Electromagnetic waves in a periodic metal-dielectric structure

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A study is made of transverse waves propagating through a sequence of alternate metal and dielectric layers with respectively strong and zero spatial dispersion. A simple proof of the unimodular nature of the transformation matrix is given. The matrix elements are calculated for electrons reflected from the boundary specularly, diffusely, or specularly at one boundary and diffusively at the other. It is shown that the properties of the superlattice are determined not as much by the purity of the metal as by interface processes and by the very fact that the motion of the electrons is restricted along the wave propagation direction.

The discovery of the giant magnetoresistance effect in multilayered Fe/Cr films[1] stimulated investigation of kinetic effects in metallic superlattices. While the early research efforts naturally concentrated on analyzing the role of magnetic interactions, it did not take long to realize that the structure of a film—and particularly of its interface(s)—also has a marked influence on the magnitude of magnetoresistance.[2] This seems to indicate that a significant role may be played by bulk layer properties as well as by those interface processes that have no direct relevance to magnetic interactions and are common to magnetic and nonmagnetic superlattices. Unfortunately, the relation between these factors and the electrodynamic properties of the film is not yet fully understood, not even for nonmagnetic structures, so it requires further effort to obtain better insight into the phenomenon.

In this paper we investigate the wave spectrum and calculate the surface impedance of a periodic structure formed by alternating nonmagnetic metal and dielectric layers. It is assumed that the magnetic field is zero and that the electron mean free path is such that in a bulk metal sample the anomalous skin effect would be observed. Under these conditions the spectrum is largely determined by scattering at the boundaries of a metal layer, and it should be realized that these boundaries may differ in their properties because of the specifics of the superlattice growth process. Since the electron is restricted to a single layer in its motion, calculations are relatively simple and the final results quite manageable.

1. PROBLEM FORMULATION

We direct the z axis along the normal to the interface planes and define the metal and dielectric layers by the respective conditions $nd\leqslant z<nd+a$ and $-nd+a\leqslant z<nd+a+b$, where $a(b)$ is the thickness of a metal (dielectric) layer, $d=a+b$ is the period of the structure, and n an integer. Suppose the superlattice carries a z-propagating wave whose electric field $\mathbf{E}(z,t)$ (t being the time) is directed along the x axis; the time variation will be taken in the form

$$\mathbf{E}(z,t)=\mathbf{E}(z)e^{-i\omega t}$$

and the polarization index will be dropped.

To find the spectrum of such waves it is convenient to employ the transfer matrix m (Ref. 3) which relates the field E and its derivative $E'=\partial E/\partial z$ at one end of a certain interval (say, $0<z<d$) to their counterparts at the other:

$$(E(0),E'(0))=m\begin{pmatrix}E(d)\\E'(d)\end{pmatrix}. \tag{1}$$

Here m_a and m_b are the transfer matrices for metal and dielectric layers, respectively ($m=m_am_b$). Note that these matrices are unimodular: this can be deduced from the reciprocity theorem of Ref. 4 which is essentially a consequence of the symmetry of kinetic coefficients. A direct proof of the unimodular nature of the transfer matrices is obtained in the next section in connection with the theory.

By Floquet's theorem,[3]

$$[E(d),E'(d)]=e^{iQd}[E(0),E'(0)].$$

Substituting this into (1) and noting that $\|m\|=1$, we obtain the familiar expression for determining the Bloch wave vector $Q=Q'+iQ''$:

$$\cos Qd=\tfrac{1}{2}(m_{11}+m_{22}). \tag{2}$$

For a superlattice occupying the half-space $z\geqslant 0$ the surface impedance is

$$Z=\frac{4\pi i\omega}{c^2}\frac{E(0)}{E'(0)}=\frac{4\pi i\omega}{c^2}\frac{m_{12}}{e^{-iQd}-m_{11}}, \tag{3}$$

taking that solution to (2) for which $Q''>0$.

For a dielectric layer the transfer matrix is given by[3]

$$m_{b11}=m_{b22}=\cos(q_{\rm em}b),\quad m_{b12}=q_{\rm em}^{-1}\sin(q_{\rm em}b),$$

$$m_{b21}=-q_{\rm em}\sin(q_{\rm em}b),$$

where $q_{\rm em}$ is the vector of the electromagnetic wave in the dielectric. Thus we see that the problem reduces to that of finding the matrix m_a.

2. TRANSFER MATRIX FOR A METAL LAYER

Consider a metal layer $0 \leqslant z < a$. The spatial dependence of the electric field in the layer is found from the equation

$$\frac{\partial^2 E}{\partial z^2} + \frac{4\pi i\omega}{c^2} j = 0, \tag{4}$$

with the current density

$$j(z) = \frac{2}{a} \int_0^a K(z, z') E(z') dz'. \tag{5}$$

Expanding $E(z)$ in a Fourier series

$$E(z) = \sum_{n=0}^{\infty} \left(1 - \frac{1}{2}\delta_{N0}\right) E_N \cos(q_N z),$$
$$q_N = \frac{\pi N}{a}, \quad N = 0,1,2,..., \tag{6}$$

we find

$$j_N = \sum_{N'=0}^{\infty} \left(1 - \frac{1}{2}\delta_{N'0}\right) K_{NN'} E_{N'}, \tag{7}$$

$$q_N^2 E_N = \frac{2}{a} [(-1)^N E'(a) - E'(0)]$$
$$+ \frac{4\pi i\omega}{c^2} \sum_{N'=0}^{\infty} \left(1 - \frac{1}{2}\delta_{N'0}\right) K_{NN'} E_{N'} \tag{8}$$

Singling out the term $N' = N$ in the sum over N' and denoting

$$P_{NN'} = \frac{4\pi i\omega}{c^2} (1 - \delta_{NN'}) K_{NN'}, \tag{9}$$

we rewrite Eq. (8) as

$$\left[q_N^2 - \frac{4\pi i\omega}{c^2}\left(1 - \frac{1}{2}\delta_{N0}\right) K_{NN}\right] E_N$$
$$= \frac{2}{a} [(-1)^N E'(a) - E'(0)]$$
$$+ \sum_{N'=0}^{\infty} \left(1 - \frac{1}{2}\delta_{N'0}\right) P_{NN'} E_{N'}. \tag{10}$$

Iterating Eq. (10) now yields E_N in series form which we must substitute into (6) to obtain $E(0)$ and $E(a)$. The result may be written as

$$E(0) = A_{11} E'(0) + A_{12} E'(a),$$
$$E(a) = A_{21} E'(0) + A_{22} E'(a), \tag{11}$$

where the elements of the matrix A are given by the expansions

$$A_{11} = -\frac{2}{a}\left[\sum_{N=0}^{\infty}\left(1 - \frac{1}{2}\delta_{N0}\right)\frac{1}{\Delta_N} + \sum_{N,N'=0}^{\infty}\left(1 - \frac{1}{2}\delta_{N0}\right)\right.$$
$$\left.\times\left(1 - \frac{1}{2}\delta_{N'0}\right)\frac{P_{NN'}}{\Delta_N \Delta_{N'}} + ...\right],$$

$$A_{12} = \frac{2}{a}\left[\sum_{N=0}^{\infty}\left(1 - \frac{1}{2}\delta_{N0}\right)\frac{(-1)^N}{\Delta_N} + \sum_{N,N'=0}^{\infty}\left(1 - \frac{1}{2}\delta_{N0}\right)\right.$$
$$\left.\times\left(1 - \frac{1}{2}\delta_{N'0}\right)\frac{(-1)^{N'} P_{NN'}}{\Delta_N \Delta_{N'}} + ...\right],$$

$$A_{21} = -\frac{2}{a}\left[\sum_{N=0}^{\infty}\left(1 - \frac{1}{2}\delta_{N0}\right)\frac{(-1)^N}{\Delta_N} + \sum_{N,N'=0}^{\infty}\left(1 - \frac{1}{2}\delta_{N0}\right)\right.$$
$$\left.\times\left(1 - \frac{1}{2}\delta_{N'0}\right)\frac{(-1)^N P_{NN'}}{\Delta_N \Delta_{N'}} + ...\right], \tag{12}$$

$$A_{22} = \frac{2}{a}\left[\sum_{N=0}^{\infty}\left(1 - \frac{1}{2}\delta_{N0}\right)\frac{1}{\Delta_N} + \sum_{N,N'=0}^{\infty}\left(1 - \frac{1}{2}\delta_{N0}\right)\right.$$
$$\left.\times\left(1 - \frac{1}{2}\delta_{N'0}\right)\frac{(-1)^{N+N'} P_{NN'}}{\Delta_N \Delta_{N'}} + ...\right],$$

$$\Delta_N = q_N^2 - \frac{4\pi i\omega}{c^2}\left(1 - \frac{1}{2}\delta_{N0}\right) K_{NN}.$$

From (11),

$$m_a = \begin{pmatrix} A_{11}/A_{21} & -\|A\|/A_{21} \\ 1/A_{21} & -A_{22}/A_{21} \end{pmatrix}. \tag{13}$$

The elements of the matrices A and m_a are not independent. The symmetry of the kinetic coefficients implies that $K(z, z') = K(z', z)$ (cf. Ref. 5, § 103) and hence $P_{NN'} = P_{N'N}$. With this knowledge, it is readily shown that in the series for A_{12} and A_{21} the terms of the same order differ only in sign, so that $A_{21} = -A_{12}$ and $\|m_a\| = -A_{12}/A_{21} = 1$. Thus the unimodularity of the transformation matrix follows from the symmetry of kinetic coefficients.

The relation between A_{11} and A_{22} may be of any form, but if $K_{NN'} = (-1)^{N+N'} K_{NN'}$ then $A_{22} = -A_{11}$.

The Fourier transform of the current density is commonly written in the form

$$j_N = \sigma_N E_N + \sum_{N'=0}^{\infty}\left(1 - \frac{1}{2}\delta_{N'0}\right)\sigma_{NN'} E_{N'}, \tag{14}$$

where $\sigma_N = \sigma(q_N)$ is the bulk conductivity and $\sigma_{NN'}$ the surface conductivity of the metal. Comparing (7), (9), and (14) we obtain

$$\left(1 - \frac{1}{2}\delta_{N0}\right) K_{NN} = \sigma_N + \left(1 - \frac{1}{2}\delta_{N0}\right)\sigma_{NN},$$
$$P_{NN'} = \frac{4\pi i\omega\sigma_{NN'}}{c^2}(1 - \delta_{NN'}). \tag{15}$$

As is well known,

$$\frac{4\pi i\omega\sigma_N}{c^2}=\begin{cases}2i/\delta_0^2, & q_N l\ll 1,\\ i/q_N\delta^3, & q_N l\gg 1,\end{cases} \tag{16}$$

where $\delta_0=c(2\pi\omega\sigma_0)^{-1/2}$ and δ are the skin depths in the normal and anomalous skin effect regimes, respectively; $\delta^3\sim\delta_0^2 l$, l is the mean free path, and σ_0 the bulk conductivity of the metal when in a uniform field. To calculate $\sigma_{NN'}$ it is necessary to specify boundary conditions for the electron distribution function. In the present study the *integral* boundary conditions[6] are replaced—admittedly at the expense of consistency—by the simpler *Fuchs conditions*. It is assumed, namely, that the electrons are scattered from the boundaries either purely specularly or purely diffusely, or else specularly from the plane $z=0$ and diffusely from the plane $z=a$. Even though in a somewhat simplistic fashion, these boundary conditions make it possible to analyze the role of imperfect boundaries—including their possible dissimilarity. It is assumed that spatial dispersion is strong (i.e., $\delta_0\ll\delta\ll l$) and that the electronic spectrum is parabolic and isotropic.

Within these approximations the calculation of the surface conductivity poses no problems of principle and may be carried out by standard methods (see Refs. 6–8 for a discussion and references), so that in what follows we summarize the results without presenting details.

When the electrons are reflected specularly from both boundaries of the layer, then the surface conductivity is zero. For diffuse reflection,

$$\sigma_{NN'}=-\frac{3}{2}\sigma_0\frac{a}{l}F_{NN'}^{dd}\left(\frac{a}{l}\right), \tag{17}$$

where the function $F_{NN'}^{dd}(x)$ is given by

$$F_{NN'}^{dd}(x)=\int_x^\infty \frac{t^2-x^2}{t}\times\frac{1+(-1)^{N+N'}-[(-1)^N+(-1)^{N'}]e^{-t}}{[(\pi N)^2+t^2][(\pi N')^2+t^2]}\,dt. \tag{18}$$

For the case where the electrons are reflected specularly from the plane $z=0$ and diffusely from the plane $z=a$,

$$\sigma_{NN'}=-6\sigma_0\left(\frac{a}{l}\right)F_{NN'}^{sd}\left(\frac{2a}{l}\right), \tag{19}$$

where

$$F_{NN'}^{sd}(x)=(-1)^{N+N'}\int_x^\infty \frac{t^2-x^2}{t}\times\frac{1-e^{-t}}{[(2\pi N)^2+t^2][(2\pi N')^2+t^2]}. \tag{20}$$

Note that $A_{22}\neq -A_{11}$ in this last case.

Although Eqs. (17)–(20) hold for an arbitrary ratio of the metal thickness to mean free path, we will assume $a\ll\delta$, because otherwise superlattice vibrations would die down across a single metal layer.

The transversely averaged steady-state conductivity of the film is given by

$$\bar\sigma_0=\sigma_0+\sigma_{00}/2. \tag{21}$$

In the limiting case $a\ll l$, Eqs. (17) through (21) yield the familiar results[8,9]

$$\bar\sigma_0^{dd}=\frac{3a}{4l}\sigma_0\left[\ln\left(\frac{l}{a}\right)+1-C\right],$$
$$\bar\sigma_0^{sd}=\frac{3a}{2l}\sigma_0\left[\ln\left(\frac{l}{2a}\right)+1-C\right], \tag{22}$$

where $C=0.577...$ is Euler's constant.

The definitions above show that as $x\to 0$, the functions $F_{NN'}(x)$ are only singular at $N=N'=0$, so that, turning back to the series expansions (12) for the A_{ij}, we see that for $a\ll\delta\ll l$ the first sum in the brackets is larger than the second. Neglecting small terms we have

$$A_{11}=-A_{22}=-\frac{i\bar\delta_0^2}{2a}-\frac{a}{3},$$
$$A_{12}=-A_{21}=\frac{i\bar\delta_0^2}{2a}-\frac{a}{6}, \tag{23}$$

where $\bar\delta_0$ is obtained from δ_0 by replacing σ_0 with $\bar\sigma_0$.

Substitution of (23) into (13) now gives the matrix m_a.

3. SOLUTION OF THE DISPERSION EQUATION

We are now able to be more specific when writing Eq. (2). If we multiply m_a, Eq. (13), by m_b and substitute the expressions obtained for m_{11} and m_{22}, Eq. (2) becomes

$$\cos Qd=\frac{A_{11}-A_{22}}{A_{21}}\cos(q_{\rm em}b)-\frac{1}{2}\left(\frac{\|A\|}{A_{21}}q_{\rm em}+\frac{1}{q_{\rm em}A_{21}}\right)\sin(q_{\rm em}b). \tag{24}$$

We can simplify this equation by noting, first, that for typical frequencies and layer thicknesses $q_{em}b\ll 1$ and, second, that we always have $q_{em}\delta\ll 1$. This gives

$$\cos Qd=\frac{A_{11}-A_{22}-b}{2A_{21}}, \tag{25}$$

or, substituting A_{11}, A_{22}, and A_{21} from (23),

$$\cos Qd=\frac{i\bar\delta_0^2/a^2+2/3+b/a}{i\bar\delta_0^2/a^2-1/3}. \tag{26}$$

Note that to take the limit $a\to 0$ in Eqs. (25) and (26) would be incorrect because we have assumed $\cos(q_{em}b)=1$ in their derivation. This assumption is justified if the (not overrestrictive) condition $a/b\gg q_{em}\bar\delta_0$ is fulfilled.

Let us consider the limiting cases which can arise. If

$$\bar\delta_0^2/a^2\gg(2/3+b/a)$$

(thin-layered medium), then

$$Q^2=2ia/\bar\delta_0^2 d. \tag{27}$$

If $b\gg a$, then the condition

$$b/a \gg \bar{\delta}_0^2/a^2 \gg 1/3$$

may be fulfilled, which makes cos (Qd) almost pure imaginary and quite large in absolute value

$$\cos(Qd) = -i\, ab/\bar{\delta}_0^2, \tag{28}$$

This means that the wave is virtually damped out across one period of the structure. A similar situation occurs for $\bar{\delta}_0^2/a^2 \ll 1/3$, but in this case cos (Qd) turns out to be almost pure real:

$$\cos(Qd) = -(2+3b/a). \tag{29}$$

4. SURFACE IMPEDANCE

By following the same approximations used in proceeding from (24) to (25) we obtain

$$m_{11} = A_{11}/A_{21}, \quad m_{12} = -\frac{\|A\|}{A_{21}} - \frac{bA_{11}}{A_{21}}. \tag{30}$$

Using (3) and (30) it is simple to calculate Z for all the cases discussed. For a thin-layered medium

$$Z = \frac{2\pi\omega\bar{\delta}_0}{c^2}\sqrt{\frac{d}{a}}\,(1-i). \tag{31}$$

For $b/a \gg \bar{\delta}_0^2/a^2 \gg 1/3$ the impedance is essentially real

$$Z = \frac{1}{a\bar{\sigma}_0}, \tag{32}$$

to be contrasted with the case $\bar{\delta}_0^2/a^2 \ll 1/3$ in which it is almost pure imaginary

$$Z = -i\frac{2\pi\omega a}{c^2}\frac{a+4b}{3b+\sqrt{(2a+3b)^2-a^2}}. \tag{33}$$

It is found that the variation of the impedance with ω (frequency) and $\bar{\sigma}_0$ (conductivity of the metal layer) is strongly situation-dependent: for a thin-layered structure we have $Z \propto (\omega/\bar{\sigma}_0)^{1/2}$; in the second case we have $Z \propto \bar{\sigma}_0^{-1}$ and it does not depend on ω; in the third case the impedance does not depend on the conductivity but is proportional to the frequency. Note that the last case, $\bar{\delta}_0^2/a^2 \ll 1/3$, may only occur when the electrons are reflected specularly from layer boundaries. Even if only one boundary reflects diffusely, we have

$$\bar{\delta}_0^2/a^2 \sim (\delta_0^2 l/a^3)/\ln(l/a) \sim (\delta/a)^3/\ln(l/a)$$

and our assumption $a \ll \delta \ll l$ suggests $\bar{\delta}_0^2 \gg a^2$.

Our results indicate that if the metal layers are thin, then the electrodynamic properties of the periodic structure considered depend not as much on the purity of the metal as on the nature of interface processes and on the very fact that the freedom of motion of the electrons is restricted in the wave propagation direction. A consequence of this restriction is that the electric field of the wave, rather than interacting with a relatively small group of grazing electrons (as it does in the anomalous skin effect regime in a bulk metal), interacts with all the totality of electrons available in the layer; with a result, for example, that in a thin-layered medium the skin effect turns out to be normal. For the same reason the limiting process $b \to 0$ does not take us back to the bulk metal situation—as it would in the absence of spatial dispersion.

Measurements of the surface impedance may be used to determine, in a contactless manner, the thickness dependence of the conductivity of the metal layer; if found, such a dependence would signify a strong interface scattering of electrons. If the variation of the thickness is impossible for some reasons, then data on the surface impedance and the steady-state conductivity may be used to understand the nature of the interface reflection: if the conductivity is large and the skin effect normal, then the reflection of the electrons from the boundary planes is close to specular.

Finally, it is felt that the basic results of this study should remain valid when a dielectric layer is replaced by a layer of a low-conductivity metal. To see this, note the assumptions we have made concerning the properties the dielectric layer: 1) spatial dispersion is absent, 2) the wave undergoes a small phase shift and remains almost undamped on traversing the layer, and 3) high-mobility electrons are unable to escape from their layer. Clearly these conditions may be fulfilled if the mean free path l_b and the skin depth δ_{0b} in the "bad" metal satisfy the inequalities $l_b \ll b \ll \delta_{0b}$ and $a \gg l_b$.

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