with a velocity $\mathbf{u} = \{-u,0,0\}$ $(u > 0)$. We change to a frame fixed to the wave. In that frame the unperturbed values of the variables are equal to

$$\rho=\rho_0;\ p=p_0,\ \mathbf{V}=\mathbf{V}_e=\{u,\ 0,\ 0\},\ E_{y0}=0;\ E_{z0}=-u/cB_0\sin\theta;$$
$$\mathbf{B}_0=\{B_0\cos\theta,\ B_0\sin\theta,\ 0\}. \tag{6}$$

Using (3) to eliminate $\mathbf{V}_e$ and assuming that the infinitesimal dissipation is resistive and does not change the plasma momentum we get

$$dB_z/dx=4\pi\rho e(V_xB_y+cE_z)/m_icB_x, \tag{7}$$

$$dB_y/dx=-4\pi\rho e(V_xB_z-cE_y)/m_icB_x, \tag{8}$$

$$B_x=B_0\cos\theta,\ E_y=0,\ E_z=-u/cB_0\sin\theta, \tag{9}$$

$$\rho V_x=\rho_0 u, \tag{10}$$

$$\rho V_xV_y-B_x(B_y-B_0\sin\theta)/8\pi=0, \tag{11}$$

$$\rho V_xV_z-B_xB_z/8\pi=0, \tag{12}$$

$$p-p_0+\rho V_x^2-\rho_0u^2+(B_y^2+B_z^2-B_0^2\sin^2\theta)/8\pi=0, \tag{13}$$

$$\rho V_x(V_x^2+V_y^2+V_z^2)/2-\rho_0u^3/2+pV_x\ln(\rho/\rho_0)$$
$$+c(E_yB_z-E_zB_y)/4\pi-uB_0^2\sin^2\theta/4\pi=S. \tag{14}$$

Equations (10) to (14) express the conservation of the fluxes of matter, of the three momentum components, and of the energy of the plasma. We introduced in them constant terms which reduce these fluxes to zero in the equilibrium state (6).

The only parameter in (7) to (14) which does not depend on the unperturbed state (6) is thus the energy flux S of the wave. This parameter determines the amplitude of the wave by analogy with the total energy in the case of a mechanical oscillator. Since there is no energy flux in the limiting case of linear waves, the wave amplitude must tend to zero as $S\to 0$.

Introducing the dimensionless variables

$$n=N/N_0=\rho/\rho_0,\ v=V_x/u,\ b_y=B_y/B_0,\ b_z=B_z/B_0 \tag{15}$$

and eliminating the transverse components of the velocity and the magnetic field we get from (7) to (14)

$$nv=1, \tag{16}$$

$$(v-1)(1-1/M^2v)+(b_y^2+b_z^2-\sin^2\theta)/2M_A^2=0, \tag{17}$$

$$v^2-1+b_z^2\cos^2\theta/M_A^4+(b_y-\sin\theta)^2\cos^2\theta/M_A^4$$
$$-2(\ln v)/M^2+2(b_y-\sin\theta)\sin\theta/M_A^2=-2\varepsilon, \tag{18}$$

$$b_y(v-\cos^2\theta/M_A^2)-(1-\cos^2\theta/M_A^2)=db_z/d\xi, \tag{19}$$

$$b_z(v-\cos^2\theta/M_A^2)=-db_y/d\xi. \tag{20}$$

Here ξ is a dimensionless Lagrangian coordinate:

$$d/d\xi=v\Delta d/dx,\ \Delta=V_A^2\cos\theta/u\omega_i$$
$$(\omega_i=eB_0/m_ic,\ V_A^2=B_0^2/4\pi\rho_0), \tag{21}$$

and we also introduced the dimensionless parameters

$$M_A^2=u^2/V_A^2,\ M^2=u^2\rho_0/p_0=u^2/V_S^2,\ \varepsilon=-S/\rho_0u^3.$$

Substituting (20) into (17) and (19) and using (18) we find

$$(db_y/d\xi)^2+2U=0, \tag{22}$$

$$d^2b_y/d\xi^2+dU/db_y=0, \tag{23}$$

where

$$U={}^1/_2(v-\cos^2\theta/M_A^2)^2[b_y^2-\sin^2\theta+2M_A^2(v-1)(1-1/M^2v)], \tag{24}$$

while the b_y dependence of v is determined implicitly by the relations

$$b_y=b^*-M_A^2(v-\cos^2\theta/M_A^2)^2[2(1-\cos^2\theta/M_A^2)\sin\theta]^{-1}$$
$$-[(v-1)\cos^2\theta-M_A^2v\ln(v)]/[M^2v(1-\cos^2\theta/M_A^2)\sin\theta], \tag{25}$$

$$b^*=\sin\theta+(M_A^2-\cos^2\theta)/2\sin\theta-\varepsilon M_A^4/[(M_A^2-\cos^2\theta)\sin\theta]. \tag{26}$$

Equations (17) to (20) are thus reduced to the equation of motion (23) and the energy integral (22) in a field with an effective potential $U(b_y)$. We note that this formulation of the problem differs from the one used in Refs. 7 to 9 since in those papers the effective potential depends on two parameters, b_y and b_z.

3. EFFECTIVE POTENTIAL AND DIFFERENT KINDS OF WAVE

We start our considerations with linear waves for which $|b_y - \sin\theta| \ll 1$. Expanding (24) in a series in the vicinity of the point $b_y = \sin\theta$ we get

$$U\simeq A(b_y-\sin\theta)^2+C, \tag{27}$$

where

$$A=[M^2(M_A^2-1)-M_A^2+\cos^2\theta]/2(M^2-1)(M_A^2-\cos^2\theta),$$
$$C=-\varepsilon(M_A^2-\cos^2\theta).$$

According to (27) this kind of potential determines periodic waves with an amplitude which tends to zero as $\varepsilon\to 0$ provided $A > 0$, $C < 0$. In the (M^2, M_A^2) plane this condition determines four regions where periodic waves exist (Fig. 1):

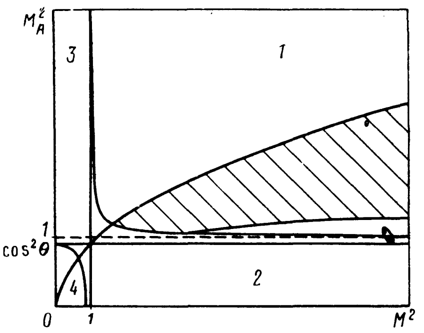

FIG. 1. Regions in the (M_A^2, M^2) plane where the fast (*1*), intermediate (*2,3*), and slow (*4*) waves exist. The hatched part of region *1*, determined by the inequality $M_{Acr}^2 < M_A^2 < M \cos\theta$, corresponds to FMS waves with an internal rotational discontinuity ($\theta = 18°$).

$$\begin{aligned} &1.\ M^2>1,\ M_A^2>(M^2-\cos^2\theta)/(M^2-1),\ \varepsilon>0,\\ &2.\ M^2>1,\ M_A^2<\cos^2\theta,\ \varepsilon<0,\\ &3.\ M^2<1,\ M_A^2>\cos^2\theta,\ \varepsilon>0,\\ &4.\ M^2<1,\ M_A^2<(M^2-\cos^2\theta)/(M^2-1),\ \varepsilon<0. \end{aligned} \tag{28}$$

Changing to dimensional quantities one checks easily that the inequalities 1 to 4 are equivalent to the following inequalities: 1. $u > V_F$; 2. $u < V_A \cos\theta < V_S$; 3. $V_S < V_A \cos\theta < u$; 4. $u < V_{SL}$, where

$$V_{F,\,SL} = \{V_A^2 + V_S^2 \pm [(V_A^2+V_S^2)^2 - 4V_A^2 V_S^2 \cos^2\theta]^{1/2}\}/2$$

are the velocities of the fast (V_F) and the slow (V_{SL}) sound. The effective potential thus enables us to describe all known kinds of low-frequency electromagnetic waves: fast (1), slow (4), and intermediate in the case $V_S > V_A \cos\theta$ (2) and $V_S < V_A \cos\theta$ (3), respectively. The sign of ε in (28) then corresponds to the sign of the dispersion of the wave. Indeed, for positive dispersion the group velocity of the wave is larger than the phase velocity in the frame fixed to the wave and the energy is carried away against the flow of the plasma current, i.e., $\varepsilon > 0$. The signs of ε determined from (28) agree with the well known form of the dispersion curves for waves of the kinds 1 to 4.

In the remaining part of the (M^2, M_A^2) plane in Fig. 1 there are no periodic waves. One can show that this is the region where solitary waves exist. Below we restrict ourselves to considering nonlinear fast magnetosonic waves (region 1).

4. NONLINEAR FMS WAVES

We consider the change in the profile of the FMS waves described by the effective potential (24) when the parameter ε increases. One must in this case take into account that the b_y dependence of v given by Eqs. (25) and (26) is not single-valued. Differentiating b_y with respect to v we find

$$db_y/dv = -M_A^4 (v - \cos^2\theta/M_A^2)(1 - 1/M^2v^2)/[(M_A^2 - \cos^2\theta)\sin\theta]. \tag{29}$$

In the general case there are therefore three values of v corresponding to a single value of b_y (Fig. 2) and the potential (24) has three branches. One can avoid this ambiguity by

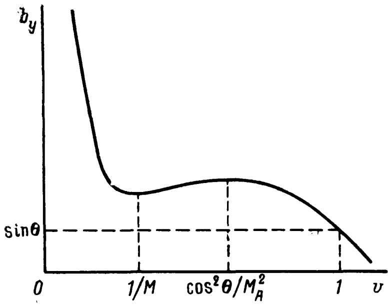

FIG. 2. The b_y dependence of v, determined by Eqs. (25) and (26).

using (25) to change in (22) from the b_y variable to the v variable:

$$(dv/d\xi)^2 + 2W = 0, \tag{30}$$

$$W = 2\{(1-\cos^2\theta/M_A^2)\sin\theta/[2M_A^2(1-1/M^2v^2)]\}^2 [b_y^2 - \sin^2\theta + 2M_A^2(v-1)(1-1/M^2v)], \tag{31}$$

and to use (25) and (26) to express b_y in terms of v. However, one must bear in mind that the Jacobian (29) of the transformation of the variables vanishes for $v = \cos^2\theta/M_A^2$ and for $v = 1/M$ and when v has one of these values Eqs. (30) and (31) may have solutions which do not satisfy Eqs. (22) and (24).

Let ε take on values which are so small that the inequality $v > \max(\cos^2\theta/M_A^2, 1/M)$ is satisfied during the whole of the wave period. In that case b_y is within the limits of one branch of the potential $U(b_y)$, and db_y/dv does not vanish. The forms of the potentials $U(b_y)$ and $W(v)$ for that case and the solution of Eq. (22) are shown in Fig. 3 [b_z^2 is found from (17) and the sign of b_z from (20)]. The waves are right-handedly elliptically polarized, and for small θ circularly, the compression of the plasma is small for quasilongitudinal propagation, and $M_A \cos\theta < M$, but the perturbation of the magnetic field may be appreciable.

When ε increases the deviations of the plasma velocity

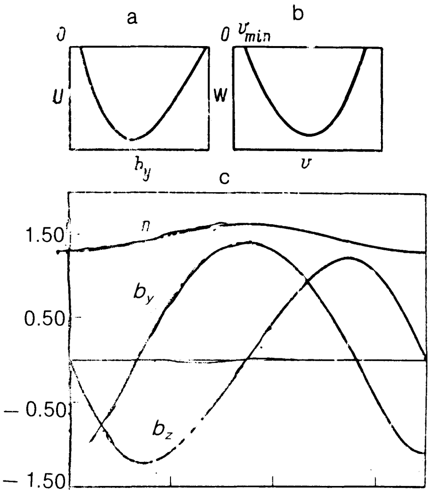

FIG. 3. The effective potentials $U(b_y)$ (a) and $W(v)$ (b) and the profile of a quasiharmonic wave for $\varepsilon < \varepsilon_{cr}$ ($M_A^2 = 3$, $M^2 = 10$, $\theta = 5°$, $\varepsilon = 0.13$) (c).

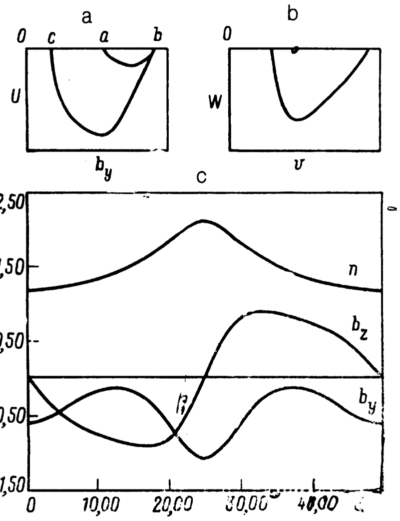

FIG. 4. The effective potentials $U(b_y)$ (a) and $W(v)$ (b) and the profile of a wave with mixed polarization for $\varepsilon > \varepsilon_{cr}$, $M_A^2 < M_{Acr}^2$ ($M_A^2 = 1.4$, $M^2 = 9$, $\theta = 5°$, $\varepsilon = 0.04$) (c).

from its equilibrium value $v = 1$ increase and, starting from some value $\varepsilon = \varepsilon_{cr}$ the minimum value of the velocity v_{min} satisfies the inequality $v_{min} < \max(\cos^2\theta / M_A, 1/M)$. We first consider the case

$$1/M < v_{min} < \cos^2\theta / M_A^2. \tag{32}$$

The potentials $U(b_y)$ $W(v)$, and the profiles of $n = 1/v$, b_y, and b_z are shown in Fig. 4. One shows easily that the solutions satisfy also Eq. (22) and the half-period of the wave corresponds here to the motion of the material point in Fig. 4a from point a to point b along the upper branch of the potential and from the point b to the point c on the lower branch. The continuity of b_z at the point b when it changes from one branch to another is, according to (20), guaranteed by the simultaneous change of the signs of $v - \cos^2\theta / M_A^2$ and $db_y/d\xi$ [we note that $W(v)$ has no singularities at all at the point $v = \cos^2\theta / M_A^2$]. The effective potentials describe in that case an unpolarized wave with a continuous profile and a significant compression of the plasma.

The inequality (32) is satisfied only in a small part of the region satisfying the condition $1/M < \cos^2\theta / M_A^2$ in the (M^2, M_A^2) plane (Fig. 1). In the remaining part of that region the inequality $v_{min} < 1/M$ is satisfied when ε is larger than the critical value ε_{cr} and the potentials $U(b_y)$ and $W(v)$ have the form shown in Fig. 5. There are in that case no periodic waves with a continuous profile. This is particularly clear from Fig. 5a; we shall show below that b_z is continuous in the point b if one changes in that point from one branch of the potential $U(b_y)$ to another, but a finite motion of the effective material point along the line bcd is impossible because the b_y dependence of U is not single valued.

We shall seek for Eqs. (17) to (20) discontinuous solutions satisfying them everywhere except at the discontinuity and also satisfying the appropriate conditions at the discontinuity. The solution shown in Fig. 5c corrsponding to the motion from the point a to the point b on the ab branch in Fig. 5a and returning from b to a along the same branch satisfies these requirements. It satisfies Eq. (22), and hence Eqs. (17) to (20), everywhere, except at the point b. According to (20), b_z has a discontinuity at the point b, changing its sign, while the component b_y is continuous so that the absolute magnitude of the magnetic field is also continuous. Finally, from the relation $v = \cos^2\theta / M_A^2$ which is satisfied in the point b we get, returning to dimensional variables and using (9) and (10), $V_{xb} = B_x/(4\pi\rho_b)^{1/2}$ where V_{xb} and ρ_b are the plasma velocity and density at the point b. The discontinuity at the point b is thus rotational. The solution is thus a right-hand polarized wave with an appreciable compression of the plasma, and it includes the case of quasilongitudinal propagation. Its structure includes an internal rotational discontinuity rotating the tangential component of the magnetic field through not more than 180°.

In conclusion we consider the case $1/M > \cos^2\theta / M_A^2$. In that case it is impossible to introduce at $\varepsilon > \varepsilon_{cr}$ an internal rotational discontinuity which "does not admit" an effective mass at the point $v = 1/M$. In that case there are therefore no periodic waves with $\varepsilon > \varepsilon_{cr}$.

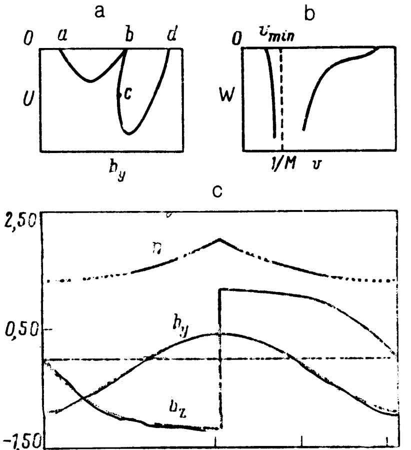

FIG. 5. The effective potentials $U(b_y)$ (a) and $W(v)$ (b) and the profile of a wave with an internal rotational discontinuity for $\varepsilon > \varepsilon_{cr}$, $M_{Acr}^2 < M_A^2 < M \cos^2\theta$ ($M_A^2 = 2$, $M^2 = 10$, $\theta = 5°$, $\varepsilon = 0.1$) (c).

5. DISCUSSION OF THE RESULTS AND CONCLUSION

We have considered how the form of the nonlinear periodic FMS wave depends on four dimensionless parameters: the Alfvén (M_A) and sound (M) Mach numbers, the angle θ between the propagation direction of the wave and the external magnetic field, and the parameter ε which is proportional to the energy flux of the wave in the reference frame fixed to it and which is determined by the wave amplitude. The main results are the following:

1. When $\varepsilon < \varepsilon_{cr}(M_A^2, M^2, \theta)$ the profile of the magnetic field of the wave is quasiharmonic and the wave is polarized elliptically, and for small θ circularly. The compression of the plasma is small, especially for small θ, but the perturbation of the magnetic field may be considerable.

2. When $\varepsilon > \varepsilon_{cr}$ the shape of the wave depends on M_A^2, M^2, and θ. If $M_A^2 < M_{Acr}^2(M^2, \theta)$ the compression of the plasma lies within the range $M_A^2/\cos^2\theta < \rho/\rho_0 < M$. The polarization of the wave is mixed and the profile of the magnetic field differs appreciably from harmonic.

3. In the range of parameter values $M_{A\mathrm{cr}}^2 < M_A^2 < M \cos^2\theta$ the ion dispersion cannot compensate for the nonlinear steepening of the wave, and a rotational discontinuity appears in its structure. This discontinuity occurs there where the wave velocity relative to the plasma is equal to the local Alfvén velocity. The maximum compression of the plasma, $\rho/\rho_0 = M_A^2/\cos^2\theta$, is reached at the same point. Only the z component of the magnetic field has a discontinuity, so that the angle of rotation of the magnetic field component orthogonal to the propagation direction is not more than 180°. The wave is right-handedly polarized and part of the total rotation of the transverse component of the magnetic field occurs in this case at the discontinuity just mentioned. The plasma density at the discontinuity is continuous, but has a vertex. The characteristic wave frequency is, as in the preceding two cases, of the order of $0.1\omega_i$ for $M_A^2 \sim 1$.

4. There are no periodic solutions for $\varepsilon > \varepsilon_{\mathrm{cr}}$ in the $M_A^2 > M \cos^2\theta$ case. This puts a restriction on the plasma parameter β at which observation of the waves considered here is possible: $\beta = 2M_A^2/M^2 < 2 \cos^4\theta/M_A^2$.

Even though the model is obviously crude attention is called to the similarity of the waves considered and the LFW in the vicinity of the Earth and of comets. The properties of the quasiperiodic waves are practically the same as the properties of the waves considered for $\varepsilon < \varepsilon_{\mathrm{cr}}$. The transition from LFW of this kind to shocklets downstream in the solar wind can be connected in a natural way with the increase of the energy flux of the wave due to the interaction with the ion current and to its exceeding the critical value. This explanation of the change in the shape of the LFW does not call for including refraction at the boundary between the regions where two kinds of waves exist,[14] and it can be applied to LFW in the vicinity both of the Earth and of comets.

The observed properties of shocklets are close to the properties of nonlinear FMS waves with an internal rotational discontinuity. We note that the angle of rotation of the magnetic field over the length of the rotational discontinuity in the structure of the FMS wave is less than or equal to 180°, which agrees with observations.[13] The appearance of a high-frequency wave precursor ahead of the front of the rotational discontinuity can be explained by the wave emission of the surface current connected with the discontinuity, similar to what was done in Ref. 15 when the precursor of a shock wave was considered.

The internal structure of the rotational discontinuity cannot be described in the approximation considered here since we neglect dissipation. In the presence of resistive dissipation the width δ of the discontinuity increases with time like $\delta \sim (c^2 t/4\pi\sigma)^{1/2}$,[19] where $\sigma = Ne^2/m\nu$ is the effective resistivity of the plasma and ν the effective collision frequency of the electrons. One can therefore neglect the spreading of the discontinuity, provided $k\delta \ll 1$ where $k \sim \omega_i/V_s$ is the wave number of the LFW and V_s the solar wind velocity. If we assume that one may consider the conductivity in the present case to be resistive, it follows from the observation of the upstream damping of the precursors of the discontinuities with a frequency of the order of ω_i that $\nu \lesssim \omega_i$. Using the fact that the time for the interaction of the LFW with the ion current in front of the Earth's bow shock wave is $t \sim 100$ s,[14] we find $k\delta \sim 10^{-1}$.

Nonlinear MHD waves have also been studied using a model DNLS equation which is obtained through a partial linearization of the equations of the plasma dynamics that removes the nonlinear coupling between the plasma density and the magnetic field oscillations.[20] The modification of this equation, taking dissipation and an external applied force into account, made it possible to obtain a periodic solution containing in its structure an intermediate shock wave.[17] The solution given above in the form of quasiharmonic waves or waves with a mixed polarization are close to the solutions of the DNLS equation obtained for an appropriate choice of the parameters, but the solution with an internal rotational discontinuity does not have an analog amongst the solutions of the DNLS equation. The reason is that this discontinuity appears when the plasma is significantly compressed and then the nonlinear coupling between the oscillations of its density and of the magnetic field becomes important.

The main difference between the results of a numerical simulation[15] and those obtained in the present paper is that the steepening of the wave profile leads to a fast shock wave and not to a rotational discontinuity. This discrepancy cannot be attributed to neglect of the beam ions in the solutions considered above, since "deflecting" the beam during the calculation made practically no change in its results. It is apparently connected with the fact that in the numerical calculation a constant value of the magnetic field was maintained at the boundaries of the chosen range.

The model considered thus made it possible to explain the appearance of rotational discontinuities in the structure of the LFW observed in Ref. 13. To obtain more detailed information one must consistently take into account the interaction of the waves with ion currents and also dissipation.

[1] C. F. Kennel, J. P. Edmiston, and T. Hada, Geophys. Monogr. Ser., R. G. Stone and B. T. Tsurutani (eds.) Vol. 34, AGU, Washington (1985), p. 1.
[2] R. Z. Sagdeev, *Reviews of Plasma Physics*, M. A. Leontovich (ed.), Vol. 4, Consultants Bureau (1966), p. 23.
[3] D. A. Tidman and N. A. Krall, *Shock Waves in Collisionless Plasmas*, Wiley, New York (1971).
[4] A. P. Kazantsev, Zh. Eksp. Teor. Fiz. **44**, 1283 (1963) [Sov. Phys. JETP **17**, 865 (1963)].
[5] K. W. Morton, Phys. Fluids **7**, 1800 (1964).
[6] S. M. Churilov, Fiz. Plazmy **6**, 684 (1980) [Sov. J. Plasma Phys. **6**, 377 (1980)].
[7] L. H. Lyu and J. R. Kan, J. Geophys. Res. **94**, 6523 (1989).
[8] A. Cavaliere and F. Engelmann, Nucl. Fusion **7**, 137 (1967).
[9] W. F. Crevier and D. A. Tidman, Phys. Fluids **13**, 2275 (1970).
[10] F. V. Coroniti, Nucl. Fusion **11**, 261 (1971).
[11] E. W. Greenstadt, I. M. Green, G. T. Inouye *et al.*, J. Geophys. Res. **73**, 51 (1968).
[12] M. M. Hoppe and C. T. Russell, J. Geophys. Res. **88**, 2021 (1983).
[13] B. T. Tsurutani, R. M. Thorne, E. J. Smith *et al.*, J. Geophys. Res. **92**, 11074 (1987).
[14] T. Hada, C. F. Kennel, and T. Terasawa, J. Geophys. Res. **92**, 4423 (1987).
[15] N. Omidi and D. Winske, J. Geophys. Res. **95**, 2281 (1990).
[16] S. R. Spangler, Phys. Fluids **29**, 2535 (1986).
[17] C. F. Kennel, M. A. Mal'kov, R. Z. Sagdeev *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 75 (1988) [JETP Lett. **48**, 79 (1988)].
[18] A. F. Alexandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics*, Springer, Berlin (1984).
[19] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon, Oxford (1960).
[20] C. F. Kennel, B. Buti, T. Hada, and R. Pellat, Phys. Fluids **31**, 1949 (1988).

Translated by D. ter Haar