Residual surface resistance of hard superconductors

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The surface resistance of a superconductor due to oscillations of Abrikosov vortices is analyzed. The resistance is derived as a function of the amplitude of the alternating field for various values of the pinning parameters, the constant magnetic flux, and the frequency. This function may be either increasing or nonmonotonic.

1. There is much research interest in the surface resistance of both high-T_c superconductors[1–6] and conventional superconductors.[7–9] One particular reason for this interest is the problem of developing superconducting cavities for accelerators. The resistance R_s is usually quite sensitive to the presence in the sample of a constant magnetic flux comprising a set of Abrikosov vortices which are confined in the sample by pinning or by a static external field. At sufficiently low temperatures, the vortex motion driven by an alternating field dominates the energy loss. This loss should therefore depend on the constant flux, the pinning force, and the nature of the pinning.

Although the surface impedance of a vortex structure has been calculated in several places,[1,10–12] the effect of pinning has either been ignored or treated in a phenomenological way. Specifically, the pinning force has been assumed to be a linear function of the displacement, or the vortices have been classified arbitrarily as free or bound. When this approach is taken, it is difficult to determine the behavior of R_s as a function of the amplitude (H_0) of the alternating field. In particular, it is difficult to explain magnetic breakdown: the sharp increase in R_s when H_0 exceeds a certain H^*.

In this paper we examine the high-frequency dynamics of vortices. We calculate the surface resistance as a function of the amplitude of the alternating field, using various pinning models.[13,14] Magnetic breakdown is interpreted in the following way: When the external agent reaches a sufficient strength, the pinning entities are no longer able to effectively confine the vortices. The behavior of the vortices becomes progressively more like that of free vortices, which are slowed only by the relatively weak viscous force and which cause an energy dissipation which is high in comparison with that caused by the pinned vortices. Breakdown has previously been explained as resulting from a thermal instability.[15] That explanation could be appropriate only in the case of poor heat transfer.

2. We first consider the case in which the density of the constant magnetic flux, B, is small in comparison with the first critical field of the superconductor, H_{c1}. In this case, each vortex can be regarded as isolated. We assume that the penetration depth of the alternating magnetic field is much greater than the London depth λ. The equation of motion for a vortex in the interior of the superconductor can then be written

$$\frac{\Phi_0 H_{c1}}{4\pi}\frac{\partial^2 u}{\partial z^2}-\eta\dot{u}+F_p(u)=0. \qquad (1)$$

Here u is the vortex displacement along the z axis from its equilibrium position by the external field (the superconductor fills the half-space $z>0$), $\eta\approx\sigma\Phi_0 H_{c2}/c^2$ is the vortex viscosity (σ is the normal conductivity), and $F_p(u)$ is the pinning force per unit length of the vortex. The alternating magnetic field $H_0\sin\omega t$, directed along the surface, imposes a boundary condition on Eq. (1):

$$H_{c1}\frac{\partial u}{\partial z}\bigg|_{z=0}=-H_0\sin\omega t, \qquad (2)$$

This condition corresponds to continuity of the tangential component of the magnetic field.[12] The electric field induced by the moving vortex is

$$E=\frac{B}{c}\dot{u}, \qquad (3)$$

and the surface resistance is given by

$$R_s=\frac{8\pi}{c}\frac{\overline{E(0)H(0)}}{H_0^2}=\frac{8\pi B}{c^2H_0}\overline{\dot{u}(0)\sin\omega t}, \qquad (4)$$

where the superior bar means a time average.

For an isolated vortex, two models of the pinning force warrant consideration.[13] The first corresponds to a collective pinning by point defects of density n, each of which can exert a maximum force f on a vortex. The pinning force summed over the randomly arranged defects and the corresponding critical current density j_p is found by introducing a correlation length L_c, over which the mean square displacement of a vortex reaches the value of the coherence length ξ:

$$L_c=[\Phi_0^4/nf^2\lambda^4]^{1/3},\quad j_p=cH_{c1}\xi/L_c^2. \qquad (5)$$

If the displacement (u) of the pinned vortex from its equilibrium position is small, the restoring force is evidently linear in this displacement: $|F_p|\propto u$. If the displacement is instead the size of the vortex core, $\sim\xi$, the force F_p should be on the order of the maximum force $\Phi_0 j_p/c$. The same estimate of F_p is valid for larger displacements (since the pinning centers are distributed uniformly and act on a vortex for any value of u). We thus adopt the following model for the pinning force:

$$F_p(u)=-\frac{\Phi_0H_{c1}\xi}{L_c^2}\,\mathrm{th}\frac{u}{\xi}=-\frac{\Phi_0}{c}j_p\varphi_1\left(\frac{u}{\xi}\right). \qquad (6)$$

As a second model we consider a pinning by extended defects (grain boundaries, twin boundaries, etc.) which are parallel to the z axis. The equilibrium positions are the points at which these defects intersect. If the pinning is determined by a deviation of the electron-phonon coupling constant from its average value by an amount g_1 in a region of thickness $d\lesssim\xi$, the pinning force is proportional to the square of the absolute value of the order parameter Δ. We can thus use the approximation

$$F_p(u)=-\nu g_1\Delta^2 d\frac{\xi^3u}{(\xi^2+u^2)^2}=-\frac{\Phi_0}{c}j_p\varphi_2\left(\frac{u}{\xi}\right). \qquad (7)$$

If the displacements are small, expressions (6) and (7) are formally the same. They have been used in previous studies in the form $F_p = -pu$.

We rewrite (1) and (2) in dimensionless units:

$$\frac{\partial^2 v}{\partial z^2} - \alpha\dot{v} - \beta^2\varphi_{1,2}(v) = 0, \quad \varphi_{1,2}(v \ll 1) = v, \tag{8}$$

$$\left.\frac{\partial v}{\partial z}\right|_{z=0} = -\gamma \sin \omega t, \tag{9}$$

$$v = \frac{u}{\xi}, \quad \alpha = \frac{4\pi\eta}{\Phi_0 H_{c1}}, \quad \beta = \left(\frac{4\pi j_p}{c\xi H_{c1}}\right)^{1/2}, \quad \gamma = \frac{H_0}{H_{c1}\xi}.$$

In weak alternating fields, the equation of motion for a vortex is always linear, and its solution is

$$v = \mathrm{Re}\frac{i\gamma}{(\beta^2 - i\alpha\omega)^{1/2}} \exp\{-(\beta^2 - i\alpha\omega)^{1/2} z - i\omega t\}. \tag{10}$$

We find the following expression for the complex impedance Z [determined from the ratio of the complex amplitudes $E(0)$ and $H(0)$ with the help of expressions (3), (9), and (10)]:

$$Z = \frac{4\pi B}{c^2 H_{c1}} \frac{-i\omega}{(\beta^2 - i\alpha\omega)^{1/2}}, \tag{11}$$

$$R_s = \mathrm{Re}\, Z = \begin{cases} \dfrac{4\pi B}{c^2 H_{c1}}\left(\dfrac{\omega}{2\alpha}\right)^{1/2} \equiv R_0, \quad \alpha\omega \gg \beta^2, & \text{(12a)} \\ \dfrac{2\pi B}{c^2 H_{c1}} \dfrac{\alpha\omega^2}{\beta^3} = \dfrac{R_0}{\sqrt{2}}\left(\dfrac{\alpha\omega}{\beta^2}\right)^{3/2}, \quad \alpha\omega \ll \beta^2. & \text{(12b)} \end{cases}$$

It can be seen from expressions (11) and (12) that in the absence of pinning ($\beta = 0$) the alternating field penetrates a depth

$$\delta = \left(\frac{\Phi_0 H_{c1}}{2\pi\eta\omega}\right)^{1/2} \approx \delta_n \left(\frac{H_{c1}}{H_{c2}}\right)^{1/2}, \tag{13}$$

and the surface resistance is

$$R_s = R_0 = \frac{B}{H_{c1}}\left(\frac{2\pi\Phi_0 H_{c1}\omega}{\eta c^4}\right)^{1/2} \approx R_n \frac{B}{(H_{c1}H_{c2})^{1/2}}, \tag{14}$$

where δ_n and R_n are the corresponding values for the skin effect in a normal metal. If the pinning is strong, and the fields weak, the penetration depth is $\delta = \beta^{-1}$, and we find $R_s \ll R_0$ [Eq. (12b)]. In the following section of this paper, we examine the latter limiting case in fields of substantial strength.

3. As the amplitude of the external field H_0 [or the parameter γ in condition (9)] increases, Eqs. (1) and (8) become nonlinear. At $v \sim 1 (u \sim \xi)$, the pinning force stops increasing linearly with the displacement and becomes less influential. The result is an increase in the dissipation (R_s). From (10) we find $v \sim 1$ at $\gamma \sim \beta$; i.e., the field at which magnetic breakdown occurs is given in order of magnitude by

$$H^* \approx (4\pi j_p H_{c1}\xi/c)^{1/2}. \tag{15}$$

The behavior of R_s as a function of the field amplitude H_0 is determined by the nature of the pinning.

In the first of the models introduced above, the increase in R_s occurs over a broad range of γ values. At $\gamma \gg \beta$, the condition $v \gg 1$ holds over most of the oscillation period, and Eq. (8) becomes

$$\frac{\partial^2 v}{\partial z^2} - \alpha\dot{v} - \beta^2 \,\mathrm{sgn}\, v = 0 \tag{16}$$

with the boundary conditions $v' = 0$ at $v = 0$ [at some point z_0 in the interior of the superconductor at which we have $v \sim 1 \ll v(0)$, the derivative v' must also be small for a rapid vanishing of v] and also under condition (9). Solving Eq. (16) by perturbation theory, we find, in zeroth order in $\alpha\omega/\beta^2$,

$$\begin{aligned} v^{(0)} &= v_0^{(0)} - \gamma \sin \omega t z + \frac{\beta^2 z^2}{2} \,\mathrm{sgn} \sin \omega t \\ &= \frac{\gamma^2}{2\beta^2} \sin^2 \omega t \,\mathrm{sgn} \sin \omega t - \gamma \sin \omega t z + \frac{\beta^2 z^2}{2} \,\mathrm{sgn} \sin \omega t \end{aligned} \tag{17}$$

(The distance $z_0 = \gamma|\sin\omega t|/\beta^2$ over which v and v' both vanish is large in comparison with the penetration depth of a weak alternating field, β^{-1}.) Calculating R_s from (4) and (17), we find $R_s = 0$: If the vortex viscosity is ignored, the coupling of the alternating electric and magnetic fields is purely inductive. It is thus necessary to solve Eq. (16) in the next order in α:

$$\partial^2 v^{(1)}/\partial z^2 = \alpha \dot{v}^{(0)}.$$

As a result, we find an increment in the displacement at the surface:

$$v_0^{(1)} = -\frac{\alpha\gamma^4}{12\beta^6} \frac{d}{dt} \sin^4 \omega t \,\mathrm{sgn} \sin \omega t. \tag{18}$$

Our final expression for the surface resistance is

$$R_s = \frac{8\pi}{c^2} \frac{B\xi}{H_0} \overline{\dot{v}_0^{(1)} \sin \omega t} = \frac{R_0}{2^{1/2}} \frac{16}{45\pi} \left(\frac{\alpha\omega}{\beta^2}\right)^{3/2} \left(\frac{\gamma}{\beta}\right)^3. \tag{19}$$

Expression (19) is valid under the conditions $\beta \ll \gamma \ll \beta^2(\alpha\omega)^{-1/2}$. At the lower boundary, R_s joins with expression (12b), while at the upper boundary it joins with (12a). It can be seen from (16)–(18) that for $\gamma \sim \beta^2(\alpha\omega)^{-1/2}$ the second term in (16) ceases to be a small perturbation, and the penetration depth becomes comparable in magnitude to that in (13).

If the pinning is by an extended defect which is extended along the z axis, there is a sharp increase in the resistance near $\gamma = \beta$ because of the strong $\varphi_2(v)$ dependence at $v \gtrsim 1$ [in a sense, Eq. (15) is an exact equality]. At $\gamma - \beta \gtrsim \beta$, R_s approaches the value in (14). It follows from the analysis in the Appendix that the resistance is dominated at $\gamma \approx \beta$ by those time intervals in which a vortex near the surface makes a large excursion from its equilibrium position: $1 \ll v \ll \gamma(\alpha\omega)^{-1/2}$ [the upper boundary here corresponds to the displacement in the absence of pinning; see (10)]. The types of limiting behavior for which analytic calculations were carried out are

$$R_s = \begin{cases} \dfrac{R_0}{\sqrt{2}}\left(\dfrac{\alpha\omega}{\beta^2}\right)^{3/2} \dfrac{2}{3\pi} \dfrac{\beta}{\beta - \gamma}, \quad \left(\dfrac{\alpha\omega}{\beta^2}\right)^{2/3} \ll \dfrac{\beta - \gamma}{\beta} \ll 1, & \text{(20a)} \\ \dfrac{2^{21/4} 3}{7\pi} R_0 \left(\dfrac{\gamma - \beta}{\beta}\right)^{7/4}, \quad \left(\dfrac{\alpha\omega}{\beta^2}\right)^{2/7} \ll \dfrac{\gamma - \beta}{\beta} \ll 1. & \text{(20b)} \end{cases}$$

Figure 1 shows R_s as a function of the amplitude of the alternating field for various mechanisms for the pinning of a single vortex.

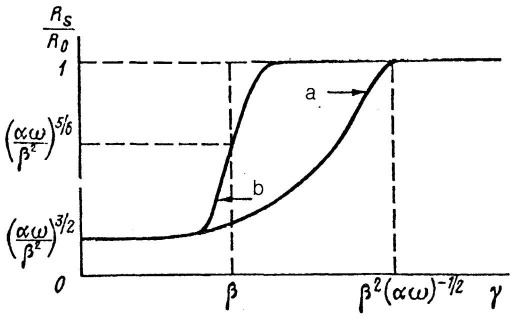

FIG. 1. Surface resistance as a function of the normalized amplitude of the alternating field for the case of strong pinning ($\alpha\omega \ll \beta^2$) at low flux densities ($B \ll H_{c1}$). a—Pinning by uniformly distributed centers; b—pinning of a vortex by an extended defect.

4. We now consider the case of comparatively strong constant fields: $B \gg H_{c1}$. The corresponding equation of motion for the Abrikosov vortices takes the following form after we average over distances large in comparison with the scale dimensions of the pinning entities but small in comparison with the penetration depth of the alternating field:

$$C_{44}\frac{\partial^2 u}{\partial z^2} - \Phi_0^{-1}\eta B\dot{u} + \Phi_0^{-1}BF_p(u) = 0; \tag{21}$$

The boundary condition on (21) is

$$C_{44}\frac{\partial u}{\partial z}\bigg|_{z=0} = -\frac{BH_0}{4\pi}\sin\omega t. \tag{22}$$

Since the torsional modulus is $C_{44} = B^2/4\pi$, these equations differ from (1) and (2) only in that H_{c1} is replaced by B, and the function $F_p(u)$ may be different. This function is the average force exerted on a unit length (along the field direction) of the vortex lattice. In the case of pinning by small defects, for example, F_p is again given by an expression like (6), with a suitably defined j_p (Ref. 13; here we can speak in terms of high-frequency oscillations of entire correlated volumes). We can apply expressions (8)–(19), replacing H_{c1} by B.

The situation is a bit more complicated if the pinning is by large defects. In this case the lattice is confined because vortex layers adhere to the surface of the defect, and the displacement far from the adhesion region may be much greater than the period of the Abrikosov lattice,[14] $b = (2\Phi_0/3^{1/2}B)^{1/2}$. If the dimensions of the defects along the field direction are large in comparison with the penetration depth for the alternating field, we have a two-dimensional pinning. In this case the displacement u far from a defect is a linear function of the macroscopic current which is flowing up to a critical value $\sim j_p$, which corresponds to a detachment of the layers from the positions in which they adhere to the defect. The pinning force can then be written as

$$F_p = -(\Phi_0 j_p/c)\,\mathrm{th}(u/u_m),$$

where $u_m \approx P_c^2 R/C_{66}^2$ (P_c is the critical pressure for the defect boundary, R is the radius of curvature of the defect, and C_{66} is the shear modulus of the vortex lattice) and $j_p \approx cB^{-1}P_c^2/RC_{66}$. The corresponding results of Secs. 2 and 3 hold for R_s [Eqs. (8)–(19), with H_{c1} replaced by B, and ξ by u_m].

In the three-dimensional case (with many pinning defects at the penetration depth), there is a nonlinear relationship between the displacement u of most of the lattice and the bulk pinning force $F_p B\Phi_0^{-1}j$, according to Ref. 14. This nonlinear relationship is

$$F_p \approx -(\Phi_0 j_p/c)(u/u_m)^{1/2},$$

where the maximum displacement is

$$u_m \approx P_c^2 R/C_{66}^{1/2}C_{44}^{1/2},$$

and the critical current density is $j_p \approx P_c^3 c/BRC_{44}^{7/4}C_{66}^{1/4}$. If the pinning is strong, and the second (viscous) term in (21) is small, this equation of motion can be solved by perturbation theory, as in Sec. 3. In the zeroth (steady-state) approximation, we find from dimensional estimates for Eqs. (21) and (22) that the displacement near the surface is

$$u^{(0)} \sim u_m[H_0/H^*]^{4/5} \gg b, \quad H^* \approx (j_p B u_m/c)^{1/2}, \tag{23}$$

and the field penetrates a depth $z_0 \sim (Bu_m c/j_p)^{1/2}(H^*/H_0)^{1/5}$. We find the following expression for the increment due to the viscosity, which leads to a finite surface resistance R_s:

$$u^{(1)} \sim \varkappa(u_m u^{(0)})^{1/2}, \quad \varkappa = c\eta\omega u_m/\Phi_0 j_p \ll 1. \tag{24}$$

Also using (4), we see that the surface resistance should fall off with increasing amplitude of the alternating field ($R_s \propto H_0^{-3/5}$) in a certain region. The reason is that a strengthening of the external agent causes an effective hardening of the vortex lattice, corresponding to a faster than linear increase in $F_p(u)$. The region in which R_s decreases is bounded from above by the magnetic breakdown field H^* (at which we have $u \sim u_m$). It is also bounded from below, either by the condition that the viscous force is relatively weak, or by the condition $u > b$ [at $u \ll b$, the $F_p(u)$ dependence is obviously linear, and R_s is given by (12) after a renormalization of the parameters α, β, and R_0—through the replacement of H_{c1} by B, of ξ by b, and of j_p by $j_p(b/u_m)^{3/2}$]. We thus find the following estimate of the surface resistance as a function of the amplitude of the alternating field (Fig. 2) at $H_0 < H^*$:

$$\begin{gathered} R_s \approx R_s(0)\min\{1, (H'/H_0)^{3/5}\}, \\ R_s(0) \approx R_n(B/H_{c2})^{1/2}\min\{\varkappa^{3/2}(b/u_m)^{-3/4}, 1\}, \\ H' \approx H^*\max\{(b/u_m)^{5/4}, \varkappa^{5/2}\}. \end{gathered} \tag{25}$$

At $H_0 > H^*$, as in (19), R_s increases in proportion to $(H_0/H^*)^3$ until it reaches a value

$$R_s \approx R_n(B/H_{c2})^{1/2}, \tag{26}$$

which corresponds to the behavior of a free vortex lattice at $B \gg H_{c1}$.

5. If the pinning is very strong, or the frequency high, the alternating field may penetrate to a depth on the order of the London depth λ. We would then need to incorporate the Meissner current

$$j_M = (cH_0/4\pi\lambda)\sin\omega t\,\exp(-z/\lambda)$$

on the right side of the equations of motions of the vortices, (1) and (21). In the limit

$$\max(H_{c1}, B)/4\pi\lambda^2 \ll \max(\eta\omega, \Phi_0 j_p/c\xi),$$

which is the opposite of that discussed in Secs. 2–4, we can

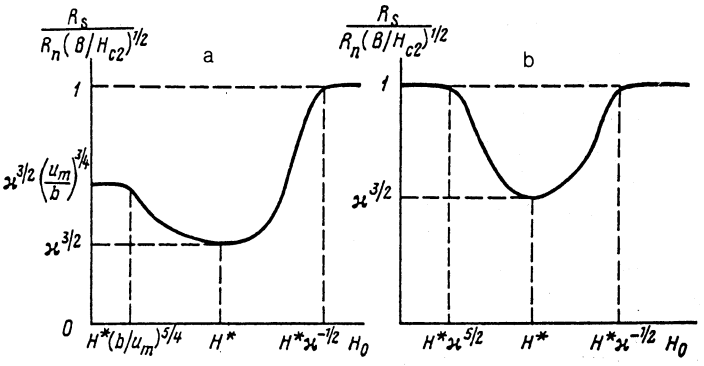

FIG. 2. Surface resistance in the case of three-dimensional pinning by large-scale defects ($B \gg H_{c1}$). a—$\varkappa \ll (b/u_m)^{1/2}$; b—$(b/u_m)^{1/2} \ll \varkappa \ll 1$.

ignore the gradient terms in (1) and (21). The equation of motion then becomes

$$\eta\dot{u} + \frac{\Phi_0}{c} j_p \varphi_{1,2}\left(\frac{u}{\xi}\right) = \frac{\Phi_0}{c} j_M. \qquad (27)$$

In weak fields (with $u \ll \xi$), a calculation yields a surface resistance (see, for example, Refs. 1 and 10)

$$R_s = \frac{\Phi_0 B}{2\eta c^2 \lambda}\left[1 + \left(\frac{\Phi_0 j_p}{\xi\eta\omega c}\right)^2\right]^{-1} \qquad (28)$$

[In the case in which the Abrikosov vortex is pinned by large defects, we must obviously replace ξ by u_m in (27) and (28).] In the case of strong pinning, magnetic breakdown occurs when the external field H_0 reaches a value at which Eq. (27) no longer has steady-state solutions:

$$H_0 = H^* = j_p \lambda \max\varphi \qquad (29)$$

(The Meissner current density at the surface is comparable to the pinning critical current density.) At $H_0 > H^*$, the resistance R_s is given by (28) with $j_p = 0$ if

$$\omega \gg \Phi_0 \max(B, H_{c1})/4\pi\lambda^2\eta \sim T_c \max(B, H_{c1})/H_{c2}.$$

In the opposite case of low frequencies, it is given by an expression like (19).

We should bear in mind that, if the pinning is strong enough that the oscillations of the vortices are substantially slowed, the surface resistance may have a large component from the motion of normal electrons (that problem requires further study). We should also bear in mind that we have ignored the effect of thermal fluctuations on R_s; such fluctuations become important with increasing temperature.[16-18]

In summary, we have calculated the surface resistance of superconductors which results from oscillations of vortices driven by an alternating field, for various pinning mechanisms. The effect of pinning is seen clearly at low frequencies, specifically, at frequencies low in comparison with the so-called depinning frequency

$$\omega_0 = c\Phi_0 j_p/\sigma\xi H_{c2}.$$

At such frequencies, the effective pinning of vortices suppresses the surface resistance R_s in comparison with that of a system of free vortices. This is true as long as the amplitude of the external field is small in comparison with the magnetic breakdown field, which pulls vortices out of the potential wells created by inhomogeneities in the material. The shape of the $R_s(H_0)$ curves is determined by the particular pinning mechanisms and can indeed be exploited in order to identify these mechanisms experimentally.

I am deeply indebted to A. I. Larkin for a discussion of these results and to V. Palmieri (National Institute of Nuclear Physics, Legnaro, Italy) for the successful collaboration which stimulated the present study.

APPENDIX

To solve the equation of motion of a vortex near an extended defect which is extended in the direction parallel to the vortex,

$$\frac{\partial^2 v}{\partial z^2} - \alpha\dot{v} - \beta^2 \frac{v}{(1+v^2)^2} = 0 \qquad (A1)$$

under boundary condition (9), we take the approach we took for Eq. (16), adopting the assumption $\alpha\omega \ll \beta^2$. The first integral of the steady-state equation (with $\alpha = 0$) is as follows, when we allow for the decrease in $|v|$ in the interior of the superconductor:

$$(v')^2 = \beta^2 \frac{v^2}{1+v^2}, \qquad (A2)$$

The solution is thus given implicitly by

$$z = \beta^{-1} \int_v^{v_0^{(0)}} \frac{dv_1 (1+v_1^2)^{1/2}}{v_1}, \qquad (A3)$$

where

$$v_0^{(0)} = \frac{\gamma \sin\omega t}{(\beta^2 - \gamma^2 \sin^2\omega t)^{1/2}} \qquad (A4)$$

is the displacement at the surface. As can be seen from (A4), the maximum absolute value of the derivative at the boundary for which steady-state solutions of Eq. (A1) exist is β. At $\beta - \gamma \equiv -\Delta\gamma \ll \beta$, R_s is dominated by time intervals near $\omega t = \pm\pi/2$, in which the relation $|v_0^{(0)}| \gg 1$ holds [see (4)]. For definiteness below, we consider the neighborhood of $\omega t = \pi/2$, with $v > 0$ (the behavior of a vortex in the case $\sin\omega t \approx -1$ is absolutely symmetric and makes an identical contribution to R_s). In this case the $v(z)$ dependence is linear up to $z = z_0 \simeq v_0^{(0)}\gamma^{-1}$:

$$v \approx v^{(0)}(z) = v_0^{(0)} - \gamma \sin\omega t z \qquad (A5)$$

[In the region $|z - z_0| \sim \beta^{-1} \ll z_0$, v rapidly vanishes according to (A3).] The third term in Eq. (A1) is small at $z < z_0$ in this case. Working from the equation

$$\frac{\partial^2 v^{(1)}}{\partial z^2} - \alpha\dot{v}^{(0)} = 0 \qquad (A6)$$

with the boundary conditions $v^{(1)}(z_0)=0$ and $v^{(1)\prime}(0)=0$, we then find the following expression for the increment $v^{(1)}$, due to the finite viscosity:

$$v^{(1)}=v_0^{(1)}\left(\frac{z^2\gamma^2}{v_0^{(0)2}}-1\right), \tag{A7}$$

$$v_0^{(1)}=\frac{\alpha}{2\gamma^2}\dot{v}_0^{(0)}v_0^{(0)3}\ll v_0^{(0)},\quad v_0^{(0)}\approx\frac{\gamma}{(\beta^2-\gamma^2\sin^2\omega t)^{1/2}}\gg 1. \tag{A8}$$

Calculating R_s from (4) and (A8), we see that the electric field $B\xi\dot{v}_0^{(0)}/c$ does not contribute to the resistance (this is what we expected to find). The basic contribution from $\dot{v}_0^{(1)}$ is indeed made by the intervals $\omega t-\pi/2=\theta\sim[(\beta-\gamma)/\beta]^{1/2}\ll 1$:

$$\begin{aligned} R_s&=2^{3/2}R_0\left(\frac{\alpha\omega}{\gamma^2}\right)^{1/2}\overline{\omega^{-1}\sin\omega t\left(-\frac{d^2}{dt^2}\frac{\alpha v_0^{(0)3}}{6\gamma^2}\right)}\\ &=\frac{2^{1/2}}{3}R_0\left(\frac{\alpha\omega}{\beta^2}\right)^{3/2}\frac{1}{\pi}\int_{-\pi/2}^{\pi/2}\frac{d\theta}{[2(|\Delta\gamma|/\beta)+\theta^2]^{3/2}}\\ &=\frac{2^{1/2}}{3\pi}R_0\left(\frac{\alpha\omega}{\beta^2}\right)^{3/2}\frac{\beta}{|\Delta\gamma|},\quad\frac{|\Delta\gamma|}{\beta}=\frac{\beta-\gamma}{\beta}\ll 1 \end{aligned} \tag{A9}$$

[see (20a)]. The range of applicability of this expression is limited on the small $|\Delta\gamma|$ side by the condition for the applicability of expression (A8), i.e., $v_0^{(1)}\ll v_0^{(0)}$. The maximum value of expression (A9), which corresponds to $|\Delta\gamma|/\beta\sim(\alpha\omega/\beta^2)^{2/3}$, determines R_s in order of magnitude at the "critical" point $\gamma=\beta$:

$$R_s(\gamma=\beta)\approx R_0(\alpha\omega/\beta^2)^{5/6}. \tag{A10}$$

With a further increase in γ, Eqs. (A4)–(A8) remain valid for values of the phase θ greater than $\theta_0=(2\Delta\gamma/\beta)^{1/2}$ in absolute value. As θ approaches θ_0, the displacement $v_0\approx(\theta^2-\theta_0^2)^{-1/2}$ is cut off at

$$|\theta-\theta_0|_{min}\sim(\beta/\Delta\gamma)^{1/6}(\alpha\omega/\beta^2)^{1/3},$$

at which the condition for the applicability of (A8) is violated. If $\Delta\gamma/\beta$ is not too large, the corresponding displacement at the surface,

$$v_{0\,max}\sim(\theta_0|\theta-\theta_0|_{min})^{-1/2}\sim\left(\frac{\Delta\gamma}{\beta}\right)^{-1/3}\left(\frac{\alpha\omega}{\beta^2}\right)^{-1/6} \tag{A11}$$

is the maximum order of magnitude of the displacement of a vortex over the oscillation period. Using the relation

$$\overline{\dot{v}_0\cos\theta}\sim\omega v_{0\,max}\theta_0^2,$$

we find the estimate

$$R_s\approx R_0\left(\frac{\alpha\omega}{\beta^2}\right)^{1/2}v_{0\,max}\theta_0^2\approx R_0\left(\frac{\alpha\omega}{\beta^2}\right)^{1/3}\left(\frac{\Delta\gamma}{\beta}\right)^{2/3}, \tag{A12}$$

At $\Delta\gamma/\beta\sim(\alpha\omega/\beta^2)^{2/3}$, this estimate joins (A9) and (A10).

For sufficiently large $\Delta\gamma$, the displacement of a vortex can reach values much larger than (A11) over the time required for the phase to vary by an amount up to $\sim\theta_0$. Specifically, these values $v_{0\max}$ now determine the resistance R_s [see (A12)—the first estimate]. On the other hand, as long as the condition $\Delta\gamma\ll\beta$ holds, the second term in (A1) remains a small perturbation in the region $v(z)\gg 1$. A relation like (A5) holds for $v(z)$, with v_0 determined from the following condition: At $z\approx v_0/\gamma$, the derivative $v'=\alpha\dot{v}_0z-\gamma\sin\omega t$ [found from Eq. (A1), with allowance for the circumstance that the dependence $v_0(t)$ is fast in comparison with the sinusoidal dependence $\dot{v}(z)\approx\dot{v}_0\gg\omega v_0$] is equal to the limiting value $v'=-\beta$, which corresponds to the pinning of a vortex line by a pinning defect. For v_0 we then find the equation

$$\frac{\alpha\omega}{\gamma}v_0\frac{dv_0}{d\theta}=\gamma\sin\omega t-\beta\approx\frac{\gamma}{2}(\theta_0^2-\theta^2),\quad\theta_0=\left(\frac{2\Delta\gamma}{\beta}\right)^{1/2}\ll 1 \tag{A13}$$

with the initial condition $v_0(-\theta_0)=0$. Hence

$$v_0=\frac{\gamma}{(\alpha\omega)^{1/2}}(\theta+\theta_0)\left(\frac{2\theta_0-\theta}{3}\right)^{1/2}. \tag{A14}$$

For the surface resistance

$$R_s=2^{3/2}R_0\left(\frac{\alpha\omega}{\gamma^2}\right)^{1/2}\frac{1}{\pi}\int_{-\theta_0}^{2\theta_0}v_0(\theta)\theta\,d\theta$$

we find expression (20b). The lower boundary of the range of applicability of that expression is found by comparing expressions (A14) and (A11)

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