The effect of thermal fluctuations on the FMR line shape in dispersed ferromagnets

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We investigate magnetic resonance in a superparamagnet, and derive the dynamic susceptibility of an ensemble of randomly dispersed single-domain anisotropic ferromagnetic particles. We show that as the temperature rises, the orientational fluctuations of the magnetic moment simultaneously weaken the inhomogeneous broadening of the FMR line arising from the distribution in the directions of anisotropy axes of the particles and create an even stronger homogeneous (superparamagnetic) broadening. A particular result of the combined action of these effects is the nonmonotonic temperature dependence of the FMR linewidth in dispersed ferromagnets, which is well known in experiments.

INTRODUCTION

Continuing interest in the study of the physical properties of ultradispersed ferromagnets and ferrites by ferromagnetic resonance (FMR) is fueled by the appearance of entirely new media belonging to this class: glasses and zeolites that incorporate transition metals,\(^1\)\(^\text{2}\) magnetic liquids,\(^3\) heterogeneous metal polymers,\(^4\) biogenic magnetite,\(^5\) etc. However, theoretical interpretation of the FMR spectra obtained remains a complicated problem.

In an ultradispersed ferromagnet, the characteristic size of a particle is 50 to 200 Å. Even taken by itself, the smallness of the particle size causes peculiarities in the magnetic behavior. First of all, it is well-known\(^6\) that a ferromagnetic microrystal with a volume \(V \sim 10^{-18} \text{cm}^3\) has no domain structure, i.e., it is uniformly magnetized and possesses a constant magnetic moment \(\mu = M_V\), where \(M\) is the magnetization of the material that makes up the particle. Secondly, for \(V \sim 10^{-18} \text{cm}^3\) and \(M \lesssim 10^{-6} \text{G}\) the random magnetic field induced by thermal fluctuations, whose amplitude \(H\), should equal \(h_k T/\mu\) in order of magnitude, reaches room temperature values \(H \gtrsim 10^5 \text{Oe}\), and consequently becomes commensurate with the field \(H\), for the magnetic anisotropy of the material. It is this orientational diffusion of the vector \(\mu\) under the action of \(H\), that causes, among other things, the superparamagnetism of dispersed ferromagnets.

The theoretical study of FMR taking into account the superparamagnetic effects is the subject of this paper. In Ref. 8, equations were obtained that describe the motion of the magnetic moment of a single-domain particle at finite temperatures, and the temperature and frequency dependences of the natural FMR parameters were studied for particles with the "easy axis" type of anisotropy. In Ref. 9, the dynamic susceptibility of an isotropic superparamagnet was calculated; in Refs. 10, 11, FMR was investigated in an anisotropic particle placed in a constant external field \(H\). It is this latter problem that is closest in its formulation to the situation encountered in experiment. Actually, the overwhelming majority of data published to date on FMR in dispersed ferromagnets (see, e.g., Refs. 1,2,4,10,12–15) were obtained from measurements in a magnetizing field \(H\) at a fixed resonant excitation frequency \(\omega/2\pi \sim 10 \text{GHz}\), which corresponds to \(H \sim 3 \times 10^5 \text{Oe}\). Thus, the condition \(H \gg H_k\), is fulfilled for any dispersed ferromagnet in which the anisotropy field is a few hundred oersteds.

The goal of this paper is to apply the theoretical results of Refs. 9 and 11 to FMR in solid dispersed ferromagnets. Calculating the dynamic susceptibility using these results requires averaging not only over the statistical ensemble of particles with a given direction of the anisotropy axis, but also over the orientational texture of the sample, i.e., over the angular distribution of the anisotropy axes in such a particle. In what follows, we show that this second stage of averaging changes the observed dynamic susceptibility in a nontrivial way, and in particular allows us to obtain a temperature and field dependence of the FMR parameters that is qualitatively close to what is observed in experiment. Specifically, the discussion will center around the following phenomena: an increase in asymmetry of the absorption line as the temperature drops, a decrease in the linewidth \(\Delta H\) as \(T\) increases, and an increase or decrease of the resonant field \(H_\text{res}\), as the temperature changes, depending on the sign of the magnetic anisotropy constant of the dispersed particles.

1. DYNAMICS OF THE MAGNETIC MOMENT OF A SUPERPARAMAGNETIC PARTICLE

In investigating the motion of the magnetic moment \(\mu\) of a single-domain particle at temperatures that are nonzero but not too close to the Curie point, it is natural to assume \(\mu = M \chi = \text{const.}\)

This allows us to study only the rotation of the magnetic moment, which we write in the form \(\mu = \mu e\), where \(e = 1\).

In a statistical ensemble, the macroscopic (observed) magnetic moment of a particle should be defined as the average of the "microscopic" vector \(\mu\):

\[
\langle \mu \rangle = \frac{1}{V} = \frac{1}{V} \int W(\mu, t) \, d\mu,
\]

where \(W(\mu, t)\) is an orientational distribution function which satisfies the kinetic equation\(^6\)

\[
\frac{dW}{dt} + \left( \frac{1}{2} \frac{\partial W}{\partial \phi} \right) = 0.
\]

Here, \(\phi = \left[ e \text{V} \right]^{1/2} \) is the infinitesimal rotation operator, and \(\phi = \frac{\partial}{\partial \mu} \) is the gradient evaluated at the surface of the unit sphere. The angular velocity \(\Omega\) of the unit vector \(e\) is split up

\[
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into a regular (field-induced) and diffusive (fluctuation) part: \( \Omega = \Omega_0 + \Omega_f \). The expression for the regular component follows from the Landau-Lifshitz equation:

\[
\hat{\partial} \mu / \partial t = - \gamma [\mu H] - \left( \gamma a / \mu [\mu H] \right),
\]

(3)

where \( \gamma \) is the gyromagnetic ratio, \( a \) is the dimensionless relaxation time, and \( H = - \partial U / \partial \mu = - (1 / \mu) VU \) is the magnetic field defined by the orientation-dependent portion \( U(\epsilon) \) of the total energy of the particle. Writing (3) in the form of a kinematic relation \( \dot{\Omega} = \Omega + \Omega_f \) and introducing the operator \( \hat{F} = \dot{J} + (1 / \alpha) \partial U / \partial \mu \), we obtain

\[
\dot{\Omega} = - \gamma [\mu H] \hat{F} U + \gamma a [\mu H] \hat{F} U.
\]

(4)

We find the rate of random motion \( \dot{\Omega}_f \) (see Ref. 8) by replacing the regular field \( H \) by the random field \( H_f = - (k_b T / \mu) \ln W \) in (4). From this the total rate of rotation of the magnetic moment is

\[
\Omega = - \gamma [\mu H] \hat{F} U + \gamma a [\mu H] \hat{F} U.
\]

(5)

Substituting (5) into (1) leads to a kinetic equation in the form

\[
2 \pi \Omega \partial W / \partial \Omega = \left[ - \gamma \hat{F} (U / k_b T + \ln W) \right] \Omega + \gamma a \hat{F} U,
\]

(6)

where we introduce the notation \( \tau = \mu / 2 \gamma k_b T \) for the characteristic orientational diffusion time of the magnetic moment.

Under steady-state conditions, for which the magnetic field is \( H = \text{const}(\epsilon) \), the solution to Eq. (6) corresponding to equilibrium is a Gibbs distribution

\[
W_{\text{eq}} = Z_{\text{eq}} \exp \left( - U_{\text{eq}} / k_b T \right) = \exp \left( - U_{\text{eq}} / k_b T \right) d \Omega,
\]

(7)

where \( U_{\text{eq}} = - \mu (\epsilon H(\epsilon)) \). Switching on a variable field \( h(\epsilon) \) perturbs the thermodynamic equilibrium and causes the magnetic moment to move, which is described by the kinetic equation (6) with \( U = U_0 - \mu (\epsilon h(\epsilon)) \). The quantity of interest in magnetic resonance is the nonequilibrium portion of the macroscopic magnetic moment

\[
m = \mu (\epsilon - \epsilon_{\text{eq}}).
\]

(8)

Here angle brackets without labels correspond to statistical averaging with the function \( W(\Omega) \) from (6), whereas the label \( 0 \) denotes averaging with respect to the equilibrium distribution (7). The motion of the vector \( m \) is obtained after multiplying (6) on the left by \( \mu \) and integrating over angles:

\[
2 \pi \partial m / \partial t = - \gamma \left\{ \hat{F} U + \langle \hat{F} U \rangle \right\} + \gamma a \hat{F} U d \Omega
\]

(9)

the parentheses in (9) indicate that each operator acts only on the function standing alongside it.

We note that Eq. (9) is an unclosed system of equations with respect to the quantities \( m_0 = \langle \epsilon \rangle \) to be found whenever the energy function \( U(\epsilon) \) differs from a constant, since moments of higher order appear on its right side in addition to \( \langle \epsilon \rangle \), e.g., \( \langle \epsilon^2 \rangle \), etc. The problem of decoupling a system of nonstationary moment equations does not have an exact solution, which is a source of considerable difficulty even in calculating the linear response. In what follows, we will use the closure procedure (the so-called effective-field method) first applied to the study of superparamagnet dynamics in Refs. 8, 9, and 16. The mathematical nature of the effective-field approximation identifies it as one of the variants of the well-known Galerkin method.

Let us write the solution to the kinetic equation (6) in the form

\[
W = \exp \left[ - U / k_b T + (\epsilon - \epsilon_{\text{eq}}) \right] W_{\text{eq}}
\]

(10)

where we assume that the vector parameter (the effective field) \( \epsilon - \mu (\epsilon h(\epsilon)) / k_b T \) is determined independent of \( \epsilon \). Linearization and normalization of the function (10) leads to a form

\[
W = W_{\text{eq}} \left[ 1 + (4 \pi / 3) \langle \epsilon h \rangle (\epsilon_{\text{eq}} - \langle \epsilon h \rangle) \right],
\]

(11)

where the spherical harmonics \( Y_n^m \) appear in Eq. (11) because we have chosen to represent vectors in terms of their spherical components (\( k = 0 \), \( \pm 1 \)) and in what follows. In particular, we have for the unit vector \( \epsilon \)

\[
\epsilon_n = (4 \pi / 3) Y_n^m (\theta, \phi), \quad \langle \epsilon h \rangle = (4 \pi / 3) \langle \epsilon h \rangle Y_n^m W d \Omega,
\]

(12)

where \( d \Omega = \sin \theta d \theta d \phi \) is the element of solid angle.

We close the equations of motion of the magnetic moment (9) using the effective-field approximation. Substituting (11) into (8) gives a linear relation between the vectors \( m \) and \( h(\epsilon) \):

\[
W = W_{\text{eq}} \left[ 1 + \sum_{n \geq 2} \sum_{m \geq 2} \langle \epsilon h \rangle Y_n^m (\epsilon_{\text{eq}} - \langle \epsilon h \rangle) \right],
\]

(13)

where the matrix coefficients

\[
A_{nm} = W_{\text{eq}} \left[ 1 + \sum_{n \geq 2} \sum_{m \geq 2} \langle \epsilon h \rangle Y_n^m (\epsilon_{\text{eq}} - \langle \epsilon h \rangle) \right],
\]

(14)

contain averages only with respect to the equilibrium distribution.

It is obvious that Eq. (11) is the simplest type of approximation for the distribution function that is linear in the perturbations. If we use a multiparameter function to solve Eq. (6), e.g.,

\[
W = W_{\text{eq}} \left[ 1 + \sum_{n \geq 2} \sum_{m \geq 2} \langle \epsilon h \rangle Y_n^m (\epsilon_{\text{eq}} - \langle \epsilon h \rangle) \right],
\]

(15)

where all the \( a_{nm} \) are \( - \mu (\epsilon h) / k_b T \), then a closure procedure analogous to (12) and (14) leads to a \((\nu + 2) \times 2\) matrix equations for the coefficients \( a_{nm} \). There is interest in the problem of choosing the minimum value of \( \nu_{\text{eq}} \) that provides a correct description of FMR in a single-domain particle. Comparing the results of Refs. 8 and 9 shows that the necessary order of approximation depends on the symmetry of the magnetizing field \( H = - \partial U / \partial \mu \), which determines the equilibrium orientation of the magnetic moment. Thus, in studying FMR in an external field \( H = H \) (i.e., dipole symmetry), it is permissible to retain only terms with \( \nu = 1 \) in the sum (15) (see Refs. 9 and 11). For the case of natural FMR i.e., a magnetically uniaxial particle with \( H = 0 \) and

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hence a field $H$, with quadrupole symmetry, it is necessary (see Ref. 8) to save terms with both $\nu=1$ and 2 in (15) in order to obtain the correct description. In their numerical solution to this problem, the authors of Ref. 17 also included higher harmonics ($\nu<20$). However, they observed that, contrary to what was expected, the solution in this way led only to a quantitative refinement of their results compared to those found analytically in Ref. 8 for $\nu_{\text{res}}=2$.

The considerations we have outlined here clarify this reason for choosing the approximation (11) and determine the range of applicability of Eq. (13) based on it. This equation remains correct provided that the anisotropy field of the particle $H_a$ is small compared to the magnetizing field $H$. We now turn to the case of particles that possess a nonzero magnetic anisotropy (due either to crystallography or an anisotropic shape), i.e., we consider an energy function of the form $U=U_0+U_a$. For the anisotropy contribution $U_a$ it is convenient to use the representation

$$U_a = \sum_j K_{ja}(x', \theta) \phi_j \Phi_i(x),$$

where the $K_{ja}$ are anisotropy constants, $\phi_j$ are numerical coefficients, and $\theta$ and $\phi'$ are angles the magnetic moment vector of the particle makes with the anisotropy axis. In the approximation of weak anisotropy, we have $|U_a|/H \ll 1$, the first-order corrections to the eigenvalues of Eq. (13) can be obtained if we use for the equilibrium distribution function the expression

$$W^{eq} \approx W_r \left(1 + \sum_j K_{ja}(x', \theta) \Phi_i(x) \phi_j \phi' \right),$$

where $W_r$ is the "Langevin-like" distribution function from (7). Taking into account contributions linear in $K_a$ does not alter the diagonal nature of the matrix $\Lambda_{\text{eq}}$, since the calculation of the decay rates $\lambda_k$ reduces to finding the equilibrium moments ($x$, $\phi$), and $\phi'$), with the distribution function (22) and substituting them into Eq. (16) in place of $\langle x \rangle$, $\langle \phi \rangle$, and $\langle \phi' \rangle$.

Let us note that in calculating the moments $\langle \ldots \rangle$, it is necessary to transform the angular coefficients $\Phi_i(x)$ of the expansion (21) to the coordinate system in use, for which the polar axis is directed along the magnetizing field $H$. Since we require only the convolution of $\Phi_i(x)$ with functions that do not contain any azimuthal dependence, it is sufficient for their calculation to use the integral relations obtained in Ref. 18:

$$\langle P_{ij}(x) \Phi_i(x) \phi_j \phi' \rangle = \int d^3x \langle P_{ij}(x) \phi_j \phi' \rangle \Phi_i(x).$$

Average with the distribution function (22) gives the following expression for the eigenvalues of Eq. (13):

$$\lambda_k = \lambda_k^{eq} + \Delta \lambda_k = \lambda_k^{eq} + \left(\Delta R_{k} - \lambda_k^{eq} N_{\text{eq}} \right)/N_{\text{eq}},$$

where $\lambda_k^{eq}$ is the frequency of free precession $\omega_0$ and relaxation time $\tau_r$ for the component of the magnetic moment perpendicular to $H$:

$$\omega_0 = \frac{1}{2\pi} \frac{1}{\tau_r} = \frac{1}{2\pi} \frac{1}{\tau_r} = \omega_L H,$$

and $\Delta R_{k}$ is introduced.

Equation (19) reproduces the results of Ref. 9. This equation implies that the precession frequency of an isotropic superparamagnet always remains the same as in the bulk crystal, while the relaxation time decreases without bound as the argument of the Langevin function $\xi$ decreases, i.e., with increasing temperature or decreasing particle size. In what follows it is convenient to write the relaxation time in the form $\tau_r = (\alpha y H)^{-1}$, introducing an effective damping rate for the precession

$$\alpha = \alpha \left(3 - L_3 \right) / 2L_2 \left( \xi < 1 \right),$$

$$\alpha = \alpha \left( \xi \right)$$

where $L_2$ and $L_3$ are the second and third moments of the magnetic moment perpendicular to $H$.

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where \( A_{p}^{*} \) and \( N_{iO}^{*} \) are defined by Eqs. (17) and (18). Calculating the corrections \( \Delta A \) and \( \Delta N \) using Eq. (23) gives

\[
\Delta A = -\frac{V}{k_{B}T} \sum_{i} K_{i} \Phi_{i}(\Gamma) \langle z^{*}P_{i} \rangle, \\
\Delta N = \frac{V}{k_{B}T} \sum_{i} K_{i} \Phi_{i}(\Gamma) \langle (z^{2} - 2z)P_{i} \rangle, \quad \Delta N = \Delta R/2.
\]

Substituting (25) into (24) gives

\[
R_{\alpha} = \left( 1 - \frac{L_{a}}{3} \right) - \frac{2}{24} \sum_{i} \kappa_{i} \Phi_{i} \left( \frac{1}{2} \frac{dL_{a}}{dz} - \frac{1}{24} \frac{dL_{a}}{dz} \right), \tag{26}
\]

which allows us to represent the eigenvalues in the form

\[
\lambda_{\alpha} = \left\{ \frac{1}{L_{a}} + \frac{V}{k_{B}T} \sum_{i} \frac{1}{2} L_{a} \sum_{i} \frac{1}{2} \frac{dL_{a}}{dz} \right\} K_{i} \Phi_{i} \right) \tag{27}
\]

where \( L_{a} \equiv (P_{i})_{a} \); the explicit expression for \( Ro \) will not be given here due to its complexity. In deriving Eqs. (26) and (27) we made use of the relation

\[
\langle (1-z)xP_{i}(z) \rangle_{a} = \frac{2}{L_{a}} \langle z xP_{i}(z) \rangle = \int \frac{f(\Gamma)}{F(z)} d\Gamma, \tag{28}
\]

obtained by averaging the equations for the Legendre polynomials with the distribution function \( f(\Gamma) \) from (7).

The free precession parameters of an anisotropic superparamagnetic particle can be found by substituting (27) into the definition (19). Let us do this for the two best-known types of anisotropy, uniaxial \((J = 2, K_{2} = K_{a})\) and cubic \((J = 4, K_{4} = K_{c})\). In these cases the sums entering into expressions (26)-(27) contain only one nonzero term:

\[
\frac{2}{15} K_{a} \Phi_{a}(\cos \theta) + \left( \frac{2}{15} (Y_{a}(0, \phi) + Y_{a}(\theta, 0, \phi)) \right) K_{a} \Phi_{a} \tag{28}
\]

Here the angle \( \Gamma = (\theta, \phi) \) gives the position of the anisotropy axis of the particle in a spherical system of coordinates with the polar axis along the field \( H \). Using Eq. (28), we obtain from (27) the eigenfrequencies of oscillation of the magnetic moment:

\[
a_{\alpha}^{[\Gamma]} = \frac{\Gamma + 2}{M} K_{a} L_{a} \] \\
\[ a_{\alpha}^{[\Gamma]} = \frac{\Gamma + 2}{M} K_{a} L_{a} \tag{29}
\]

It follows from Eq. (27) that in small particles a thermal-fluctuation-induced “dressing” of the anisotropy constant of the form \( K_{\alpha} = K_{a}(L_{a}/L_{a}) \) takes place, as the temperature increases, for \( \xi < 1 \) the anisotropy “melts” according to the law \( K_{\alpha} \propto \xi^{-1} \). For crystallographic anisotropy this dependence was predicted from intuitive considerations in Ref. 10. However, for uniaxial anisotropy caused by the shape of a particle the authors of Ref. 10 obtained a different result: \( K_{\alpha} = K_{a} L_{a}^{-1} \). This is obviously an error, since according to the rigorously obtained expressions (25)-(29) the renormalization \( K_{\alpha} = K_{a}(L_{a}/L_{a}) \) should not depend on the specific nature of the anisotropy constants.

3. Dynamic Susceptibility of an Ensemble of Superparamagnetic Particles with a Random Distribution of Anisotropy axes

In order to calculate the response of a particle to a weak time-dependent field \( h \) it is necessary to use the inhomogeneous equation (13). As is well-known, the fundamental characteristic of FMR is the susceptibility \( \chi_{a}^{[\Gamma]} \) with respect to a circularly polarized field.
...are complex quantities. Equation (33) allows us to take various limits that lead to familiar expressions. Let us do this, e.g., for the imaginary part of the dynamic susceptibility $\chi''$, which is $\chi'' = \chi'' - \chi''''$.

1) For $K_s = 0$ we immediately obtain the result of Ref. 9

$$
\chi'' = \frac{n \eta J}{k T} - \frac{1}{C - \text{Im} A} \frac{1}{\text{Re} C} \frac{1}{\text{Re} C}
$$

where $M = n \eta L, (\xi, \alpha_B = \gamma H)$, and the effective damping rate is determined by Eq. (20). Since the parameter $\alpha_B$ reduces to $\alpha = \gamma H - \alpha_B$, the classical result for $\chi''$, of an isotropic ferromagnet at low temperatures (or for large-volume particles) follows from (35).

2) In the limit of particles made of a material with a vanishingly small (\textit{c---0}) intrinsic FMR linewidth, only those particles for which the resonance condition (29) holds contribute to the observed susceptibility (32). Particles oriented with anisotropy axes transverse to the magnetizing field ($\theta = \pi/2$) have the lowest resonance frequency, $\omega = \omega_{\text{pl}} (1 - \epsilon, L_2, L_1)$, where $\epsilon = K_s / M H$. This quantity determines the left-hand edge of the absorption curve on the frequency axis. The right-hand edge of the function $\chi''$, i.e., the largest resonance frequency, is given in this case by $\omega = \omega_{\text{pl}} (1 + 2L_2/L_1)$, which corresponds to $\theta = 0$. Thus, the intrinsic inhomogeneous broadening of the absorption line is

$$
\Delta\omega = \Delta \omega_{\text{pl}} |L_2/L_1|.
$$

Taking the limit $\alpha = 0$ in Eq. (33), in that frequency interval we obtain

$$
\chi'' = \frac{2M}{2H} \left[ 1 \left( 1 - L_2/L_1 \right) \left( 1 - \omega/\omega_{\text{pl}} \right) \right].
$$

It is clear from (37) that the absorption curves for particle ensembles with positive and negative anisotropies ($\varepsilon > 0, \varepsilon < 0$) are mapped into each other under reflection with respect to the axis $\omega = \omega_{\text{pl}}$. At low temperatures ($\xi + \alpha$) there follows from (37) the expression

$$
\chi'' = \frac{2M}{2H} \left[ 1 \left( 1 + L_2/L_1 \right) \left( 1 - \omega/\omega_{\text{pl}} \right) \right].
$$

$\xi$ and $\omega$ are defined in the interval $1 - \varepsilon < \omega < \varepsilon < 1 + 2\varepsilon$. Eq. (38), which is obtained from (33) in the limits $\varepsilon \rightarrow 0$ and $\xi \rightarrow \alpha$, corresponds to the results of Morrison and Karayianis\textsuperscript{19} for an ensemble of independent uniaxial weakly anisotropic ferromagnetic grains.

Figures 1 and 2 show the results of calculations of $\chi''$ for finite values of $\alpha, \varepsilon$, and $\xi$ when the particles have uniaxial and cubic anisotropy. The situation here is typical of experiments in which the frequency $\omega$ and amplitude $h_0$ of the AC field are fixed. The parameters that vary are the dimensionless intensity of the magnetizing field $\gamma H/\omega$ and the quantity $\xi = M H_0 / \gamma h_0$. This is analogous to $\xi$ and is determined by the particle size and temperature. Let us take $\alpha = 10^{-4}$ for the FMR linewidth of the particle material, and choose the anisotropy parameter in the form $\varepsilon = K_s / M H$, where $K_s$ is the uniaxial or cubic anisotropy constant respectively.

Let us indicate the characteristic ranges of values of $\xi$ and $\varepsilon$, assuming that $M_s = 500$ G and $K = 10^{3}$ erg/cm (magnetite), and $\omega = 2 \times 10^{12}$ rad/sec (i.e., a spectrometer wavelength of 3 cm). From this we find that when $\gamma = 2 \times 10^{12}$ erg/Hz, $\xi$ varies from $\sim 1$ (particles of diameter $50 \, \AA$, temperature $\sim 400$ K) to $\sim 100$ (particle diameter $150 \, \AA$, temperature $\sim 100$ K). For the same values of the magnetic characteristics we find $|\varepsilon| \approx 0.1$, and the absorption line is no longer visible.

For convenience and clarity we present not only the family of absorption lines $\chi''$, but also their derivatives $d\chi''/dH$. The latter are directly proportional to the signal recorded by the spectrometer, and are also convenient for measurements.
FIG. 1. Absorption line shape $\Delta\gamma$ (left column) and $d\Delta\gamma/dH$ (right column) for an ensemble of particles with positive uniaxial anisotropy ($\epsilon = 0.1$) as functions of the Langevin parameter: $\xi_0 = 30$ (a), $5$ (b), $2$ (c), and $1$ (d); for all the curves the precession attenuation parameter $\alpha = 10^{-2}$.

finding the exact width $\Delta H$ of the FMR line. The quantity $\Delta H$ is defined as the distance along the field axis between the two extrema (maximum and minimum) of the function $d\Delta\gamma/dH$ located on different sides of the point $d\Delta\gamma/dH = 0$ that are farthest away from this point. We note that according to this definition the width of a symmetric Lorentzian line is $\Delta H = \frac{2\Delta\gamma}{\gamma}$.

Figures 1 and 2 allow us to trace how the line shape changes as the temperature increases, i.e., as $T$ decreases. A curve for $\Delta\gamma$, that is initially asymmetric with clearly marked steep edges and a width $\Delta H = 3\Delta\gamma/\gamma$ gradually is transformed into a smooth symmetric Lorentzian curve. In this case the maximum absorption shifts to the point $\gamma H/\alpha = 1$, corresponding to magnetic resonance in an isotropic superparamagnet. The smoothing of the curve is easy to understand if we recall that as the temperature increases the inhomogeneous broadening of the line (36) decreases and the effective attenuation constant $\alpha_e$ given in (20) increases, and that the latter determines the homogeneous broadening. To sum up, we are led to conclude that in a randomly oriented disperse ferromagnet the absorption linewidth turns out to be a nonmonotonic function of temperature. For low temperatures $\Delta H$ is large due to the scatter in directions of the anisotropy fields of the particles (inhomogeneous broadening); as the temperature increases the thermally induced tendency to make the magnet isotropic causes $\Delta H$ to decrease, but in the fluctuation region it once again begins to increase. This effect is illustrated in Figs. 3a and 3b, which show our numerical calculations of the dependence of $\Delta H$ on $\xi_0 = 1/T$ for randomly oriented uniaxial and cubic magnets. In the same figures we compare the behavior of this function with the following asymptotically exact solutions, shown as dashed curves: $(\gamma/\alpha)\Delta H = 2\Delta\gamma/3\sqrt{2}$ for superparamagnetic broadening (as $K \rightarrow 0$), $(\gamma/\alpha)\Delta H = 3\Delta\gamma/L_e$ for inhomogeneous broadening as $\alpha \rightarrow 0$ for the case of uniaxial anisotropy, and $(\gamma/\alpha)\Delta H = 10/3\sqrt{2}$ for inhomogeneous broadening as $\alpha \rightarrow 0$ in the case of cubic anisotropy. It is remarkable how rapidly the exact solution approaches the corresponding asymptotic curves on both sides of the minimum point, independent of the type of magnetic anisotropy. It is clear from Fig. 3 that the position of this point can be estimated using the relations $\Delta H = \Delta H$ and $\Delta H = \Delta H$. It is especially simple to obtain estimates for the case of uniaxial anisotropy.
Absorption line shape (left column) and $d^2 H/d^2$ (right column) for an ensemble of particles with positive cubic anisotropy ($E = 0.1$) as functions of the Langevin parameter: $\xi = 50$ (a), 10 (b), 2 (c), and 1 (d). For all the curves the precession attenuation parameter $\alpha = 10^{-1}$. For definiteness, let us use the asymptotic forms (35) and (36) to write the expressions for the superparamagnetic and inhomogeneous contributions to the linewidth of an ensemble of magnetically uniaxial particles:

$$
\Delta \omega = \frac{4}{3} \omega \frac{a}{b^3} \frac{b}{b_0}, \quad \Delta \omega = \frac{3}{5} \omega \frac{a}{b^3} \frac{b}{b_0}.
$$

and investigate these expressions when $\xi < 1$. An expansion gives

$$
\Delta \omega = \frac{4}{3} \omega \frac{a}{b^3} \frac{b}{b_0}, \quad \Delta \omega = \frac{3}{5} \omega \frac{a}{b^3} \frac{b}{b_0}.
$$

Setting $\Delta \omega = \Delta \omega$, we obtain the relation

$$
\Delta \omega = \frac{4}{3} \omega \frac{a}{b^3} \frac{b}{b_0}, \quad \Delta \omega = \frac{3}{5} \omega \frac{a}{b^3} \frac{b}{b_0},
$$

which is in satisfactory agreement with the results of the numerical calculations.

The results of including the effects of both superpara-
FIG. 4. Position of the maximum of the curve $\frac{d\gamma}{dH}$ (resonance field) as a function of the Langevin parameter $\frac{1}{kT}$ for uniaxial (curve 1) and cubic (curves 2) anisotropy when $\lambda = 0.1$ (solid curves) and $\lambda = -0.1$ (dashed curves).

magnetic and inhomogeneous broadening on the value of the resonance field $H_{\text{res}}$, which we determine based on the position of the point $\frac{d\gamma}{dH} = 0$, i.e., that of the peak in the absorption curve, are illustrated in Fig. 4. It turns out that heating the sample can lead to either an increase or a decrease in the resonance field, depending on the sign of the magnetic anisotropy constant of the particles.

4. CONCLUSIONS

In randomly oriented dispersed ferromagnets and ferrites, superparamagnetism can weaken or entirely suppress the inhomogeneous broadening of the FMR line caused by the scatter in orientations of the anisotropy axis.

In such systems the combined influence of the orientational texture and the superparamagnetism can lead to a nonmonotonic temperature dependence of the FMR linewidth, of the sort that has been noted in a number of experiments when the latter is measured in a magnetizing field, i.e., $\Delta H(T)$ passes through a minimum for $T \sim \left(\frac{\sqrt{K}}{kT}\right)\left(\frac{K_{\text{eff}}}{\gamma A T^2}\right)^{1/2}$.

The character of the temperature dependence of the resonance field is connected in a simple way with the nature and sign of the magnetic anisotropy of the particles in the dispersed ferromagnet: as the temperature decreases, $H_{\text{res}}$ increases for systems with $K > 0$ and decreases for the case $K < 0$.


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