Superconductor-ferromagnet structures

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A method is proposed for analytically solving the Usadel equations for an interleaved system of superconducting (S) and ferromagnetic (F) metals. In the case of a strong exchange field $I \gg T_c$ (where T_c is the transition temperature of the bulk superconductor), the transition temperature of the superlattice, T_c^*, is a nonmonotonic function of the thickness of the F layers and of the quantity I. In SFS Josephson junctions, the exchange field in the F layer leads to oscillations of the critical current I_c. This current vanishes at certain values of the effective thickness of the layers. The possibility of experimentally observing these effects is briefly discussed.

1. INTRODUCTION

It has now become possible to fabricate superconducting structures of a new type: superlattices in which layers of a superconductor alternate with layers of a magnetic material.[1-5] Such structures are extremely interesting for research into the mutual effects of superconductivity and magnetism. Another promising direction is the use of a ferromagnet as an interlayer material in Josephson junctions.

The results of a numerical solution of a self-consistent equation for the superconducting parameter in the case of a superconductor-ferromagnet (SF) superlattice were reported in Ref. 6. It was pointed out there that the transition temperature might be an oscillatory function of the thickness of the F layers and of the strength of the ferromagnetic exchange field. Unusual properties of SFS Josephson junctions were studied in Refs. 7 and 8. It was pointed out that the critical current can oscillate as a function of the parameters of the F interlayer in the clean limit. In the present paper we develop a general analytic formalism for describing SF structures on the basis of the Usadel equations.[9] The approach offered here makes it possible to describe the oscillation effects and to formulate requirements on the parameters of suitable structures for experimental observation of these effects. Some of the results in the present paper have been published previously as brief communications.[10,11]

The origin of these effects lies in an oscillatory spatial variation of the anomalous Green's function in the presence of an exchange field. Essentially the same factor is responsible for the appearance of an inhomogeneous Larkin-Ovchinnikov-Fulde-Ferrell state[12,13] when the paramagnetic effect is taken into account in superconductors: In this case a spatial modulation of the superconductivity order parameter corresponds to the ground state in the strong Zeeman field. An exact solution[14] for a model SF system of alternating atomic superconducting and ferromagnetic layers also shows that an inhomogeneous superconducting state, in which the sign of the order parameter alternates from one superconducting layer to the neighboring one, corresponds to the ground state in sufficiently strong exchange fields.

2. BASIC EQUATIONS

We assume that the dirty-limit conditions hold for the S and F metals which make up the SF structure, and we assume that the transition temperature of the ferromagnet is zero (i.e., the Cooper pairing constant is $\lambda = 0$ in an F region). A strong exchange field $I \gg T_c$, where T_c is the transition temperature of the bulk superconductor, is applied to the electrons in the F regions. The value of I in a ferromagnetic metal is typically 10^2–10^3 K, so the latter requirement does not impose any serious restriction. It is essentially always satisfied in typical experimental situations.[15] As in Ref. 15, we will ignore the orbital effect of the magnetic field in the ferromagnet in comparison with the exchange effect in the calculations.

Directing the quantization axis for the electron spin along the direction of the exchange field, we write the Usadel equations for the S and F metals:

$$-\frac{D_{n,s}}{2}\nabla[G_{n,s}(\mathbf{r},\omega)\nabla F_{n,s}(\mathbf{r},\omega)-F_{n,s}(\mathbf{r},\omega)\nabla G_{n,s}(\mathbf{r},\omega)]$$
$$=G_{n,s}(\mathbf{r},\omega)\Delta_{n,s}(\mathbf{r})-\tilde{\omega}^{n,s}F_{n,s}(\mathbf{r},\omega), \qquad (1)$$

$$G^2_{n,s}(\mathbf{r},\omega)+F_{n,s}(\mathbf{r},\omega)\tilde{F}_{n,s}(\mathbf{r},\omega)=1, \qquad (2)$$

where n labels a ferromagnetic region, s labels a superconducting region, D_n and D_s are the diffusion coefficients in the F and S regions, $\tilde{\omega}^s = \omega = \pi T(2n+1)$ are the Matsubara frequencies, $\tilde{\omega}^n = \omega + iI$, $\Delta_s(\mathbf{r}) \equiv \Delta(\mathbf{r})$, $\Delta_n(r) \equiv 0$, and Δ is the superconductivity parameter. As can be seen from (1), the presence of the exchange field acting on the electron spins is described in the Usadel equations by the replacement $\omega \to \omega + iI$.

The function $\tilde{F}$ in (2) is determined by the condition $\tilde{F}(\omega) = F^*(-\omega)$. It is important to note that we have $F(-\omega) \neq F(\omega)$ in our case, in contrast with the standard situation with $\tilde{F}(\omega) = F^*(\omega)$.

As usual, Eqs. (1) and (2) are to be supplemented with a self-consistency equation in the S region,

$$\Delta(\mathbf{r})\ln\left(\frac{T_c}{T}\right)=\pi T\sum_{\omega}\left[\frac{\Delta(\mathbf{r})}{|\omega|}-F(\mathbf{r},\omega)\right], \qquad (3)$$

and with boundary conditions at the SF boundaries,[16]

$$F_s=F_n, \qquad \sigma_s\frac{d}{dx}F_s=\sigma_n\frac{d}{dx}F_n. \qquad (4)$$

Here σ_s and σ_n are the conductivities in the S and F regions in the normal state, and the x axis runs along the normal to the boundary.

Near the transition temperature T_c we can linearize Eqs. (1) and (2) in terms of $\Delta \to 0$. The normal function G is replaced by its value in the absence of superconductivity, i.e., $G = \mathrm{sign}\omega$. In an S region, the Usadel equation is written in the form

$$|\omega|F_s(\mathbf{r},\ \omega)-{}^1/{}_2D_s\nabla^2F_s(\mathbf{r},\ \omega)=\Delta(\mathbf{r}). \tag{5}$$

In an F region, by virtue of the condition $I\gg T_c$ we can ignore the frequency $|\omega|$ in comparison with I, and we can write an equation for F as follows:

$$iI\ \mathrm{sign}\ \omega F_n(\mathbf{r},\ \omega)-{}^1/{}_2D_n\nabla^2F_n(\mathbf{r},\ \omega)=0. \tag{6}$$

The term I signω in (6) makes it convenient to switch from the ordinary anomalous Usadel functions to the functions

$$F^{\pm}_{n,s}=F_{n,s}(\omega)\pm F_{n,s}(-\omega). \tag{7}$$

As a result, the system of equations for the functions $F^{\pm}$ becomes

$$\pi T_c(\xi_n)^2\frac{d^2}{dx^2}F_n{}^{\pm}\mp iIF_n{}^{\mp}=0,\qquad -2d_n<x<0, \tag{8}$$

$$-\pi T_c(\xi_s)^2\frac{d^2}{dx^2}F_s{}^{\pm}+|\omega|F_s{}^{\pm}=2\Delta\delta^{\pm},\qquad 0<x<2d_s, \tag{9}$$

$$\Delta\ln\left(\frac{T_c^*}{T_c}\right)-\pi T_c^*\sum_{\omega>0}\left(\frac{2\Delta}{\omega}-F_s{}^+\right)=0. \tag{10}$$

Here $\delta^+=1$, $\delta^-=0$, $2d_n$ and $2d_s$ are the thicknesses of the F and S regions, $\xi_{n,s}=(D_{n,s}/2\pi T_c)^{1/2}$ are the correlation lengths in the S and F regions, and T_c^* is the transition temperature of this SF system.

3. TRANSITION TEMPERATURE OF AN SF SUPERLATTICE

In the calculation of the transition temperature of a superlattice of alternating S and F layers, shown schematically in Fig. 1, the order parameter Δ and the functions F must satisfy "Bloch" conditions for a translation equal to the period of the structure, $\Lambda=2\,(d_s+d_n)$, by virtue of the translational invariance of the problem:

$$\Delta(x+\Lambda)=e^{i\varphi}\Delta(x),\qquad F^{\mp}_{n,s}(x+\Lambda)=e^{i\varphi}F^{\mp}_{n,s}(x). \tag{11}$$

In other words, they differ only by a constant phase factor $\exp(i\varphi)$.

Assuming that the superconducting properties of the structure in the plane of the layers are spatially uniform, we can easily write a general solution of Eqs. (8) and (9) for the case under consideration here:

$$\begin{aligned}F_n{}^+&=A_1\,\mathrm{ch}[k_n(x+d_n)]+A_2\,\mathrm{sh}[k_n(x+d_n)]+B_1\,\mathrm{ch}[k_n^*(x+d_n)]\\&\quad+B_2\,\mathrm{sh}[k_n^*(x+d_n)],\\F_n{}^-&=A_1\,\mathrm{ch}[k_n(x+d_n)]+A_2\,\mathrm{sh}[k_n(x+d_n)]\\&\quad-B_1\,\mathrm{ch}[k_n^*(x+d_n)]-B_2\,\mathrm{sh}[k_n^*(x+d_n)],\\F_s{}^-&=C_1\,\mathrm{ch}[k_s(x-d_s)]+C_2\,\mathrm{sh}[k_s(x-d_s)].\end{aligned} \tag{12}$$

where $k_n=(1+i)(I/D_n)^{1/2}$, $k_s=(2|\omega|/D_s)^{1/2}$, and the constants $A_{1,2}$, $B_{1,2}$, and $C_{1,2}$ are determined from the boundary conditions at the SF boundaries,

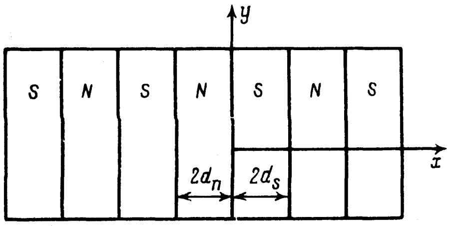

FIG. 1.

$$\begin{gathered}F_s{}^{\pm}(\pm d_s+k\Lambda)=F_n{}^{\pm}(\pm d_s+k\Lambda),\\\xi_s\frac{d}{dx}F_s{}^{\pm}(\pm d_s+k\Lambda)=\gamma\xi_n\frac{d}{dx}F_n{}^{\pm}(\pm d_s+k\Lambda),\\\gamma=\sigma_n\xi_s/\sigma_s\xi_n,\end{gathered} \tag{13}$$

and from requirement (11).

We seek a solution of the Usadel equations for the functions F_s^+ in the class of functions

$$F_s{}^+=\frac{2\Delta}{|\omega|+\rho}=D_1\cos[k(x-d_s)]+D_2\sin[k(x-d_s)]. \tag{14}$$

The parameter

$$\rho={}^1/{}_2D_sk^2 \tag{15}$$

in (14) is related to the superlattice transition temperature, T_c^* by the self-consistency condition

$$\ln\left(\frac{T_c^*}{T_c}\right)=\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}+\frac{\rho}{2\pi T_c^*}\right) \tag{16}$$

(ψ is the digamma function). In general, this parameter depends on the phase factor $\exp(i\varphi)$.

Among the many solutions corresponding to the various values of φ, that which is realized is the one which corresponds to the maximum value of T_c^* for the structure.

Substituting the sought solutions (12) and (14) into boundary conditions (13), we find a system of linear equations for the coefficients A, B, C, and D. The condition under which this system is compatible leads to the following equation for the wave vector k in expression (14):

$$\begin{vmatrix}a_{11}&a_{12}&-a_{11}^*&a_{12}^*\\a_{21}&a_{22}&a_{21}^*&-a_{22}^*\\a_{31}&a_{32}&-a_{31}^*&a_{32}^*\\a_{41}&a_{42}&a_{41}^*&-a_{42}^*\end{vmatrix}=0, \tag{17}$$

where

$$\begin{aligned}a_{11}&=i\,\mathrm{tg}(\varphi/2)\,[\gamma\xi_nk_n\,\mathrm{th}(k_nd_n)+\xi_sk\,\mathrm{ctg}(kd_s)],\\a_{12}&=\gamma\xi_nk_n\,\mathrm{cth}(k_nd_n)+\xi_sk\,\mathrm{ctg}(kd_s),\\a_{21}&=\gamma\xi_nk_n\,\mathrm{th}(k_nd_n)-\xi_sk\,\mathrm{th}(kd_s),\\a_{22}&=i\,\mathrm{tg}(\varphi/2)\,[\gamma\xi_nk_n\,\mathrm{cth}(k_nd_n)-\xi_sk\,\mathrm{th}(kd_s)],\\a_{31}&=\gamma\xi_nk_n\,\mathrm{th}(k_nd_n)+\xi_sk_s\,\mathrm{th}(k_sd_s),\\a_{32}&=i\,\mathrm{tg}(\varphi/2)\,[\gamma\xi_nk_n\,\mathrm{cth}(k_nd_n)+\xi_sk\,\mathrm{th}(kd_s)],\\a_{41}&=i\,\mathrm{tg}(\varphi/2)\,[\gamma\xi_nk_n\,\mathrm{th}(k_nd_n)+\xi_sk\,\mathrm{cth}(kd_s)],\\a_{42}&=\gamma\xi_nk_n\,\mathrm{cth}(k_nd_n)+\xi_sk_s\,\mathrm{cth}(k_sd_s).\end{aligned} \tag{18}$$

This approach to the solution of the problem is self-consistent only if k is independent of the Matsubara frequencies ω. It follows from the structure of expressions (18) that at small values of γ, specifically,

$$\gamma\ll\frac{1}{|k_n|\xi_n}\min\left(1,\frac{d_s}{\xi_s}\right), \tag{19}$$

all the elements of the third and fourth rows of the matrix contain an identical factor [$k_s\tanh(k_sd_s)$ and $k_s\coth(k_sd_s)$, respectively] in the zeroth approximation in this small parameter. Canceling this factor out, and ignoring the terms with γ in the first row, we find for k an equation which does not contain ω:

$$\xi_sk\,\mathrm{tg}(kd_s)\approx\xi_sk^2d_s=\frac{\rho d_s}{\pi T_c\xi_c}=\frac{1}{2}\gamma\frac{\xi_n}{d_n}\mathscr{F}_0(\varphi,k_nd_n), \tag{20}$$

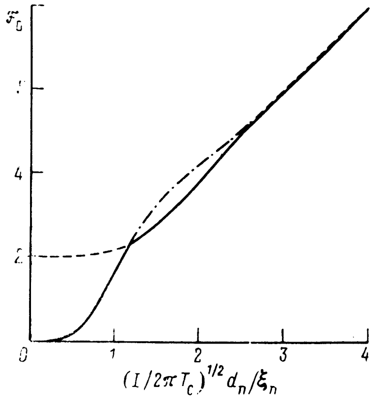

FIG. 2. Parameter $3\mathscr{F}_0 = (d_s d_n/\gamma\xi_s\xi_n)(T_c - T_c^*)/T_c$ vs effective thickness; solid line—main state; dot-and-dash line—O-phase; broken line—Π-phase.

where

$$\mathscr{F}_0(\varphi, k_n d_n) = \sin^2(\varphi/2)[k_n d_n \operatorname{cth}(k_n d_n) + k_n^* d_n \operatorname{cth}(k_n^* d_n)] + \cos^2(\varphi/2)[k_n d_n \operatorname{th}(k_n d_n) + k_n^* d_n \operatorname{th}(k_n^* d_n)].$$

The superlattice transition temperature T_c^* corresponds to the smallest value of k. Depending on which of the expressions in square brackets in (20) is larger, this will be either the 0 phase ($\varphi = 0$) or the π phase ($\varphi = \pi$). Working from (15) and (20), we easily find an expression for the transition temperature T_c^*:

$$\frac{d_n d_s}{\gamma\xi_n\xi_s}\frac{T_c - T_c^*}{T_c} = 3\mathscr{F}_0(\varphi, k_n d_n).$$

It is plotted in Fig. 2.

In the thickness interval

$$\xi_n\left(\frac{\pi T_c}{2I}\right)^{1/2}(-\theta+\pi n) < d_n < \xi_n\left(\frac{\pi T_c}{2I}\right)^{1/2}[-\theta+\pi(n+1)], \quad (21)$$

where

$$\theta = \operatorname{arctg}\left\{\operatorname{th}\left[\frac{d_n}{\xi_n}\left(\frac{2I}{\pi T_c}\right)^{1/2}\right]\right\}.$$

The π phase is realized if n is an odd integer, while the zero phase is realized if n is even. The oscillatory behavior of ρ leads to oscillations of the superlattice transition temperature as a function of the parameter $|k_n|d_n$. Our approximation in (19) corresponds to a small change in T_c ($T_c - T_c^* \ll T_c$), but it is clear that the oscillatory $T_c^*(|k_n|d_n)$ behavior persists at a qualitative level at values $\gamma \sim d_s/|k_n|\xi_n\xi_s$, at which the deviation of T_c^* from T_c is not small. Numerical calculations[6] also indicate that the oscillations in T_c^* persist in this regime.

Expression (20) is evidence that the transition between the 0 phase and the π phase occurs abruptly as d_n is varied. However, if we take the terms of the next order in γ into account in solving (17), we find that a narrow transition region with $0 < \varphi < \pi$ appears between the 0 and π phases, and the plot of Fig. 2 is smoothed out slightly. This result again is seen in the numerical calculations of Ref. 6.

At large values of γ,

$$\gamma \gg \frac{1}{|k_n|\xi_n}\max\left(1, \frac{\xi_s}{d_s}\right), \quad (22)$$

we again find for k an equation which is independent of ω in a first approximation:

$$\xi_s k \operatorname{tg}(kd_s) = 2\gamma\xi_n \Big/ \left\{\sin^2\left(\frac{\varphi}{2}\right)\left[\frac{\operatorname{th}(k_n d_n)}{k_n} + \text{c.c.}\right] + \cos^2\left(\frac{\varphi}{2}\right)\left[\frac{\operatorname{cth}(k_n d_n)}{k_n} + \text{c.c.}\right]\right\}, \quad (23)$$

We also find an oscillatory $T_c^*(|k_n|d_n)$ dependence associated with transitions between the 0 and π phases:

$$\left(\frac{\rho}{\pi T_c}\right)^{1/2} = \frac{\pi}{2}\frac{\xi_s}{d_s}\left\{1 - \frac{\xi_s}{2\gamma\xi_n d_s}\left[\sin^2\left(\frac{\varphi}{2}\right)\left(\frac{\operatorname{th}(k_n d_n)}{k_n} + \text{c.c.}\right) + \cos^2\left(\frac{\varphi}{2}\right)\left(\frac{\operatorname{cth}(k_n d_n)}{k_n} + \text{c.c}\right)\right]\right\}. \quad (24)$$

At large values of γ, only superlattices with $d_s \gtrsim \xi_s$ need be considered, since the superconductivity will be completely suppressed if the S layer is thin. In the case $d_s \gg \xi_s$, the shift of T_c^* is small, as can be seen from (23). In this case the π phase corresponds to the interval

$$\xi_n\left(\frac{\pi T_c}{2I}\right)^{1/2}(\theta+\pi n) < d_n < \xi_n\left(\frac{\pi T_c}{2I}\right)^{1/2}[\theta+\pi(n+1)], \quad (25)$$

is n is an odd integer, while the 0-phase region corresponds to even values of n in (25).

Superlattices with $d_s < \xi_s$ and small values of $\gamma \sim d_s/|k_n|\xi_s\xi_n < 1$ and $d_n \sim 1/|k_n|$ are the most promising for an experimental test of this oscillation effect in the $T_c^*(d_n)$ dependence. The reason is that in this case we should see a significant suppression of the transition temperature ($\Delta T_c \sim T_c$). Our theory is applicable only at a qualitative level in this case, but the numerical calculations of Ref. 6 confirm that in this case there are again significant oscillations in T_c^*.

4. POSSIBLE METHODS FOR ESTIMATING THE PARAMETER γ

We see that T_c^* depends strongly on not only the geometric factors but also on the transport properties of the materials making up the superlattice, i.e., on the parameter γ. This coefficient can be estimated directly by substituting the numerical values of σ and ξ into (13). Table I shows the results of such an analysis for the most suitable materials from our standpoint. We see that the parameter γ is not large ($\gamma \approx 0.3$). Consequently, the oscillation effects which we had been discussing here might be observed experimentally.

A more accurate value of this parameter can be estimated from indirect experiments, e.g., from the change of the transition temperature of a superconducting film in contact with a bulk ferromagnetic metal. It is not difficult to derive corresponding expressions for the transition temperature as a function of the film thickness. Letting $d_n \to \infty$ in (20), and setting $\varphi = 0$, we find the following equation for small values of γ:

$$\xi_s k \operatorname{tg}(2kd_s) = \gamma\xi_n \operatorname{Re}(k_n). \quad (26a)$$

Correspondingly, for large values of γ we find from (23)

$$\xi_s k \operatorname{tg}(2kd_s) = 2\gamma\xi_n \operatorname{Re}(k_n). \quad (26b)$$

Substituting k from (26a) and (26b) into (16), we find

TABLE I. Estimates of the parameter γ for Nb/RE superlattices, where RE = Ho, Dy, or Gd.

RE	T_c, T_N, Θ, K	$1/\sigma$, $\mu\Omega\cdot$cm	I, K	ξ_s, ξ_n, Å	γ
Nb	$T_c=9{,}2$	1	–	162	–
Ho	$T_N=130$ Θ=20	3,21	767	178	0,33
Dy	$T_N=180$ Θ=88	5,77	737	117	0,3
Gd	Θ=293	2,62	1094	145	0,25

Here Θ is the Curie point, T_N is the Néel point, $1/\sigma$ is the residual resistivity, and I is the exchange integral.

an equation for the reduced temperature $\tau = T_c^*/T_c$. Figure 3 shows the results of a calculation of τ as a function of d_s for various values of γ. The dashed lines show the result in the region in which the theory presented here is only qualitatively valid.

5. JOSEPHSON JUNCTION WITH A FERROMAGNETIC INTERLAYER

Yet another interesting physical consequence of an oscillatory behavior of the anomalous function F in a ferromagnet [see Eq. (8)] is the existence of oscillations in the critical current of a Josephson SFS junction, with this current vanishing at certain points.

Let us examine an SFS sandwich with the geometry shown in Fig. 4 for temperatures near the transition temperature. The x axis runs perpendicular to the interfaces, and the origin of coordinates is at the middle of the F layer. Switching to an analysis of the Usadel equations for the functions $F^{\pm}$, as in Secs. 2 and 3, we can immediately write solutions for $F_n^{\pm}$ [see (11)]. In the superconducting regions the solutions for the functions F_s^- are

$$F_s^-=C\exp(-k_sx),\quad x>d_n,$$
$$F_s^-=\tilde{C}\exp(k_sx),\quad x<-d_n. \tag{27}$$

Using the joining conditions at the S — F interfaces, we find that we can immediately write boundary conditions on the superconducting parameter Δ for the case in which the interlayer material has a resistivity high enough so that condition

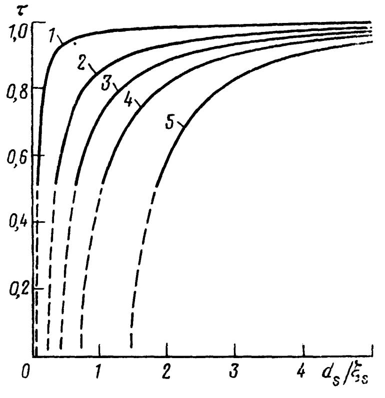

FIG. 3. Dependence of given temperature $\tau = /T_c$ of a film on its thickness for the selection of ξ_s and ξ_n parameters, which correspond to the situation with Nb film on Dy: 1—$\gamma = 0.01$; 2—$\gamma = 0.05$; 3—$\gamma = 0.1$; 4—$\gamma = 0.2$; 5—$\gamma = 100$.

(19) holds and the boundary conditions on the functions F_s^+ are independent of ω:

$$\frac{\Delta'(d_n)+\Delta'(-d_n)}{\Delta(d_n)-\Delta(-d_n)}=\frac{\sigma_n}{2\sigma_s}[k_n\operatorname{cth}(k_nd_n)+\text{c.c.}],$$
$$\frac{\Delta'(d_n)-\Delta'(-d_n)}{\Delta(d_n)+\Delta(-d_n)}=\frac{\sigma_n}{2\sigma_s}[k_n\operatorname{th}(k_nd_n)+\text{c.c.}]. \tag{28}$$

The primes here mean differentiation with respect to the coordinate x. Using the first integral of the Ginzburg-Landau equations, as in Ref. 17, we find a sinusoidal dependence of the supercurrent on the difference between the phases φ of the order parameter at the junction:

$$j=j_0\gamma\frac{|\Delta(d_n)|^2}{\Delta_0^2}\sin\varphi\left[\frac{k_n\xi_n}{\operatorname{sh}(2d_nk_n)}+\text{c.c.}\right], \tag{29}$$

where Δ_0 is the magnitude of the order parameter deep in the superconducting regions, and $j_0 = \pi\Delta_0^2\sigma_s/4eT_c\xi_s$. The critical current of the junction can conveniently be written in the form

$$I_cR_n=V_0y\frac{|\operatorname{sh}y\cos y+\operatorname{ch}y\sin y|}{\operatorname{sh}^2y\cos^2y+\operatorname{ch}^2y\sin^2y},\quad y=\frac{d_n}{\xi_n}\left[\frac{2I}{\pi T_c}\right]^{1/2}, \tag{30}$$

where R_n is the resistance of the junction, and $V_0 = \pi\Delta^2(d_n)/4eT_c$. Note that negative values of the expression between the absolute value bars in (30) correspond to a π junction[7] in which a phase difference of π corresponds to the minimum energy. Figure 2 in Ref. 11 shows a plot of $I_c(y)$ calculated from expression (30). The critical current oscillates with increasing y and vanishes at $y\approx 3\pi/4 + n\pi$. This condition corresponds precisely to a transition from a zero phase to a π phase in an SF superlattice [see (24)]. Consequently, oscillations of the critical current as the parameter y is varied should also be characteristic of SF superlattices. It follows from Fig. 2 in Ref. 11 that the value of I_cR_n at the second maximum is about 30% of

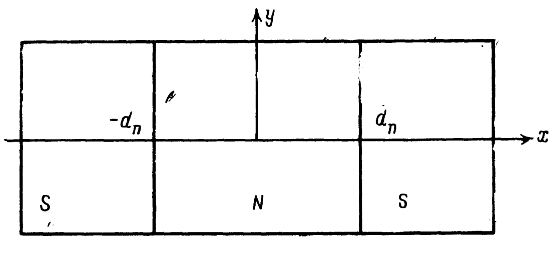

FIG. 4.

$R_n I_c(y=0)$, so the nonmonotonic $I_c(y)$ behavior could definitely be observed experimentally.

In calculating I_c at nonzero temperatures we need to bear in mind that the anomalous Usadel functions F and $\widetilde{F}$ are related by $\widetilde{F}(\omega) = F^*(\omega \to -\omega)$ in an F metal, and the normalization condition reduces to $G^2 + F\widetilde{F} = 1$.

In the limit of large thicknesses of the F layer we have $y > 1$, and under inequality (27)—which guarantees the validity of rigid boundary conditions at the SF boundaries—the solution of the Usadel equations can be written in the form

$$F=\exp(-i\varphi/2)\sin\alpha^- + \exp(i\varphi/2)\sin\alpha^+,$$

$$\alpha^{\pm}=4\,\mathrm{arctg}\{A\exp[\pm(1+i\,\mathrm{sign}\,\omega)(I/D_n)^{1/2}(x\mp d_n)]\}, \quad (31)$$

$$A=|\Delta|/\{\Omega+|\omega|+[2\Omega(\Omega+|\omega|)]^{1/2}\}, \quad \Omega=(\omega^2+|\Delta|^2)^{1/2}.$$

Substituting (31) into the expression

$$j(\varphi)=i\pi N(0)D_nT\sum_{\omega=-\infty}^{\infty}\left[F\frac{d}{dx}\widetilde{F}-\widetilde{F}\frac{d}{dx}F\right] \quad (32)$$

for the supercurrent, and making use of the symmetry properties of the functions F under the replacement of ω by $-\omega$, as mentioned above, we find the following result for the critical current I_c:

$$I_cR=32\cdot 2^{1/2}(\Delta/e)\mathscr{F}(\Delta/T)\,y\exp(-y)\sin[y+\pi/4], \quad (33)$$

$$\mathscr{F}\left(\frac{\Delta}{T}\right)=\pi T\sum_{\omega=0}^{\infty}\frac{\Delta}{(\Omega+\omega)[(2\Omega)^{1/2}+(\Omega+\omega)^{1/2}]}$$

$$=\begin{cases}\dfrac{\pi}{128}\dfrac{\Delta}{T_c}, & T\approx T_c,\\ 0{,}071, & T\ll T_c.\end{cases}$$

Figure 3 in Ref. 11 shows the temperature dependence of the function $\mathscr{F}$. Near the transition temperature, expression (33) becomes the same as (30) in the region $y \gg 1$. It can thus be concluded that this $I_c(y)$ behavior is valid over the entire temperature range $T \leqslant T_c$ and is a general property of SFS structures.

From the experimental standpoint, the most interesting approach would be to study the I_c oscillations in structures in which the Curie temperature Θ of the interlayer is close to T_c. The temperature dependence of the exchange field in the F region in this case should lead to oscillations of the critical current as a function of the temperature. This behavior might be of interest for controlling the critical currents of Josephson junctions.

The results found here also apply to junctions in the form of variable-thickness bridges, with restrictions on the parameters of the weak-link material which are much less stringent than (27).

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