Kinetic theory of nonequilibrium fluctuations and the theory of acoustoelectric domains

Yu. V. Gulyaev and V. I. Pustovoit

Institute of Radio Engineering and Electronics, Russian Academy of Sciences, 103907 Moscow, Russia
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In this paper we solve the problem of how acoustic-wave fluctuations develop in piezoelectrics under conditions of sonic instability. We obtain a system of kinetic equations in rather general form for the nonequilibrium phonons and the nonequilibrium source, and show that the unstable system automatically separates into regions where the instabilities are absolute and convective in character. In these regions the growing fluctuations depend differently on coordinates and time: in a region with convective instability, the fluctuations increase with the spatial coordinate, while in a region with absolute instability, they increase with time. The boundary between these regions translates through the crystal with a velocity close to the velocity of a sound wave. We then construct a general theory of acoustoelectric domains in semiconductors under conditions of sonic instability. We show that the large-scale properties of the medium (i.e., for spatial and temporal scales considerably larger than the corresponding wavelength and oscillation period of the equilibrium acoustic phonons) are described by different equations for the regions with instabilities of absolute and convective character. Using the saddle-point method, we also find an expression for the acoustoelectric force, which describes the reverse effect of the growing phonon fluctuations on the large-scale properties of the medium, and solve the equations for the electric field distribution in the regions with absolute and convective instability. We also find an expression for the electric current under conditions of sonic instability, and show that the electric current saturates as the instability develops. It is noteworthy that this current saturation is not connected with trapping of carriers in a potential well, but rather is explained by the collective action of the nonequilibrium phonons.

The investigation of nonequilibrium media is always a topic of current interest both from a practical and a theoretical point of view, since even today there are numerous physical phenomena that have not been explained theoretically.

Many examples of nonequilibrium media are known: laser media, in which state populations are inverted and spontaneous amplification of electromagnetic waves is possible; gas-discharge and semiconductor plasmas in a strong electric field; and semiconductor crystals subjected to electric fields strong enough that the directed drift of electrons and holes exceeds the phase velocity of sound or some other type of wave, so that growing fluctuations or oscillations of the corresponding waves are possible. It is important to note that the macroscopic properties of a nonequilibrium medium differ significantly from those of an equilibrium medium under conditions where fluctuations can develop: abrupt changes occur in the scattering of optical, x-ray and γ-radiation, the media behave differently in external electric and magnetic fields, and it is even possible for qualitatively new states to emerge, e.g., a magnetic moment (see Refs. 1–3) or the Hall constant measured in an experiment may change sign.[3–5] The experiments described in Refs. 6–8 reveal that when phonons are generated in a piezoelectric semiconductor crystal under sonic instability conditions, the electric field distribution becomes highly nonuniform, and a region of rather high electric field and nonequilibrium phonon density forms (a so-called acoustoelectric domain) which propagates through the crystal with a velocity close to that of sound. This phenomenon was observed about 30 years ago, and despite a multitude of experiments (see the review Ref. 8), there is no theory for it to this day. In our opinion, one reason why attempts to construct a theory of these acoustoelectric domains have been unsuccessful is that the theory that describes the evolution of nonequilibrium acoustic fluctuations in semiconductor crystals is itself in need of some refinement, in the direction of a rigorous derivation of kinetic equations for the nonequilibrium fluctuations (see below). Investigators have found (see Ref. 9) that when the nonequilibrium fluctuations are studied using the Langevin approach, the Langevin source itself depends on coordinates and time in the nonequilibrium system, thereby ensuring that the character of the spatial and temporal fluctuations will change. It is found that an unstable system automatically separates into regions with convective and absolute instabilities, in which the reverse effect of the growing fluctuations on the macroscopic properties of the medium depends on the spatial and temporal coordinates in different ways. This latter circumstance, which has not been taken into account previously in constructing the theory of acoustoelectric domains, turns out to play an important role in elucidating the physical nature of the evolution of acoustoelectric instability in semiconductors.

The goal of this paper is to construct a general theory of nonequilibrium fluctuations in systems described by lin-

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ear equations, and to use this general theory to create a theory of acoustoelectric instability in piezoelectric semiconductors and a theory of acoustoelectric domains.

1. KINETIC THEORY OF NONEQUILIBRIUM FLUCTUATIONS

A general method of constructing a theory of fluctuations for a system in a state of stationary and thermodynamic equilibrium was developed by Rytov[10] and Callen and Welton,[11] who showed that fluctuations in the system are determined by the dissipative part of the general response to a corresponding external perturbation (the so-called fluctuation-dissipation theorem). For nonequilibrium linear systems, as we show in this paper, a system of equations can likewise be constructed in closed form which describes these fluctuations; as it turns out, the intensity of the fluctuations is subject to a kinetic equation whose right side contains a source for which a special equation is obtained. The method of obtaining this system of equations is based on the fact that it is always possible in practice to identify three characteristic temporal and spatial scales: a small scale, which determines the period and wavelength of the fluctuating field, a medium scale over which the external Langevin forces are correlated, and finally a third large scale, which characterizes the spatial and temporal growth of the fluctuations themselves. This division into scales implies that the spatial and temporal dependences of the correlation functions of the external forces will be the same for a nonequilibrium medium and a uniform medium, since the properties of the medium that determine the correlation can change only on the large scales that determine the growth of the fluctuations themselves. Mathematically this implies that the Fourier transforms of the correlation functions of the external forces will not depend explicitly on the position and time in the nonequilibrium medium; of course, the correlation functions of the amplitudes of the fields or the mixed correlation functions of the amplitudes and fields, which determine the corresponding sources in the kinetic equations for the intensities of the fluctuating fields, do depend on position and time. Our entire discussion will be couched in a general form for the example of a piezoelectric semiconductor, in which the instability is caused by the plasma subsystem of drifting electrons; since the state of the nonequilibrium plasma subsystem is not specified, this work is completely general and can be applied to any other linear system.

In accordance with the Langevin approach, we introduce external (random) forces into the original system of equations for the theory of elasticity and the Maxwell equations; these sources determine the fluctuations in the amplitudes of the elastic displacements and electric fields. By applying the slowly-varying amplitude approximation in the usual way, we obtain kinetic equations for the bilinear averages on whose right sides appear correlation functions of the corresponding amplitudes of the elastic displacement (electric field) and the external force. In contrast to previous papers, we also obtain equations for these correlation functions in the same slowly-varying-amplitude approximation, whose right sides now contain the intrinsic correlation functions of the external forces, which should be found independently. This separation by scales allows us to use well-known methods to calculate the external forces in nonequilibrium media in order to find their correlation functions, e.g., a generalization of the theory of thermodynamic fluctuations (see Refs. 1,9) or the methods of kinetic theory (see Refs. 12,13). For the correlation functions of the external elastic stresses we use the fluctuation-dissipation theorem, since the lattice subsystem will remain in equilibrium at constant temperature (far from a phase transition). In this paper the intrinsic correlation functions of the external forces are assumed to be known.

The solutions we find for the source equations show that in a nonequilibrium medium the source itself becomes a function of position and time. This reflects the simple physical fact that the correlations between an external force and an amplitude (field) will grow in time and space in an unstable medium. Therefore it is natural that the solutions we find for the energy spectral density of the generated phonons will differ from the known expressions, although the conclusion that the phonon spectral density grows exponentially remains valid.

We begin with the equations of the theory of elasticity for a semiconducting piezoelectric medium[9,14] in their most general form:

$$\rho \frac{d^2u_i}{dt^2}-\lambda_{iklm}\frac{du_{lm}}{dr_k}-\mu_{iklm}\frac{d^2u_{lm}}{dr_kdt}-\beta_{l,ik}\frac{d^2\varphi}{dr_kdr_l}=\frac{d}{dr_k}\sigma_{ik}^{(s)}(\mathbf{r},t), \tag{1}$$

$$\varepsilon_0\frac{d^2\varphi}{dr_idt}-4\pi\int dt'\int d^3r\sigma_{ij}(\mathbf{r}-\mathbf{r}',t-t')\frac{d\varphi(\mathbf{r}',t')}{dr_j'}$$
$$-4\pi\beta_{i,kl}\frac{du_{kl}}{dt}=-\frac{dD_i^{(s)}(\mathbf{r},t)}{dt}. \tag{2}$$

Here ρ is the density of the crystal, λ_{iklm}, μ_{iklm} are the tensors for the elastic modulus and viscosity, respectively (their symmetry properties coincide), $\beta_{l,ik}$ is the tensor of piezoelectric coefficient "with respect to strain," ε_0 is the dielectric permittivity of the medium, $\sigma_{ik}(\mathbf{r},t)$ is the conductivity tensor of the medium including spatial and temporal dispersion, $u_{ik}(\mathbf{r},t)$ is the strain tensor given by the expression $u_{ik}=\frac{1}{2}(du_i/dr_k+du_k/dr_i)$, where $u_i(\mathbf{r},t)$ is the displacement vector, and $\varphi(\mathbf{r}, t)$ is the scalar potential of the electric field that accompanies the sound wave in a piezoelectric medium.

Because the elastic medium is also piezoelectric, there are two kinds of "random forces" that lead to fluctuations in the elastic displacement vector: spontaneous (more often called random) elastic oscillations of the stress $\sigma_{ik}(\mathbf{r},t)$ in the medium, and random or spontaneous oscillations of the electric displacement $D_i^{(s)}(\mathbf{r},t)$ or current $\mathbf{J}^{(s)}(\mathbf{r},t)=(1/4\pi)(d/dt)\mathbf{D}^{(s)}(\mathbf{r},t)$. These quantities play the role of "random forces" in Eqs. (1) and (2) (see Refs. 15, 16). We will assume that the correlations between these external forces at different times and at different points in space are given by known correlation functions:

$$\left\langle \frac{d\sigma_{ik}^{(s)}(\mathbf{r},t)}{dr_k} \frac{d\sigma_{jm}^{(s)}(\mathbf{r},t)}{dr_m} \right\rangle = \psi_{ij}(|\mathbf{r}-\mathbf{r}'|,\ |t-t'|), \quad (3)$$

$$\langle \operatorname{div} \mathbf{D}^{(s)}(\mathbf{r},t) \operatorname{div} \mathbf{D}^{(s)}(\mathbf{r}',t') \rangle = \phi(\mathbf{r}-\mathbf{r}',\ t-t'). \quad (4)$$

The differing dependences of the correlation functions ψ_{ij} and ϕ on their arguments reflect the fact that the correlations (3) for the elastic subsystem are always stationary and in thermodynamic equilibrium, because there is no physical basis within the framework of the problem as formulated here to assume that the correlations of the random elastic stresses of the crystalline medium (without electrons) do not satisfy the fluctuation-dissipation theorem. However, for a semiconductor in which currents of electrons and (or) holes can exist, there is no basis to assume that the correlation function (4) does satisfy the fluctuation-dissipation theorem. For our specific case of a drifting electron plasma in a semiconductor, the correlation function (4) was determined in Refs. 3, 9, 12, 13; therefore we will assume in what follows that the functions (3) and (4) are known and specified.

When the carrier drift in the semiconductor is supersonic, amplification and generation of acoustic waves become possible. In obtaining kinetic equations describing fluctuations that grow in space we can no longer make use of the usual Fourier transformations in space and time, i.e., the functions we seek diverge as t, $x \to \infty$. However, it is also obvious that the divergence of these functions has an exponential character; therefore we always can introduce a certain parameter $s > 0$ such that a function of the form $e^{-sx}u(x)$ will now be convergent as $x \to \infty$. For definiteness we will assume that the medium is semi-infinite along the direction of electron drift, i.e., in the x direction. In the y and z directions we will assume that the medium is unbounded, and that the orientation of the crystal is chosen such that the growth of the fluctuations in these directions is bounded, so that we can use the usual Fourier transformations in these directions. Along the x-direction we will use the Fourier–Laplace transformation with the necessary displacement of the contour of integration.

In Eqs. (1) and (2), we introduce the Fourier components of the quantities u and φ according to the relations:

$$\mathbf{u}(\mathbf{r},t) = \frac{1}{(2\pi)^4} \int_{-\infty+is_1}^{\infty+is_1} d\omega e^{i\omega t} \int_{-\infty+is_2}^{\infty+is_2} dq_x e^{-iq_x x} \times \int dq_y dq_z e^{-iq_y y - iq_z z} \mathbf{u}(\omega,\mathbf{q}), \quad (5)$$

$$\mathbf{u}(\omega,\mathbf{q}) = \int_0^\infty dt e^{-i\omega t} \int_0^\infty dx e^{iq_x x} \int dy dz e^{iq_y y + iq_z z} \mathbf{u}(\mathbf{r},t), \quad (6)$$

then we obtain the following linear system of algebraic equations from Eqs. (1) and (2)

$$L_{ij}(\omega,\mathbf{q}) u_j(\omega,\mathbf{q}) = Y_i(\omega,\mathbf{q}). \quad (7)$$

Here $L_{ij}(\omega,\mathbf{q})$ is the dispersion operator, which equals

$$L_{ij}(\omega,\mathbf{q}) = \rho\omega^2\delta_{ij} - \lambda_{iklj} q_k q_l - i\omega\mu_{imnj} q_m q_n - \frac{4\pi\beta_{l,ik}\beta_{p,js} q_l\, q_k\, q_p\, q_s}{q_m q_n \varepsilon_{mn}(\omega,\mathbf{q})}, \quad (8)$$

where $\varepsilon_{ij}(\omega,\mathbf{q}) = \varepsilon_0\delta_{ij} - 4\pi\sigma_{ij}(\omega,\mathbf{q})/i\omega$ is the permittivity of the medium, and $Y_i(\omega,\mathbf{q})$ is the Fourier transform of the sources of the random forces taking into account the initial and boundary conditions:

$$Y_i(\omega,\mathbf{q}) = iq_m \sigma_{mi}^{(s)}(\omega,\mathbf{q}) + i\beta_{l,ik} q_k D_l^{(s)}(\omega,\mathbf{q}) + Y_i^0(\omega,\mathbf{q}); \quad (9)$$

here $Y_i^0(\omega,\mathbf{q})$ are terms that contain the values of the initial and boundary amplitudes, i.e., the values of the Fourier components of $\mathbf{u}(\mathbf{r},t)$ at $x=0$, $t=0$ and their first derivatives. The explicit form of the terms $Y_i^0(\omega,\mathbf{q})$ depends on the specific boundary and initial conditions (see Refs. 9,14).

Let us express the system of linear equations (7) in terms of eigenvalues. Taking into account the symmetry of the matrix L_{ij} and the fact that its real part is much larger than its imaginary part [it is in this case that we can speak of wave solutions to the system (1), (2)], we obtain from Eq. (7)

$$\lambda_\alpha(\omega,\mathbf{q}) u^\alpha(\omega,\mathbf{q}) = Y_\alpha(\omega,\mathbf{q}), \quad (10)$$

where $\lambda_\alpha(\omega,\mathbf{q})$ is an eigenvalue of the matrix $L_{ij}(\omega,\mathbf{q})$ defined by the equation:

$$\det|L_{ij}(\omega,\mathbf{q}) - \lambda_\alpha(\omega,\mathbf{q})\delta_{ij}| = 0, \quad (11)$$

and $u^\alpha(\omega,\mathbf{q})$ is the amplitude of the displacement "along" the eigenvector b_i^α corresponding to the eigenvalue λ_α, i.e., such that $u_\alpha(\omega,\mathbf{q}) = \Sigma_\alpha\, b_i^\alpha u_i(\omega,\mathbf{q})$, $Y_\alpha(\omega,\mathbf{q}) = b_i^\alpha Y_i(\omega,\mathbf{q})$; the label α characterizes the polarization of the sound waves, $\alpha = 1,2,3$.

Equation (10) is the starting point for constructing the kinetic equations for phonons. We note at once that many linear equations for waves in media can be reduced to equations of the form (10), in particular the Maxwell equations, and the derivation of the kinetic equations we will carry out below can therefore be used successfully to construct kinetic equations for photons, plasmons, excitons, magnons, and other excitations in the medium.

Let us apply the slowly-varying-amplitude approximation, or, what is the same thing, the wave-packet approximation, to Eq. (10). For this we assume that the wave amplitudes $u^\alpha(\omega,\mathbf{q})$ depend weakly on the coordinates and time; then the following inequalities hold:

$$\left| \frac{1}{\omega_{\min}} \frac{1}{u^\alpha} \frac{du^\alpha}{dt} \right| \ll 1, \quad \left| \frac{1}{q_{\min}} \frac{1}{u^\alpha} \frac{du^\alpha}{dr} \right| \ll 1, \quad (12)$$

where $\omega_{\min}$ and $q_{\min}$ are the minimum frequency and wave vector of the fluctuating amplitudes, respectively. Conditions (12) imply that in the interval $\omega < \omega_{\min}$, $q < q_{\min}$ the functions u^α and φ do not need to be expanded the Fourier integral (6). Introducing the slowly varying amplitudes in Eq. (10), we obtain the equation

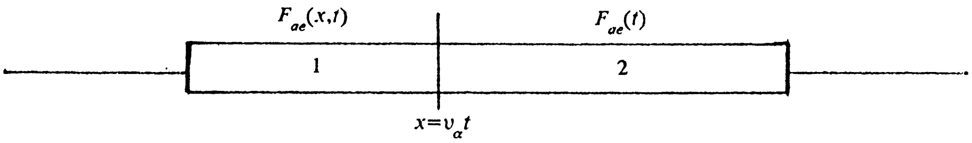

FIG. 1. Division of the sample into two regions: $1—x-v_\alpha t$, and $2—x>v_\alpha t$, in which regimes of convective and absolute instability are realized, under conditions of sonic instability; the acoustoelectric force F_{ae} depends on coordinates and time or only on time in accordance with these conditions.

$$\left[\lambda(\omega,\mathbf{q})-\frac{d\lambda'(\omega,\mathbf{q})}{d\omega}\frac{d}{dt}\pm i\frac{d\lambda'(\omega,\mathbf{q})}{d\mathbf{q}}\frac{d}{d\mathbf{r}}\right]u(\omega,\mathbf{q};\mathbf{r},t)$$
$$=Y(\omega,\mathbf{q}). \qquad (13)$$

The choice of sign in Eq. (13) is determined by the direction of propagation of the waves: the upper sign corresponds to waves propagating to the right, i.e., r increasing with t, while the lower sign corresponds to waves propagating to the left. So as not to encumber the notation, the polarization label α will be dropped from here on, and only one of the waves—that propagating to the right—will be discussed. It is clear on physical grounds that the space and time dependence in the amplitudes $u(\omega,\mathbf{q};\mathbf{r},t)$ appears because of the imaginary part of the eigenvalue $\lambda_\alpha(\omega,\mathbf{q})$, which determines the amplification and absorption of waves (in a nongyrotropic medium, of course); therefore we retain only the real part of λ, i.e., $\mathrm{Re}\,\lambda(\omega,\mathbf{q})\equiv\lambda'(\omega,\mathbf{q})$, in the derivatives $d\lambda^i/d\omega$ and $d\lambda^i/dq$.

Let us multiply Eq. (13) by $u^*(\omega,\mathbf{q};\mathbf{r},t)$, and the complex conjugate of (13) by $u(\omega,\mathbf{q};\mathbf{r},t)$, and subtract one equation from the other after statistically averaging the product of the fluctuating amplitudes over the "medium" scale. Then we obtain

$$\left[\frac{d}{dt}+\mathbf{v}_g\frac{d}{d\mathbf{r}}-2\gamma(\omega,\mathbf{q})\right]\langle u^*u\rangle=2\left(\frac{d\lambda'}{d\omega}\right)^{-1}I(\omega,\mathbf{q};\mathbf{r},t), \qquad (14)$$

where we have introduced the notation

$$\mathbf{v}_g\equiv\frac{d\lambda'}{d\mathbf{q}}\Big/\frac{d\lambda'}{d\omega},\quad \gamma(\omega,\mathbf{q})\equiv\mathrm{Im}\,\lambda(\omega,\mathbf{q})\Big/\frac{d\lambda'}{d\omega},$$
$$I\equiv\mathrm{Im}\langle Y^*u\rangle.$$

Note, however, that we can assign a physical meaning to the quantities $\mathbf{v}_g(\omega,\mathbf{q})$ and $\gamma(\omega,\mathbf{q})$ introduced above only after imposing the dispersion relation between the frequency ω and the wave vector $\mathbf{q}$, i.e., requiring that the dispersion equation $\lambda'(\omega,\mathbf{q})=0$ be satisfied. It is easy to show that $\mathbf{v}_g(\omega,\mathbf{q})$ is the group velocity of the waves, and $\gamma(\omega,\mathbf{q})$ is their growth rate.

The correlation function between the external forces $Y(\omega,\mathbf{q})$ and the amplitudes $u^*(\omega,\mathbf{q};\mathbf{r},t)$ enters into the right side of Eq. (14). In order to determine this correlation function we proceed in the following way: Let us multiply Eq. (13) by $Y^*(\omega,\mathbf{q})$ and carry out statistical averaging. Separating the real and imaginary parts of the equation obtained in this way, and noting that only the imaginary part of this correlation function enters into the kinetic equation (14), we obtain for the latter the equation

$$\left\{\left(\frac{d}{dt}+\mathbf{v}_g\frac{d}{d\mathbf{r}}\right)^2-2\gamma\left(\frac{d}{dt}+\mathbf{v}_g\frac{d}{d\mathbf{r}}\right)+\Omega^2+\gamma^2\right\}I=-\gamma\langle Y^*Y\rangle. \qquad (15)$$

Here $\Omega\equiv\Omega(\omega,\mathbf{q})=\lambda'(\omega,\mathbf{q})/[d\lambda'/d\omega]$, while the correlation function $\langle Y^*Y\rangle$ can easily be expressed in terms of the correlation functions (3) and (4). Thus, the system of kinetic equations (14) for the phonons together with the equations for the sources (15) in terms of known expressions for the correlation functions (3) and (4) form a closed system of equations that describe the kinetic properties of the nonequilibrium fluctuations. The difference between the equations derived here and those given in previous papers[1,9,12] is that the source of fluctuations in the kinetic equation (14) is itself determined by Eq. (15) found above; as we will show below, for nonequilibrium media this function also grows in time and space. In this case, the medium separates into two regions: in one region the instability is convective in character, and accordingly the source and fluctuations grow in space, while in the other region the instability is absolute in character, and the source and fluctuations grow in time.

Let us find an explicit expression for the source $I(\omega,\mathbf{q};\mathbf{r},t)$. To do this, we will construct a solution to Eq. (15). This solution has a simpler form when expressed in the variables $\xi=\mathbf{r}-\mathbf{v}_gt$ and t, which we introduce in place of the variables $\mathbf{r}$ and t:[1)]

$$\left(\frac{d^2}{dt^2}-2\gamma\frac{d}{dt}+\Omega^2+\gamma^2\right)I(\omega,\mathbf{q};\xi,t)=-\gamma\langle Y^*Y\rangle. \qquad (16)$$

The solution to Eq. (15) can be written in the form

$$I(\omega,\mathbf{q};\mathbf{r},t)=\frac{\gamma}{\gamma^2+\Omega^2}\langle Y^*Y\rangle+C_1(\mathbf{r}-\mathbf{v}_gt)e^{\gamma t}\cos\Omega t$$
$$+C_2(\mathbf{r}-\mathbf{v}_gt)e^{\gamma t}\sin\Omega t, \qquad (17)$$

where C_1 and C_2 are arbitrary functions that depend on $\xi=\mathbf{r}-\mathbf{v}_gt$. Note that the case of thermodynamic equilibrium corresponds to $\gamma<0$, i.e., the fluctuations decay with time, so that the last two terms in Eq. (17) disappear as $t\to\infty$.

For the case of convective instability, growing waves and fluctuations in the medium occupying the region of space $r>0$ are possible only in one direction; in the opposite direction neither waves nor fluctuations can grow. Therefore the region of time and space can be divided into two subregions: $\mathbf{r}-\mathbf{v}_gt<0$, where a stationary state is established, and $\mathbf{r}-\mathbf{v}_gt>0$, in which the fluctuations grow with time while remaining uniform in space (a natural consequence of the uniform initial conditions at $t=0$). Requiring that the solution to (17) describe stationary fluctuations in the region $\mathbf{r}-\mathbf{v}_g t<0$, and furthermore that the system be in thermodynamic equilibrium at time $t=0$ and at the boundary $\mathbf{r}=0$, we obtain from (17)

$$I(\omega,\mathbf{q};\mathbf{r},t)=\{\langle Y^*Y\rangle(e^{\gamma\tau}-1)\operatorname{sign}\gamma+\langle Y^*Y\rangle_{\mathrm{eq}}\}\pi\delta(\Omega)\times\theta(\xi)+\{\langle Y^*Y\rangle(e^{\gamma\tau}-1)\operatorname{sign}\gamma+\langle Y^*Y\rangle_{\mathrm{eq}}\}\pi\delta(\Omega)\theta(-\xi), \tag{18}$$

where $\theta(\xi)\equiv\theta(\xi_\alpha)$ is the theta function, $\tau\equiv\tau_\alpha\equiv\mathbf{r}\mathbf{v}_g^\alpha/v_g^2$, $\xi\equiv\xi_\alpha=\mathbf{r}-\mathbf{v}_g^\alpha t$, and $\langle Y^*Y\rangle_{\mathrm{eq}}$ are the thermodynamic equilibrium values of the correlation functions determined by the fluctuation-dissipation theorem. It is clear from Eq. (18) that the source $I(\omega,\mathbf{q};\mathbf{r},t)$ is always positive, independent of the sign of γ. In deriving (18) we have used the relation $\lim_{\gamma\to 0}[\gamma/(\gamma^2+\Omega^2)]=\pi\delta(\Omega)\operatorname{sign}\gamma$, which is a consequence of the conditions ω, $\mathbf{q}\mathbf{v}_\alpha\gg\gamma_\alpha(\omega,\mathbf{q})$. It is found that the physical requirement that the fluctuations be stationary in the region $\mathbf{r}-\mathbf{v}_g t<0$ can be satisfied provided that $\Omega\to 0$. Mathematically this implies that the coefficients C_1 and C_2 in (17) must contain a factor of $\delta(\Omega)$. This is understandable because the division of the space and time region into stationary and uniform regions is physically meaningful only when the frequency ω and fluctuation wave vector q satisfy the dispersion relation $\Omega_\alpha=0$, since the correlations arise only because of transport of the perturbation in the medium by acoustic waves.

From the expression for the source (18) it is clear that for $\gamma_\alpha<0$ (although the medium can still be out of equilibrium) the source asymptotically ceases to depend on space and time:

$$I_\alpha^{\mathrm{st}}(\omega,\mathbf{q};\mathbf{r},t)=\pi[\langle Y_\alpha^*Y_\alpha\rangle+\langle Y_\alpha^*Y_\alpha\rangle_{\mathrm{eq}}]\delta(\Omega). \tag{19}$$

It is this expression (without the last term in the square brackets) that was used previously in Refs. 1, 9, 12. [The appearance of the last term in square brackets in (19) is a consequence of the initial and boundary conditions at $\mathbf{r}=0$ and $t=0$.] Substituting Eq. (18) into the kinetic equation (14) and noting that we have $\delta(\Omega_\alpha)=\delta(\omega-\mathbf{q}\mathbf{v}_\alpha)$, i.e., we discuss only waves in one direction, we obtain the final form of the kinetic equation for fluctuations of the elastic displacement:

$$\left[\frac{d}{dt}+\mathbf{v}_g^\alpha(\mathbf{q})\frac{d}{dr}-2\gamma^\alpha(\mathbf{q})\right]\langle u_\alpha^*u_\alpha\rangle_{\mathbf{q}}=G_\alpha(\mathbf{q};\mathbf{r},t), \tag{20}$$

where $\langle u_\alpha^*u_\alpha\rangle_{\mathbf{q}}=\langle u_\alpha^*(qv_\alpha,\mathbf{q};\mathbf{r},t)$ and $u_\alpha(qv_\alpha,\mathbf{q};\mathbf{r},t)\rangle$; for the source,

$$G_\alpha(\mathbf{q};\mathbf{r},t)=\frac{\pi}{\rho q v_\alpha}\{\eta_\alpha(t)\theta(\xi_\alpha)+\eta_\alpha(\tau_\alpha)\theta(-\xi_\alpha)\},$$

$$\eta_\alpha(t)\equiv\langle Y_\alpha^*Y_\alpha\rangle\{\exp[\gamma_\alpha(\mathbf{q})t]-1\}\operatorname{sign}\gamma_\alpha+\langle Y_\alpha^*Y_\alpha\rangle_{\mathrm{eq}}. \tag{21}$$

We have already integrated over ω in Eq. (20), which reduces to replacing ω by $q\omega_\alpha$ due to the presence of $\delta(\omega-\mathbf{q}\mathbf{v}_\alpha)$. It is convenient to introduce the energy spectral density of the acoustic oscillations $\mathscr{E}_\alpha(\mathbf{q};\mathbf{r},t)=\rho\omega_\alpha^2(\mathbf{q})\langle u_\alpha^*u_\alpha\rangle_{\mathbf{q}}$ in place of $\langle u_\alpha^*u_\alpha\rangle_{\mathbf{q}}$, which also satisfies a kinetic equation like (20).

It is clear from Eq. (18) that the source consists of a sum of two functions, one of which depends only on t in the region $\xi>0$, while the other depends only on $\mathbf{r}$ for $\xi<0$. In accordance with this structure of the source, we will seek the solution to Eq. (20) in the form

$$\mathscr{E}_\alpha(\mathbf{q};\mathbf{r},t)=\mathscr{E}_\alpha(\mathbf{q};t)\theta(\xi)+\mathscr{E}_\alpha(\mathbf{q};r)\theta(-\xi),$$

then Eq. (20) splits in two:

$$\frac{d\mathscr{E}_\alpha(\mathbf{q};t)}{dt}-2\gamma_\alpha(\mathbf{q})\mathscr{E}_\alpha(\mathbf{q};t)=Q_\alpha(t),$$

$$\mathbf{v}_g\frac{d\mathscr{E}_\alpha(\mathbf{q};r)}{d\mathbf{r}}-2\gamma_\alpha(\mathbf{q})\mathscr{E}_\alpha(\mathbf{q};r)=Q_\alpha(\mathbf{r}), \tag{22}$$

where $Q_\alpha(t)$ and $Q_\alpha(\mathbf{r})$ are the factors in square brackets in Eq. (18). Note that continuity of the source (18) at the boundary $\xi=0$ also implies that the energy spectral densities of the acoustic phonons $\mathscr{E}_\alpha(\mathbf{q},t)$ and $\mathscr{E}_\alpha(\mathbf{q},\mathbf{r})$ are continuous at the boundary $\xi=0$. The solution of Eqs. (20), (22) can be written

$$\mathscr{E}_\alpha(\mathbf{q};\mathbf{r},t)=\pi\left\{\langle Y_\alpha^*Y_\alpha\rangle_{\mathrm{eq}}\frac{\exp(2\gamma_\alpha t)-1}{2\gamma_\alpha}+\langle Y_\alpha^*Y_\alpha\rangle\operatorname{sign}\gamma_\alpha\frac{[\exp(\gamma_\alpha t)-1]^2}{2\gamma_\alpha}+\mathscr{E}_1(\mathbf{q})\times\exp(2\gamma_\alpha t)\right\}\theta(\xi_\alpha)$$
$$+\pi\left\{\langle Y_\alpha^*Y_\alpha\rangle_{\mathrm{eq}}\frac{\exp(2\gamma_\alpha\tau)-1}{2\gamma_\alpha}+\langle Y_\alpha^*Y_\alpha\rangle\operatorname{sign}\gamma_\alpha\frac{[\exp(\gamma_\alpha\tau)-1]^2}{2\gamma_\alpha}+\mathscr{E}_2(\mathbf{q})\exp(2\gamma_\alpha\tau)\right\}\theta(-\xi_\alpha), \tag{23}$$

where $\mathscr{E}_1$ and $\mathscr{E}_2$ are the values of the phonon spectral density for $t=0$ and $\mathbf{r}\mathbf{v}_g=0$, respectively. For $\gamma_\alpha<0$, which corresponds to a state in which the fluctuations do not grow, Eq. (23) predicts the following value of the stationary phonon density for $t>1/|\gamma_\alpha|$:

$$\mathscr{E}_\alpha^{\mathrm{st}}(\mathbf{q})=\frac{1}{2}|\gamma_\alpha(\mathbf{q})|\{\langle Y_\alpha^*Y_\alpha\rangle_{\mathrm{te}}+\langle Y_\alpha^*Y_\alpha\rangle\}. \tag{24}$$

This distribution determines a nonequilibrium but stationary and uniform distribution function for the phonons. If, however, we have $\gamma_\alpha t\approx 1$ and therefore $\gamma_\alpha\tau\approx 1$, then the nonequilibrium phonon density is

$$\mathscr{E}_\alpha(\mathbf{q};\mathbf{r},t)=\mathscr{E}_\alpha^{\mathrm{st}}(\mathbf{q})[\exp(2\gamma_\alpha t)\theta(\xi_\alpha)+\exp(2\gamma_\alpha\tau)\theta(-\xi_\alpha)], \tag{25}$$

where we set $\mathscr{E}_1=\mathscr{E}_2=0$ here for the sake of simplicity.

The method developed here for obtaining kinetic equations for the nonequilibrium fluctuations can be applied successfully to obtain equations for the mixed correlations of the form $\langle u_i E_j \rangle$ or $\langle u_i u_k \rangle$, which determine the scattering properties of the medium among other things.

2. THEORY OF ACOUSTOELECTRIC DOMAINS

The kinetic equations we have obtained for nonequilibrium fluctuations in piezoelectric semiconductors allow us to formulate a theory of the macroscopic electric properties of the crystal in strong fields in general form, i.e., to describe the underlying physics of the appearance of acoustoelectric domains, and to give a semiquantitative picture of the phenomenon under very general assumptions.

Numerous experimental investigations have shown that under conditions of phonon generation in a piezoelectric semiconductor crystal a region of strong electric field is formed that appears very abruptly in space, in which the nonequilibrium phonon density is very high; this domain propagates through the crystal with a velocity close to the velocity of sound (see Ref. 8). Attempts to construct a theory of this phenomenon have not yet led to useful results, and the physical models proposed are unable to explain the full aggregate of observed behavior of this phenomenon.

The formation of domains is explained most often by starting from the assumption that application of an electric field pulse to the piezoelectric semiconductor crystal results in creation of a packet of phonons by an electric shock wave in the near-contact (cathode) region of the crystal. As this packet then propagates throughout the sample, it is amplified and "takes up" into itself a considerable portion of the electric field. It is claimed that this packet of amplified phonons, by analogy with the amplification of a monochromatic sound wave, traps electrons in a potential well generated by the piezoelectric potential. Under this assumption, practically all of the electrons will be trapped in the potential well for a sufficiently large wave amplitude; consequently, the current in the circuit will equal $eN_0 v_\alpha$ (where v_α is the sound wave velocity and N_0 is the equilibrium electron density). This explanation, which could possibly be correct for a monochromatic sound wave, cannot be made the basis of a model that involves wave packets with random phases. A wave packet with random phases cannot give rise to a potential well that traps electrons. This is easy to show for the example of two waves with arbitrary phases and frequencies that are either close together or nearly multiples of one another. If it is assumed that phase correlations arise due to nonlinear effects, then obviously a certain amount of time (or a corresponding spatial scale) is necessary for this process to develop; however, it is clear that this temporal scale should be considerably larger than the inverse of the value of the linear growth rate. On the other hand, the experimental data indicate that the domain forms quite rapidly in time, appearing at distances of a few hundred microns from the cathode boundary of the crystal.

The reason for the lack of success in constructing a theory of acoustoelectric domains is that the following very important feature of the development of the instability has escaped the notice of previous investigators: two unstable regions form in the sample, whose instabilities have different natures. On the left, i.e., in the near-cathode region, the instability is convective in character, whereas in the region on the right, which occupies a considerably larger portion of the crystal in the initial stage of development of the instability, the instability is absolute in character (see the system of equations obtained above for nonequilibrium phonons). It is significant that all the quantities that characterize the state of the system in the region with the convective instability depend only on the x coordinate and do not depend explicitly on time, while in the region with the absolute instability these quantities depend only on time and not on the spatial coordinates. The boundary between these regions translates through the crystal with the velocity of sound. The different forms of this dependence allow us to construct a simple system of equations to describe the macroscopic properties of the medium: in the convective region all quantities depend only on position, while in the absolute region they depend only on time (see below).

Another reason for the lack of success in constructing a theory of acoustoelectric domains is that the reverse influence of the growing acoustic-wave fluctuations on the distribution of electric field in the sample and the value of the current in the electric circuit that contains the crystal have not been successfully analyzed even qualitatively due to mathematical complexity; as a consequence of this, previous researchers have not understood the physical picture of the phenomenon.

The analysis given below leads to the following physical picture of the appearance of an acoustoelectric domain. When an electric field whose value exceeds the threshold value is switched on, an acoustoelectric force appears that acts on the electrons and is exerted by the nonequilibrium phonons; in the region with the convective instability this force depends only on position, and in the region with absolute instability only on time. The direction in which this force acts is always opposite the action of the external electric field, i.e., this force brakes the motion of electrons. As a result, the current in the circuit will decrease, and the electric field will be redistributed between the regions with convective and absolute instabilities. A simple analysis of the equations for the electric field in the region with absolute instability, where all quantities depend only on time, shows that the field decreases with time and falls to its threshold value, and that the current saturates as this occurs. It is only under these conditions that the electric field will remain uniform in the region with absolute instability. In the left-hand portion of the crystal, where the instability has a convective character, the phonon density and the field are nonuniform and increase with increasing spatial coordinate x. The acoustoelectric force in this region also increases in space, but since it depends on the electric field as well, it is clear that the electric field should be redistributed in self-consistently between the regions with convective and absolute instabilities. Mathematically this implies that the equations for the phonon density and distribution of electric field must be solved together. However, it is obvious that the phonon density will be largest at the point $x=v_\alpha t$, i.e., at the boundary farthest to the right of the region with convective instability, because the phonons arriving at this point have taken the longest route from the $x=0$ boundary, and thus have spent the entire time at the maximum of the electric field. Therefore, the electric field in the neighborhood of the point $x=v_\alpha t$ will increase, "acquiring" electric field from other regions of the crystal. Eventually, a steady state is arrived at for which the field is equal to its threshold value everywhere except in the region of the domain itself, and the current equals eN_0v_α. This qualitative picture of the phenomenon, which follows from the analysis we will give below, is in good agreement with numerous experimental results.

We now turn to a more detailed investigation, starting with the derivation of a system of equations for formulating a theory of acoustoelectric domains.

Let us identify a unit volume of the electron gas and describe all the forces that act on this volume. It is obvious that the condition for equilibrium of a unit volume of the electron gas is[2)]

$$en\mathbf{E}-T\nabla n-m\nu n\mathbf{v}=0, \qquad (26)$$

where $n=n(\mathbf{r},t)$ is the electron density, $\mathbf{E}(\mathbf{r},t)$ is the electric field, T is the electron temperature, m is the mass, e is the charge, ν is the effective collision frequency of the electrons, and $\mathbf{v}=\mathbf{v}(\mathbf{r},t)$ is the hydrodynamic velocity. In the equations of hydrodynamics (26) we have omitted terms with time derivatives, because $|d\mathbf{v}/dt|\ll\nu\mathbf{v}$. For the conditions under which the fluctuations develop we can write

$$\begin{aligned} n(\mathbf{r},t)&=n_0(\mathbf{r},t)+n_\sim(\mathbf{r},t),\\ \mathbf{E}(\mathbf{r},t)&=\mathbf{E}_d(\mathbf{r},t)+\mathbf{E}_\sim(\mathbf{r},t), \end{aligned} \qquad (27)$$

where $n_0(\mathbf{r},t)=\langle n(\mathbf{r},t)\rangle$, $\mathbf{E}_d(\mathbf{r},t)=\langle\mathbf{E}(\mathbf{r},t)\rangle$, and $n_\sim(\mathbf{r},t)$, $\mathbf{E}_\sim(\mathbf{r},t)$ are corrections that are proportional to the amplitude of the fluctuations in the acoustic waves; these quantities are not necessarily small compared with their average values. Substituting (27) into Eq. (26) and averaging with respect to fluctuations we obtain from (26)

$$en\mathbf{E}_d+e\langle n_\sim(\mathbf{r},t)\mathbf{E}_\sim(\mathbf{r},t)\rangle-T\frac{\partial n_0}{\partial\mathbf{r}}-\frac{1}{\mu}\mathbf{J}(t)=0. \qquad (28)$$

Here $\mu=e/m\nu$ is the electron mobility and $\mathbf{J}(t)=e\langle n(\mathbf{r},t)\mathbf{v}(\mathbf{r},t)\rangle$ is the current density flowing through the sample, which can depend only on time t but not on the position $\mathbf{r}$ by virtue of the condition of electrical neutrality ($\operatorname{div}\mathbf{J}=0$).

It can be shown[14] that the correlation $\langle n_\sim\mathbf{E}_\sim\rangle$ is expressible in terms of the spectral density $\mathscr{E}^\alpha(\mathbf{q};\mathbf{r},t)$ of the photons being generated in the form

$$\begin{aligned} \mathbf{F}_{\rm ae}(\mathbf{r},t)&\equiv e\langle n_\sim\mathbf{E}_\sim\rangle\\ &=2(2\pi)^{-3}\sum_\alpha\int\omega^{-1}\mathbf{q}\gamma_e^\alpha(\mathbf{q})\mathscr{E}^\alpha(\mathbf{q};\mathbf{r},t)d\mathbf{q}. \end{aligned} \qquad (29)$$

The quantity $F_{\rm ae}(x,t)$ is the force acting on a unit volume of the electron gas exerted by the increasing phonon flux (i.e., the acoustoelectric force). In (29), $\gamma_e^\alpha(\mathbf{q})$ is the electronic part of the growth rate of the phonons $\gamma^\alpha(\mathbf{q})$ [see (14)], which can be expressed in terms of the longitudinal dielectric permittivity of the medium $\varepsilon_\parallel(\omega,\mathbf{q})$ in the form[14]

$$\begin{aligned} \gamma_e^\alpha(\mathbf{q})&=\frac{1}{2}\varkappa_\alpha^2\omega\,\mathrm{Im}\left.\frac{\varepsilon_0}{\varepsilon_\parallel(\omega,\mathbf{q})}\right|_{\omega=qv_\alpha}\\ &=-\frac{\varkappa_d^2}{2}\frac{1}{\tau_M}\frac{1-\beta}{(1-\beta)^2+\omega^{-2}\tau_m^{-2}(1+q^2r_0^2)^2}, \end{aligned} \qquad (30)$$

where $\varkappa_\alpha^2=(4\pi\beta_\alpha^2/\rho v_\alpha^2\varepsilon_0)$ is the square of the electromechanical coupling constant, $\beta\equiv\mathbf{q}\mathbf{E}_d/qE_{\rm cr}$, $E_{\rm cr}=v_\alpha/\mu$ is the critical (threshold) value of the electric field, $\tau_M=\varepsilon_0/4\pi\sigma_0$ is the Maxwell relaxation time, and $\sigma_0=e^2N_0/m\nu$ is the conductivity of the electron gas. Equation (30) is for the case of low frequencies, i.e., $ql\ll1$, where l is the electron mean free path. We also can obtain an expression for the growth rate for phonon generation when $ql>1$ holds (see Ref. 14), i.e., in the high-frequency region, such that all the previous conclusions obtain for the case of high-frequency phonon generation as well.

For the latter analysis it is important that the spectral density of the phonons $\mathscr{E}^\alpha(\mathbf{q};\mathbf{r},t)$ for the region with convective instability depends only on the coordinate $\mathbf{r}$, while for the region with absolute instability it depends only on the time t [see Eqs. (23) and (25) obtained above]. In accordance with this, substituting the quantities $\mathscr{E}^\alpha(\mathbf{q};\mathbf{r},t)$ into (29) leads to an expression for the acoustoelectric force $\mathbf{F}_{\rm ae}$ that also will have different dependences on $\mathbf{r}$ and t for the regions with convective and absolute instabilities (see Fig. 1.) The latter implies that the equation for the electric field (28) breaks up into two equations for the regions with convective and absolute instabilities, respectively:

$$\left(1+\frac{\varepsilon_0}{4\pi eN_0}\frac{d^2E_d(x)}{dx^2}\right)E_d(x)-r_0^2\frac{d^2E_d(x)}{dx^2}+F_{\rm conv}(x)-\frac{1}{\sigma_0}J(t)=0, \qquad (31)$$

for $x<v_\alpha t$ and

$$E_d(t)+F_{\rm abs}(t)-\frac{1}{\sigma_0}J(t)=0 \qquad (32)$$

for $x>v_\alpha t$. Here

$$F_{\rm conv}(x)\equiv\frac{1}{eN_0}F_{\rm ae}(x);\quad F_{\rm abs}(t)\equiv\frac{1}{eN_0}F_{\rm ae}(t), \qquad (33)$$

where $F_{\rm ae}(x)$ and $F_{\rm ae}(t)$ are the values of the acoustoelectric force in the regions with convective and absolute instabilities, respectively. In deriving (32) we also made use of the Poisson equation in the form $\varepsilon_0\operatorname{div}\mathbf{E}_d=4\pi e(n_0(x)-N_0)$.

It is necessary to add the obvious boundary condition to Eqs. (31) and (32):

$$\int_0^{v_\alpha t} E_d(x)dx+(L-v_\alpha t)E_d(t)=V-J(t)(R_l+R_i), \quad (34)$$

where V is the potential difference of a source with internal resistance R_i, and R_l is the resistance of the load. In what follows we will assume for the sake of simplicity that $R_i=0$, $R_l=0$. The electric field in the region with convective instability depends explicitly on the coordinate x and depends implicitly on the time t as well due to the time dependence of the current $J(t)$.

Equations (31), (32) and the boundary conditions (34) combined with the kinetic equations for the phonons (22) are the starting system of equations for constructing a theory of acoustoelectric domains. The equations for the field (31) and (32) are exact; however, in deriving Eq. (29) for the acoustoelectric force we made use of the linear relations between the fluctuations $n_\sim(\mathbf{r},t)$ in the electron density, the electric field $\mathbf{E}_\sim(\mathbf{r},t)$, and the amplitude of the sound waves. This implies that we will not consider nonlinear effects of the decaying-instability type or harmonic generation.

For further analysis it is necessary to refine Eq. (29) for the acoustoelectric force. For this we substitute into (29) the expression (25) for the phonon spectral density in the region with absolute instability ($\xi_\alpha>0$, $\gamma t>0$), and carry out the integration over the wave vector $\mathbf{q}$. It is not possible to carry out this integration explicitly; therefore, we use approximate calculations. First of all, we note that the stationary phonon density also depends on the electric field $E_d(t)$, as is clear from (24). It is easy to show, although this is clear physically, that this dependence is very weak; therefore, we may assume $\mathscr{E}^{\rm st}(\mathbf{q})\approx\mathscr{E}_{\rm te}(\mathbf{q})$, i.e., that it equals its value in thermodynamic equilibrium. For an electron temperature equal to the temperature of the lattice, we have $\mathscr{E}_{\rm te}(q)=T$. We will also assume that viscosity absorption is small compared to the electronic absorption, i.e., $\gamma(\mathbf{q})\approx\gamma_e(\mathbf{q})$. The orientation of the crystal is chosen so that the radiation diagram of the phonons is axisymmetric in structure; the intensity maximum of the radiation is located near the direction of the drift vector, i.e., it is directed along the x axis. The axial symmetry of the radiation diagram for the phonons implies mathematically that Eq. (29) contains only the dependence on the direction x, and the dependence on the transverse coordinates y and z disappears [these are the conditions under which Eq. (31) was obtained; it was shown in Refs. 1, 3 that if the dependence on y, z is retained, vortex currents arise that lead to a magnetic moment in the sample[2]]. As a result, we obtain from (29)

$$F_{\rm abs}(t)\simeq\frac{T\zeta\pi^{-2}}{2eN_0v_s}\int q^2dq\gamma_e(q,E_d(t))\times\exp\left[2\int_0^t\gamma_e(q,E_d(t))dt\right], \quad (35)$$

where $\zeta\equiv\cos(\Delta\theta)$, where $\Delta\theta$ is the aperture angle of the Cherenkov generation cone. Strictly speaking, the time integration in the exponent of Eq. (35) does not start from zero, but rather from a certain very small but nonzero initial time, in accordance with the solution we have used for the kinetic equation for the phonons (25). This difference is insignificant, because at $t=0$ the acoustoelectric field is much smaller than the critical field $E_{\rm cr}$. The integration over the magnitude $q=\omega/v_\alpha$ of the wave vector in (35) can be carried out by using the saddle-point method, according to the well-known expression:[19]

$$\int\varphi(x)e^{pf(x)}dx\simeq\left(\frac{2\pi}{p|f''(x)|}\right)^{1/2}e^{pf(x_0)}\varphi(x_0),$$

$$\text{when } pf(x_0)>1, \quad (36)$$

where x_0 is the point at which the function $f(x)$ has a maximum. Integrating with respect to q in (35) and using Eq. (36), we obtain

$$F_{\rm abs}(t)=-F_0(\beta(t)-1)\exp\left[2\int_0^t\gamma(q_0,E(t'))dt'\right]\times\left[\int_0^t(\beta(t')-1)dt'/\tau_M\right]^{-1/2}. \quad (37)$$

Here $F_0=(2\pi)^{-1/2}e\varkappa\zeta/\{\varepsilon_0r_0\tau_Mv_\alpha\}$ is a quantity with the dimensions of electric field, and $q_0=1/r_0$, where r_0 is the Debye radius for electrons, i.e., the maximum contribution to the electric field is carried by phonons whose wave vector equals the inverse value of the Debye radius for the electrons. In deriving (37) we made use of the condition $|(\beta(t)-1|\ll4\tau_Mv_\alpha/r_0$, which is always met for large t, as we will see below. It is important to note that the acoustoelectric field (37) is proportional not to the square of the electromechanical coupling constant as occurs for the acoustoelectric effect from a discrete number of monochromatic waves (see Ref. 14), but rather is proportional simply to $\varkappa$. This is because the field (37) arises from the collective action of a very large number of waves. Mathematically this is due to the wave vector integration in (35) over a broad interval of values of q. We note also that

$$\gamma_{e0}(q_0,E(t))\left[\tau_M^{-1}\int_0^t dt\frac{d^2}{dq^2}\gamma_e(q,E(t))\Big|_{q=q_0}\right]^{-1/2}=\frac{\varkappa v_\alpha(\beta(t)-1)}{4r_0\left[\tau_m^{-1}\int_0^t dt'(\beta(t')-1)\right]^{1/2}}, \quad (38)$$

where $\gamma_{e0}=\gamma_e(q=q_0)$. This relation, which follows from Eq. (30), was used to derive Eq. (37).

Let us estimate the value of the quantity $F_{\rm abs}(t)$ for crystals such as cadmium sulfide, which are most often used in experiments involving the observation of acoustoelectric domains. For the slow transverse acoustic wave in the CdS crystals we have $v_\alpha=1.8\cdot10^5$ cm/s, $\varkappa=0.14$, $N_0=10^{14}$ cm^{-3}, $\nu=10^{13}$ s^{-1}, $\varepsilon_0=5$, $E_0=1125$ V/cm, $\tau_M=2\cdot10^{-10}$ s, and $r_0=2.7\cdot10^{-5}$ cm; then from Eq. (37) we obtain the estimate

$$F_{\rm abs}\left[\frac{\rm V}{\rm cm}\right]=28.5\frac{(\beta(t)-1)\exp[2\gamma_0(t_0)t/\tau_M]}{[(t/\tau_M)[\beta(t_0)-1]]^{1/2}}, \quad (39)$$

where t_0 is a certain time within the interval $[0,t]$ such that $0\ll t_0\ll t$, and

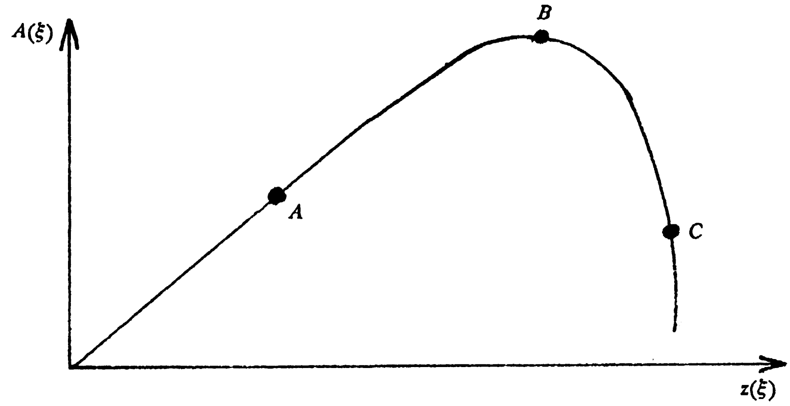

FIG. 2. The function $A(\xi)=\int_0^\xi [J(\xi)/\sigma_0 E_{cr}-1]d\xi$ versus $Z(\xi)$, in accordance with Eq. (42). The linear portion corresponds to Ohm's law, the region near the maximum to current saturation $J(t)=eN_0 v_s$.

$$\gamma_0(t_0)=0.02\frac{\beta(t_0)-I}{[\beta(t_0)-I]^2+2.25}.$$

It is clear from Eq. (39) that, as a function of time, the acoustoelectric field is initially slowly varying for small t, but grows exponentially rapidly with time for values of t such that $t\gg\tau_m/2\gamma_0$. From the estimates used to obtain (39), we find that $F_{abs}(t)$ has already reached its threshold value E_{cr} for $t=700\tau_m=1.4\cdot 10^{-7}$ s; after this time, the phonons from the boundary $x=0$ traverse a path of length $2.5\cdot 10^{-2}$ cm in all. Experiments show[8] that these are the orders of magnitudes of the time and spatial scales over which the formation of domains takes place.

Substituting the value of $F_{abs}(t)$ from (37) into Eq. (32), we finally obtain an equation for the electric field in the region with absolute instability:

$$E_d(t)-F_0\frac{\beta(t)-1}{[\tau_M^{-1}\int_0^t dt(\beta(t)-1)]^{-1/2}}\times\exp\left[2\int_0^t d t\gamma(q_0,E_d(t))\right]-\frac{J(t)}{\sigma_0}=0. \quad (40)$$

In Eq. (40) we have introduced dimensionless variables according to the definitions

$$\xi\equiv t/\tau_M,\quad \mathscr{G}\equiv F_0/E_{cr},\quad Z(\xi)\equiv\int_0^\xi(\beta(\xi)-1)d\xi.$$

Then in place of (40) we obtain the equation

$$\frac{dz(\xi)}{d\xi}[1-\mathscr{G}Z^{-1/2}(\xi)\exp(2\gamma_0(q_0)z(\xi))]=\frac{J(\xi)}{\sigma_0 E_{cr}}-1, \quad (41)$$

which does not contain the variable ξ explicitly; hence a solution can be obtained at once:

$$z(\xi)-\mathscr{G}\gamma_0^{-1/2}(q_0)\Phi(\sqrt{2\gamma_0(q_0)Z(\xi)})=\int_0^\xi(J(\xi)/\sigma_0 E_{cr}-1)d\xi, \quad (42)$$

Here $\Phi(x)=\int_0^x\exp(x^2)dx$ is the probability function of imaginary argument and $\gamma_0(q_0)=\varkappa_\alpha^2[8r_0^2]^{-1}v_\alpha^2\tau_M^2$ is the value of the growth rate for generation of phonons when $(\beta(\xi)-1)\ll v_\alpha\tau_M/(4r_0)$, divided by the factor $(1-\beta(\xi))/\tau_M$. We note that the dimensionless coefficient $\mathscr{W}\equiv\mathscr{G}\gamma^{-1/2}(q_0)$ in Eq. (42) no longer depends on the electromechanical coupling constant $\varkappa$ and has the very simple form

$$\mathscr{W}=\frac{8(2\pi)^{-1/2}\zeta e}{\varepsilon_0\tau_M^2 v_\alpha^2 E_{cr}}$$

for the estimates given above for CdS crystals we have $\mathscr{W}=0.08\zeta$. However, this quantity increases very rapidly with the electron density, since $\mathscr{W}\propto N_0^2$.

The values of the electric field at different times are determined by the derivative $dz/d\xi$, and therefore must be obtained from Eq. (41) by solving Eq. (42) to find the function $z(\xi)$. We emphasize once more that near $t\cong 0$ or $\xi\cong 0$ the saddle-point method is not really suitable, and therefore the term proportional to $\mathscr{G}$ must be dropped in accordance with the physical formulation of the problem. For small t, i.e., $t\approx 0$, it is no longer possible for the phonon generation to reach appreciable values; therefore, the phonon contribution to (32) under equilibrium conditions will be small. For the problem under discussion here this is not important, because we are interested in the behavior of the system for large t, when the phonon generation gives rise to a considerable contribution. It is easy to see directly from Eq. (41) that for small t the usual Ohm's law holds.

It is clear from the solution (42) that the maximum positive value of the current cannot exceed a certain value determined by Eq. (42), i.e.,

$$\int_0^\xi(J(\xi)/\sigma_0 E_{cr}-1)d\xi\leqslant I_0, \quad (43)$$

where I_0 is the maximum of the function of the variable z determined by the left side of Eq. (42) (see Fig. 2). It is important that Eq. (43) must be satisfied for all values of ξ, including large ones; however, this latter case is possible only in a situation where

$$J(t)=\sigma_0 E_{cr}\approx eN_0 v_\alpha,\quad \text{as } t\to\infty. \quad (44)$$

Thus, the current saturates. The value of the constant I_0 affects the character of the time dependence of the current, but not its asymptotic value (44) as $t\to\infty$. The fact that the current saturates can be proved another way, by approximating the curve in Fig. 2 as it approaches the maximum by the tangent at the corresponding point. It is obvious that in the vicinity of the point b (see Fig. 2) we can write $z(\xi)\cong A+B\alpha(\xi)$, where A and B are constants that determine the tangent at the point b. Differentiating this relation with respect to ξ, we obtain

$$J(\xi)=\sigma_0 E_{cr}[1+B^{-1}(E(\xi)/E_{cr}-1)]$$

as the point b approaches the maximum, at which B goes to infinity, the result (44) follows from the expressions derived above. Using the asymptotic value (44), we obtain from Eq. (41) the asymptotic value of the electric field as well, which corresponds to the value of the current (44):

$$E(\xi)=E_{cr}, \quad \text{as } \xi\to\infty. \tag{45}$$

The asymptotic values we have obtained for the current and electric field under conditions of intense phonon generation are in good agreement with the experimental results (see Ref. 8). We note that the condition $\xi\to\infty$, i.e., $t/\tau_M\to\infty$, is meaningful in the interval $t\ll L/v_\alpha$, where L is the size of the sample along the direction x.

Let us discuss the dependence shown in Fig. 2 in more detail. From the plot it is clear that we can identify three characteristic regions: a linear segment (near the point A), a region near the maximum (the point B), and a rapidly decaying portion. On the linear portion the following relations hold:

$$\int_0^\xi (J(\xi)/\sigma_0 E_{cr}-1)d\xi=\int_0^\xi (E(\xi)/E_{cr}-1)d\xi,$$

from which Ohm's law follows immediately. At the point B near the maximum Eq. (43) holds and the current undergoes saturation. In the decaying portion (let us say, at point C) the following relation holds:

$$\int_0^\xi (J(\xi)/\sigma_0 E_{cr}-1)d\xi=P-K\int_0^\xi (E(\xi)/E_{cr}-1)d\xi, \tag{46}$$

where P and K are constants which characterize the tangent at the point C. Differentiating (46) with respect to ξ, we obtain a relation between the current and the field in this region, from which it follows that the differential conductivity becomes negative, i.e., $dJ/dE=-K\sigma_0$. The latter implies that the system must be unstable in this region. However, this conclusion must be asserted with a certain caution, because the saddle-point method we have used may turn out to be inapplicable in this region. Actually, from Eq. (32) and the expression for the acoustoelectric force (29), which can be rewritten in the form

$$F_{abs}(t)=(1-\zeta E_d/E_{cr})F_0(E_d,t)$$

it follows immediately that the electric field E_d satisfies the relation

$$E_d(t)=\frac{\sigma_0^{-1}J(t)-F_0(E_d,t)}{1-\zeta F_0(E_d,t)/E_{cr}}$$

from which it immediately follows that $E_d\to E_{cr}/\zeta$ as $F_0\to\infty$. Thus, the exponential growth with time of the phonon density and the associated increase in the acoustoelectric field F_0 lead us to the conclusion that the electric field drops to its threshold value (we recall that $\zeta=\cos(\Delta\theta)$, where $\Delta\theta$ is the aperture of the Cherenkov cone for phonon generation, and $\zeta\to 1$ as the field E_d falls to its threshold value).

Let us now discuss the region with an instability of convective character, in which the electric field is nonuniform and the growth rate γ for generation depends implicitly on the coordinate x. The solution to the kinetic equation (25) for this case gives the following expression for the phonon spectral density:

$$\mathscr{E}(\mathbf{q};x)=\mathscr{E}_{eq}(\mathbf{q})\exp\left[2\int_0^x \gamma_e(\mathbf{q};E_d(x))v_\alpha^{-1}dx\right]. \tag{47}$$

Substituting this value of the phonon spectral density into Eq. (29) and carrying out the integration over all wave vectors, as in the derivation of Eq. (37), we obtain

$$F_{conv}(x)=F_0(\beta(x)-1)\left[v_\alpha^{-1}\tau_M^{-1}\int_0^x (\beta(x)-1)dx\right]^{-1/2}$$
$$\times\exp\left[2\int_0^x \gamma_e(q_0,E_d(x))v_\alpha^{-1}dx\right]. \tag{48}$$

Let us introduce the notation

$$Y(\chi)\equiv\int_0^\chi (\beta(\chi)-1)d\chi, \quad \chi\equiv x/(v_\alpha\tau_M)$$

and substitute (47) into the equation for the electric field (32). Assuming the electron concentration is such that the following condition holds

$$\left|\frac{\varepsilon_0}{4\pi eN_0}\frac{dE_d(x)}{dx}\right|\ll 1, \quad \left|r_0^2\frac{d^2E_d(x)}{dx^2}\right|\ll E_d(x),$$

we obtain the following equations for determining the dimensionless electric potential $Y(\chi,t)$:

$$\frac{dY(\chi,t)}{d\chi}\{1-\mathscr{G}Y^{-1/2}(\chi,t)$$
$$\times\exp[2\gamma_0(q_0)Y(\chi,t)]\}=\frac{J(t)}{\sigma_0E_{cr}}-I. \tag{49}$$

In contrast to Eq. (41) for the dimensionless electric potential in the region with absolute instability, the potential Y in Eq. (49) depends both on the coordinate x and on the time t. Equation (49) does not explicitly contain the variable χ, and, therefore, we can immediately obtain its solution:

$$Y(\chi,t)-\mathscr{W}\Phi(\sqrt{2\gamma_0(q_0)Y(\chi,t)})=\left[\frac{J(t)}{\sigma_0E_{cr}}-I\right]\chi. \tag{50}$$

It is interesting to note that the coefficient in front of the function Φ in the solution (50) no longer depends on the electromechanical coupling constant. We can find the solution for the potential $Y(\chi,t)$ from Eq. (50) numerically or graphically after first constructing the function $\chi(Y)$ and then the function $Y(\chi,t)$. The results of this analysis are shown in Fig. 3, where we show the dependence of $Y(\chi)$ on

$$\alpha(\xi)\chi\equiv\chi\int_0^t\left(\frac{J(\xi)}{\sigma_0E_{cr}}-1\right)d\xi,$$

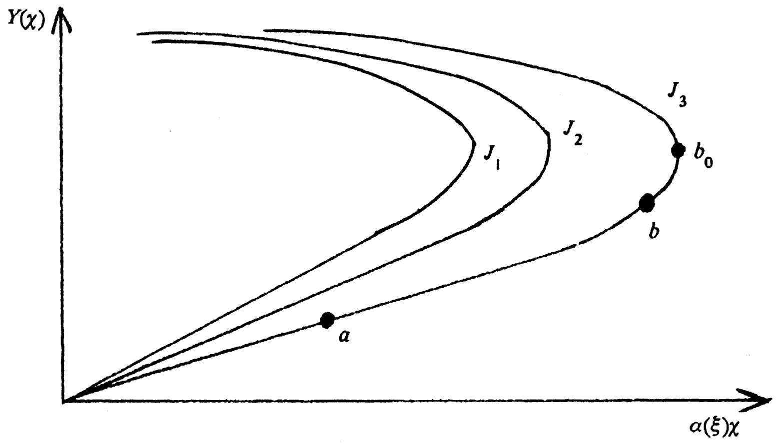

FIG. 3. Dependence of $Y(\chi)$ on $\alpha(\xi)\chi = \chi\int_0^t(J(\xi)/\sigma_0E_{cr}-1)d\xi$, which follows from Eq. (50) for different values of the current at various instants of time J_1,J_2,J_3, respectively. The linear portion corresponds to Ohm's law $J(t)=\sigma_0E(t)$, the region near the point b corresponds to the domain region; at the point b_0 the derivative $dy(\chi)/d\chi$ goes to infinity, as does the electric field. It is physically clear that this is impossible by virtue of the boundary conditions.

which follows from Eq. (50); here J_1,J_2,J_3 are values of the current at the times t_1,t_2,t_3, respectively. The linear portion (point A) corresponds to Ohm's law $J(t)=\sigma_0E(t)$, the portion near the point B corresponds to the domain region; at the point b_0 the derivative $dY(\chi)/d\chi$ goes to infinity, and accordingly the electric field also becomes infinite. It is physically clear, however, that this is impossible by virtue of the boundary conditions. From the solution (50) it is clear that in the region of small x the potential increases linearly with x. This implies that the electric field near the cathode is uniform and does not depend on x; however, as x increases the electric field grows very rapidly and at the point b_0, which can be determined from Eq. (49) it diverges. Obviously, this also is the region of the acoustoelectric domain. It is also clear that the largest value of the electric field is reached at the right boundary point of the region with convective instability, i.e., at the point $x=v_\alpha t$. This latter also implies that we can use the boundary condition (34), substituting into it the values of the electric field from formula (45). Then we obtain

$$\int_0^{v_\alpha t}\left(\frac{E_d(x)}{E_{cr}}-1\right)dx=\frac{V}{E_{cr}}-L \quad \text{for } t>t_d, \tag{51}$$

where t_d is the characteristic time in which a domain forms. From Eq. (51) it follows immediately that $Y(\chi)\leqslant(V/E_{cr}-L)v_\alpha\tau_M$ for all values of χ, and therefore we can obtain sufficient conditions for formation of a domain from (49):

$$\frac{F_0}{E_{cr}}\left[\left(\frac{V}{E_{cr}}-L\right)/v_\alpha\tau_M\right]^{-1/2} \times\exp\left[2\gamma_0(q_0)\left(\frac{V}{E_{cr}}-L\right)v_\alpha\tau_M\right]\approx 1. \tag{52}$$

Expression (52) shows that for a given field intensity, i.e., for $V/L=$const, the formation of a domain depends significantly on the length of the crystal and the value of F_0 (which is proportional to $N_0^{3/2}$), so that as the electron density N_0 and the length of the crystal L increase, the condition for domain formation (52) is relaxed. The boundary conditions (34) allow us to obtain an equation for the time dependence of the current

$$Y(\chi=t/\tau_M)\frac{v_\alpha\tau_M}{L}+\frac{dz}{d\xi}\left(1-\frac{v_st}{L}\right)=\frac{V}{E_{cr}L}-1. \tag{53}$$

Relation (53) is in fact an integral equation for the current, because Y depends functionally on the current, while $z(\xi)$ depends on the time integral of the current. This equation can be solved only by numerical methods; however, it is not difficult to show from (53) that all the asymptotic results obtained above follow as $t\to\infty$; furthermore, it follows that the current decreases as a function of time. From (53) it follows that the current is determined primarily by the function $z(\xi)$ for large L, i.e., by the character of the evolution of the absolute instability in the region of the crystal $x\geqslant v_st$.

Relation (51) may be treated as an integral equation for the function $E_d(x)$. An obvious solution to this equation is the function

$$E_d(x)=\begin{cases}E_{cr}, & 0<x<v_\alpha t-h,\\ E_{cr}+E_d(v_\alpha t-x), & v_\alpha t-h<x<v_\alpha t,\end{cases} \tag{54}$$

where $E_d(v_\alpha t-x)$ is an arbitrary nondecreasing function of its argument. Equation (54) yields a relation between the "width" h of a domain and the potential V:

$$\int_0^h E_d(x)dx=V-E_{cr}L. \tag{55}$$

This relation is also in good agreement with experiment (see Ref. 8). The stationary phonon density in the domain will be

$$\mathscr{E}(\mathbf{q},x=v_\alpha t)=T\exp\left[\gamma_e(\mathbf{q})\left(\frac{V}{E_{cr}}-L\right)\right],$$

which no longer depends either on time t or on x, but rather is determined by the applied field and the size of the crystal.

Thus, the theory of acoustoelectric domains we have formulated here answers practically all the questions posed by numerous experiments: it explains the effect of current saturation and the formation of the domain itself, and determines the translation velocity of the domain through the crystal, which turns out to be extremely close to the velocity of the phonons generated; it gives a value of the electric field in the region outside the domain that turns out to be very close to the critical (threshold) value v_α/μ, and establishes the connection between the current and the field at different instants of time. Questions about the shape of the domain remain outside the scope of this investigation; it is clear that in order to answer these questions we must discard condition (48) and solve the very complicated integrodifferential equations we have obtained for this case. Also outside our analysis are the variation and spectral content of the phonons generated, in particular the downward shift of the maximum frequency of generated phonons, and the formation of an electric double layer in the domain, which several experiments indicate. All these questions require a separate investigation; however, it is clear that they also can be solved within the framework of the equations described here.

[1)] As before, we are dealing with waves traveling in one direction and therefore $\boldsymbol{\xi}=\mathbf{r}-\mathbf{v}_g t$; however, for waves propagating in the opposite direction we use $\boldsymbol{\xi}=\mathbf{r}+\mathbf{v}_g t$, etc.

[2)] It is known[17] that taking into account temperature oscillations or electron temperature waves causes only a numerical change in the threshold for amplification (generation) of sound, while not changing the overall picture of the phenomenon. As for possible dependence of the collision frequency ν on electric field E, it can be shown that additional terms that arise in (26) due to this dependence are insignificant for all scattering mechanisms, with the sole exception of electron scattering by optical phonons in the low-temperature range $T \ll \hbar\omega_{\rm opt}$, where $\omega_{\rm opt}$ is the characteristic frequency of the optical phonons. As this case has not yet been encountered in experiments on acoustoelectric instability, it will not be discussed in this paper.

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