Order parameter in layered and filamentary superconducting structures

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It is shown that in a model of a layered or filamentary superconductor with a complex coupling coefficient between the layers (filaments) the absolute value of the order parameter can be larger than in a uniform sample.

Layered superconductors are of great interest,[1–5] since the high-temperature superconductors based on perovskites have a layered structure. Recent experiments with artificial layered superconductors have shown that in the material $YBa_2Cu_3O_{7-x}$–$PrBa_2Cu_3O_{7-x}$ the critical temperature decreases with increasing thickness of the $PrBa_2Cu_3O_{7-x}$ interlayer between the $YBa_2Cu_3O_{7-x}$ superconducting layers.[5] For a 4 nm thick interlayer the critical temperature is equal to 90 K. As the thickness of the interlayer increases the critical temperature decreases, approaching asymptotically the value 19 K, which is almost reached with an interlayer thickness of 20 nm.

These circumstances provide incentive to examine in greater detail the nature of the interaction of the order parameter in different layers of a layered supercondutor and to try to understand the role of the intermediate insulating layer.

For this purpose, we analyzed the order parameter in a model based on an extension of the Ginzburg–Landau system of equations, which is widely employed for describing the order parameter in layered superconductors.[1,2] The time-dependent form of this system, in the absence of a magnetic field, has the form

$$\frac{\partial \Psi_m}{\partial \tau} = \Delta_\perp \Psi_m + (a - b|\Psi_m|^2)\Psi_m + r(\Psi_{m+1} - 2\Psi_m + \Psi_{m-1}). \tag{1}$$

Here Ψ_m is the order parameter for the mth layer; $\tau = Dt$, where D is the diffusion coefficient; and r is the coupling constant between neighboring layers. The rest of the notation is standard.[2] Unlike the usual model (see Refs. 1 and 2), in our case the constant r is complex: $r = |r|\exp(i\nu)$.

Supposing the number of layers in the sample is quite large, we assume that the modulus of the order parameter does not depend on the layer number and we seek the solution of Eq. (1) in the form

$$\Psi_m = \Psi_{L\alpha} e^{i(\Omega t + m\alpha)}. \tag{2}$$

Substituting Eq. (2) into Eq. (1), we obtain

$$|\Psi_{L\alpha}|^2 = [a - 2|r|\cos\nu(1-\cos\alpha)]/b, \tag{3a}$$
$$\Omega = -2|r|\sin\nu(1-\cos\alpha). \tag{3b}$$

We note that for $\alpha = 0$ we obtain from Eq. (3a) the well-known equation for a uniform superconductor:

$$|\Psi_0|^2 = a/b. \tag{4}$$

Comparing $|\Psi_{1\alpha}|^2$ with $|\Psi_0|^2$ we find that $|\Psi_{L\alpha}|^2 > |\Psi_0|^2$ for $\cos\nu < 0$, i.e., if the phase of the coupling constant lies in the range $\pi/2 < \nu < 3\pi/2$. The inequality in favor of $|\Psi_{L\alpha}|^2$ becomes stronger as $|r|$ increases and as ν and α approach π.

It is quite difficult to analyze the stability of the solutions (3) in the general case. For this reason, we confine our attention below to the following assumptions. We set $\nu = \pi$; in this case the absolute value of the order parameter takes its maximum value. Assuming the solution is uniform in the plane of the layer, we neglect the diffusion term $\Delta_\perp \Psi_m$. We introduce the notation

$$x_m = \delta|\Psi_m|, \quad y_m = |\Psi_s|\delta\varphi_m, \tag{5}$$

where $\delta|\Psi_m|$ and $\delta\varphi_m$ are small variations of the absolute value of the order parameter and its phase, respectively, while $|\Psi_s|$ is determined by the equality (3a). In the linear approximation in $\delta|\Psi_m|$ and $\delta\varphi_m$ it follows from Eq. (1) that

$$\dot{x}_m = \gamma x_m - |r|[\sin\alpha(y_{m-1} - y_{m+1}) + \cos\alpha(x_{m-1} + x_{m+1})],$$
$$\dot{y}_m = 2|r|y_m\cos\alpha - |r|[\cos\alpha(y_{m-1} + y_{m+1}) - \sin\alpha(x_{m-1} - x_{m+1})], \tag{6}$$

where $\gamma = 2|r| + a - 3b|\Psi_s|^2$.

We seek the solution of Eqs. (6) in the form

$$x_m = Xe^{\lambda\tau}\sin(m\beta),$$
$$y_m = Ye^{\lambda\tau}\cos(m\beta). \tag{7}$$

Substituting Eqs. (7) into Eqs. (6) we obtain

$$(\lambda - \gamma + 2|r|\cos\alpha\cos\beta)X + (2|r|\sin\alpha\sin\beta)Y = 0,$$
$$(2|r|\sin\alpha\sin\beta)X + [\lambda - 2|r|\cos\alpha(1-\cos\beta)]Y = 0. \tag{8}$$

The quantity β is determined by the boundary conditions. For example, for periodic boundary conditions with period N

$$\beta = 2\pi s/N, \quad s = 1, 2, \ldots, N. \tag{9}$$

The determinant of the system (8) determines the Lyapunov exponent λ, and hence also the stability of the stationary solution (3).

We now examine the stability of the solution with $\alpha = 0$. In this case we have $\gamma = 2(|r| - a)$ and

$$\lambda_1 = 2(|r| - a - |r|\cos\beta), \quad \lambda_2 = 2|r|(1-\cos\beta). \tag{10}$$

The first Lyapunov exponent is related to the absolute value of the order parameter and the second exponent refers to the phase of the order parameter. For $\beta = \pi$ [see Eq. (9)] the exponent $\lambda_1 = 4|r| - 2a$ is maximum. Hence for

$$|r| > 0{,}5a \tag{11}$$

the absolute value of the order parameter is unstable. The

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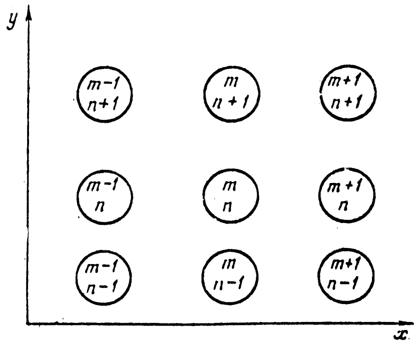

FIG. 1.

phase, however, is unstable irrespective of the values of $|r|$ or β. A similar conclusion is obtained, if a different boundary condition corresponding, for example, to the boundary of the sample with the vacuum is employed. We note that according to the analysis of the experimental data for high-T_c supercondutors performed in Ref. 2 $|r|$ is greater than a.

We now investigate the stability of the solution (3a) with $\alpha = \pi$. In this case

$$\lambda_1 = -2(5|r| + a - |r|\cos\beta), \quad \lambda_2 = -2|r|(1 - \cos\beta). \quad (12)$$

It is obvious that the exponents $\lambda_{1,2}$ are negative for any value of β: the solution (3a) for $\alpha = \pi$ is stable.

Thus in a layered superconductor with a definite phase of the coupling constant between the layers ($3\pi/2 > \nu > \pi/2$) the state in which the phases of the order parameter in neighboring layers are shifted relative to one another by an amount π is stable. For sufficiently large coupling constant the absolute value of the order parameter can be significantly greater than the analogous quantity for a uniform supercondutor. It is easy to verify that for $-\pi/2 < \nu < \pi/2$ states with the same phases of the order parameter in the superconducting layers are stable. The state with $\alpha = \pi$ is unstable.

What we have said above suggests that the nonuniformity is enhanced and it suggests studying a filamentary sample, whose structure is shown schematically in Fig. 1.

The equations describing the dynamics of a filamentary structure can be derived by a very simple extension of Eqs. (1) for layered superconductors. In the absence of a magnetic field the stationary form of the equations is

$$\frac{\partial^2 \Psi_{mn}}{\partial z^2} + (a - b|\Psi_{mn}|^2)\Psi_{mn} + r_x(\Psi_{m+1,n} - 2\Psi_{mn} + \Psi_{m-1,n}) + r_y(\Psi_{m,n+1} - 2\Psi_{mn} + \Psi_{m,n-1}) = 0. \quad (13)$$

Here Ψ_{mn} is the complex order parameter for the mth fiber along the x axis and the nth fiber along the y axis;

$$r_x = |r_x| e^{i\mu}, \quad r_y = |r_y| e^{i\nu}$$

are the coupling constants of neighboring filaments in the direction of the x and y axes, respectively.

Setting $\mu = \nu = \pi$, we seek the stationary solution of Eq. (13) in the form

$$\Psi_{mn} = \Psi_{F\pi} \exp[i\pi(m+n)]. \quad (14)$$

Substituting Eq. (14) into Eq. (13) gives the following value of $|\Psi_{F\pi}|$ for $\mu = \nu = \pi$:

$$|\Psi_{F\pi}|^2 = (a + 4|r_x| + 4|r_y|)/b. \quad (15)$$

This means that for $|r_x| \approx |r_y| = |r|$ and $4|r| \gg a$

$$|\Psi_{F\pi}|^2 / |\Psi_{L\pi}|^2 \approx 2. \quad (16)$$

The equations (13) do not contain terms corresponding to the interaction of filaments lying on the diagonals. Without discussing the necessity of introducing such terms, we note that for the order parameter in the form (14) these terms are identically equal to zero.

We conclude from the foregoing discussion that in a model of layered and filamentary structures with complex coupling constant the material binding the superconducting layers (filaments) plays an important (and sometimes decisive) role in the formation of the order parameter.

In conclusion we underscore once again that the model investigated here depends substantially on the fact that the coupling constant between the layers or filaments is complex. It seems, however, that the proposed filaments structure of a superconductor, just as a three-dimensional periodic structure, are interesting irrespective of the model studied in the present paper.

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