Frequency and field dependence of the impedance for type II superconductors in the mixed state

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The dependence of the real part Re Z of the impedance of a bulk sample of composition $Pb_{0.81}In_{0.19}$ on the external magnetic field $0<H<H_{c2}$ and frequency in the range $40<f<1000$ MHz is studied. It is shown that the field dependence of ReZ is linear for $H<H_{c1}$ and proportional to $H>H_{c1}$. The frequency dependence of the impedance is well extrapolated by the formula Re$Z\propto\omega^n$. The exponent n depends on imperfections of the superconductor surface. The results are explained in the framework of the recently suggested "two-mode electrodynamics" of Sonin, Tagantsev, and Traito [Phys. Rev. B **46**, 5830 (1992)].

1. INTRODUCTION

It is well known that the surface impedance of type II superconductors in an external magnetic field is determined by the vortex lattice dynamics. The theory of high-frequency response was first developed by Gittleman and Rosenblum.[1] Vortex lattice motion was reduced to the motion of a single vortex whose dynamic properties were given by two parameters: the viscosity coefficient η and vortex effective mass m. One of the most important results obtained in Ref. 1 was the so called depinning frequency ω_d. If the frequency of external events is larger than ω_d, the vortices can be considered free. In this case the surface resistance is determined by properties inherent in a "clean" superconductor and does not depend on the defects and treatment of the sample under study. Later on, Gor'kov and Kopnin[2,3] showed that a superconductor in the mixed state at $\omega\gg\omega_d$ behaves like an "anisotropic medium" of sorts governed by Maxwell's equations with corresponding material equations. In Ref. 3 the surface impedance Z was found in terms of the elastic vortex mode for fields $H_{c1}\ll H\ll H_{c2}$. In particular,

$$R_f=\mathrm{Re}\,Z=(2\pi\omega\rho_f/\mu c^2)^{1/2}. \qquad (1)$$

Here ρ_f is the flux-flow resistance and $\mu=B/H$. However, as noted in Ref. 3, this is valid only if the skin depth δ is large compared to the penetration depth of a constant magnetic field, λ, which imposes a limit on the electromagnetic wave frequencies. It follows from (1) that at $\omega\gg\omega_d$ and for $H_{c1}\ll H\ll H_{c2}$, a superconductor behaves like a normal metal with $\rho=\rho_f$. Therefore, here, as in the case of a normal metal, Re Z is proportional to $\omega^{1/2}$. It also follows from (1) that Re $Z\propto B^{1/2}$.

Recently Sonin, Tagantsev, and Traito[4] have developed a theory in which, besides the vortex elasticity, they allowed for vortex interaction in the lattice (the so-called nonlocality effect). The impedance was calculated for the external magnetic field perpendicular to the sample surface. It was shown that in the limit of low frequencies $\omega\ll\omega_c$ (but, as in Ref. 3, at $\omega\gg\omega_d$), where

$$\omega_c\cong H_{c1}\rho_n c^2/4\pi\lambda^2 H_{c2}\,, \qquad (2)$$

the surface impedance Re $Z\propto B$ for $H\ll H_{c1}$. For fields $H_{c1}\ll H\ll H_{c2}$, as in Ref. 3, we have Re $Z\propto B^{1/2}$. Moreover, the crossover occurs at $H\approx H_{c1}$. As for the frequency dependence, vortex pinning at the superconductor surface turns out to be important. In the case of weak surface pinning,

$$\mathrm{Re}\,Z=(2\pi\omega\rho_f\mu/c^2)^{1/2}, \qquad (3)$$

i.e. Re $Z\propto\omega^{1/2}$. For strong surface pinning, we have

$$\mathrm{Re}\,Z=(2\pi\omega\rho_f\mu/c^2)^{1/2}(\omega/\omega_c), \qquad (4)$$

therefore Re $Z\propto\omega^{3/2}$. Since $\omega\ll\omega_c$, for an imperfect surface the losses are ω/ω_c times smaller than for a perfect one. In other words, in the case of strong surface pinning, the frequency ω_c, in fact, plays the part of the depinning frequency at the surface. The estimates of Eq. (2) show that for ordinary superconductors, ω_c is of the order of hundreds of gigahertz. For comparison, the bulk depinning frequency ω_d is about 10 MHz.[1]

Thus, the theory developed in Ref. 4, first, allows us to reconstruct the field dependence of the impedance for $H^*_{c1}<B\ll H_{c2}$, where H^*_{c1} is the first critical field with the demagnetization factor allowed for. Second, even in the limit of small frequencies, when $\delta\gg\lambda$, we cannot neglect the surface properties of a superconductor. When the surface pinning increases, the impedance should decrease, and its frequency dependence will alter its exponent.

2. SAMPLES

The alloy In_xPb_{1-x} (x=5–30%) is a type II superconductor at the temperature of liquid helium. In the sixties and seventies it was widely investigated from the point of

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view of studying the mixed state of type II superconductors and the interaction of magnetic flux vortices with pinning centers (see, e.g., Ref. 5). Thus, this is a traditional alloy for the study of the mixed superconductor states and its properties are well-known. We used the alloy with $x = 0.19$. This composition is close to the one with the smallest bulk pinning,[6] which allowed us to change the vortex pinning markedly (over a wide range) while externally reducing the data.

The alloy was produced by mixing in vacuum pure pure lead and indium in the ratio required. The samples were obtained by compressing a piece of the alloy between optically polished steel plates. Upon removal of sharp angles, they were again compressed between the same plates. The resulting samples were, in their shape, close to discs 1 mm thick and about 15 mm in diameter. The alloy InPb has a low temperature of diffusion annealing,[6] therefore we did not carry out any extra thermal treatment, believing that upon being kept at room temperature, the sample comes into equilibrium both in composition and structure spoilt by compression.

After a round of impedance measurements, some samples underwent irradiation by an oxygen ion beam of energy 1 keV in doses of 10^{16} and 10^{17} ion/cm^2 at room temperature. Such energies correspond to a mean projective path of the ions in the material on the order of several tens of angstroms. Note that the irradiation did not result in apparent changes in the sample surface, which remained smooth. The other method of changing the properties of the sample surface consisted in thermal deposition of films on the surface under study, since, as a rule, they have more defects than bulk samples. The film composition corresponded to that of the initial material, i.e., In_xPb_{1-x} with $x = 0.19$. The thickness of deposited films was monitored by a quartz balance, and ranged from 300 to 1000 Å. Upon irradiation or film deposition, the samples were kept at room temperature and a round of impedance measurements for the superconducting state was carried out.

3. IMPEDANCE MEASUREMENT TECHNIQUE

As already pointed out, the impedance was measured in the frequency range 40 to 1000 MHz via two techniques. In the first case, we used as an absorbing cell a spiral resonator 4 mm in diameter and 10 mm long made of a copper wire whose fundamental frequency was about 100 MHz. To find the frequency dependences, higher resonator harmonics were used. This made it possible to measure the impedance at 6–7 discrete points of the quoted range. The sample was placed outside the spiral resonator at a distance of 0.2 mm. The resonator axis lay in the horizontal plane parallel to the sample. Such a geometry reduces the setup sensitivity with increasing harmonic number, and limits the frequency range from above for the specific resonator. The latter was excited by a standard generator through a coaxial cable. Coupling was achieved by positioning the center conductor near the resonator face (capacitive coupling). Likewise, with the help of a second coaxial cable, a signal proportional to the voltage across the resonator was fed to a measurement receiver. After detection, the resulting signal was recorded either by a strip-chart recorder or a PC.

In the second case, we used as a resonator a coaxial line of variable length shorted by an inductance. The inductance was a figure eight, so that the high-frequency currents induced magnetic fields in its loops pointing in different directions. As a result, the high-frequency magnetic field at the center was in the plane of the inductance. Placing the sample parallel to that plane, the conditions of sample irradiation by the high-frequency wave were similar to the first case. Varying the coaxial line length and using higher harmonics, we could continuously vary the measurement frequency, which enabled us to make measurements at more than 20 points of the frequency range. The resonator was excited and the desired signal extracted with the help of capacitive coupling, using the unshorted end of the coaxial resonator. It is worth noting that the two methods produce similar results.

To generate the magnetic field, we used an electromagnet that could rotate about its vertical axis, its field remaining in the horizontal plane. The sample was placed in a Dewar flask between the magnet poles. The sample plane was vertical, and we could vary the magnetic field orientation from parallel, which could be set by the impedance minimum in the field $H_{c1} < H < H_{c2}$, to normal. In the present, study we were interested in impedance measurements in a magnetic field normal to the sample plane. The temperature corresponded to the helium boiling point at atmospheric pressure, and was not controlled ($T \simeq 4.2$ K).

As is well-known, in high-frequency measurements it is quite difficult to measure the absolute value of the impedance. Usually, this is due to difficulties in finding the geometric pulse duty factor, which enters into the final signal magnitude. In our detection system, the signal at the receiver input is proportional to the resonator quality, which is determined by the losses in the resonator conductor, the sample and the parts surrounding the resonator. Estimates show that the total signal variation in the experiment does not exceed 10% of its magnitude. This makes it possible to use the expression for small deviations and to assume, in the end, that in our measurements the signal variation is proportional to the absorption in the sample, since checks using an absorbing cell without the sample do not give noticeable signal variations used throughout the magnetic field range. To compare the impedance values obtained at different frequencies, we used the following considerations. In weak magnetic fields, where the field does not penetrate into the sample (for example, when the field is parallel to the sample plane), we approximated the impedance value by $\mathrm{Re}\, Z = 0$. In fields greater than H_{c1} the error is obviously small, since at $T \approx 1/2 T_c$, $\mathrm{Re}\, Z \ll \mathrm{Re}\, Z_{\mathrm{norm.met.}}$ for $H = 0$, and the impedance value in the field range of interest is of the order of the impedance of a superconductor in the normal state.

For fields stronger than H_{c3} (or H_{c2} for H normal to the sample plane), we assumed that the normal skin-effect model was valid, i.e., $\mathrm{Re}\, Z \sim \omega^{1/2}$. Using these considerations, we could reduce the data obtained at different fre-

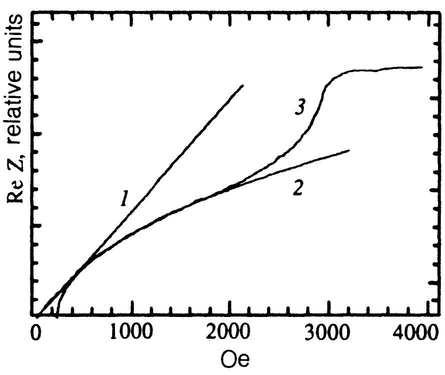

FIG. 1. Field dependence of the real part of the impedance at 170 MHz and its approximations. *1*) $\mathrm{Re}\, Z = \alpha(H - H^*_{c1})$, where α is the scale factor and $H^*_{c1} = 45$ Oe; *2*) $\mathrm{Re}\, Z = \beta(H - H_x)^{1/2}$, where β is the scale factor and $H_x = 240$ Oe is the fitting parameter; *3*) experimental data.

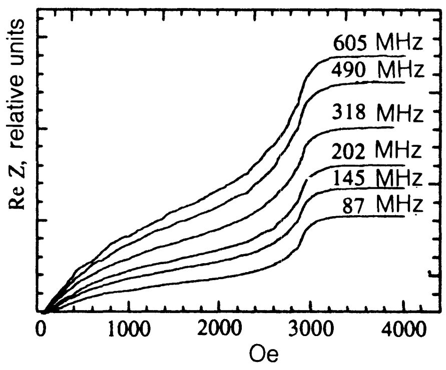

FIG. 2. Field dependence of the real part of the impedance for various frequencies.

quencies to a single scale and compare them without knowing the absolute value of the impedance.

4. EXPERIMENTAL RESULTS

Figure 1 shows the field dependence of the surface impedance for the original sample ("mirror" surface) obtained at $\omega = 170$ MHz for the external field normal to the sample surface. The curve has the form typical of the field dependence of the impedance for a type II superconductor. In the field H^*_{c1}, which corresponds to vortex penetration into the sample, the impedance grows drastically. For fields increasing up to the value H_{c2}, the impedance grows in a monotonic manner, and in the fields $H > H_{c2}$, it becomes field-independent. Clearly, for $H < 0.5$ kOe, Re Z is proportional to ($H - 45$ Oe). In large magnetic fields we have $\mathrm{Re}\, Z \propto (H - 240\ \mathrm{Oe})^{1/2}$.

Figure 2 shows the real part of the impedance of the original sample versus the normal magnetic field for several frequencies. The dependences are reduced to the same scale by the procedure described above. The field dependences of Re Z for the original sample and samples coated with films 300 and 1000 Å thick are displayed in Fig. 3. The curves are shown on the same scale. It is seen that the impedance falls with film thickness. Similar results were obtained for ion-irradiated surfaces. Since the curves Re $Z(H)$ did not differ qualitatively from those shown in Fig. 3, in the present study we considered only this example.

For all of the samples, we obtained field dependences of Re Z at different frequencies and reduced them to one scale, as in Fig. 2. Examining the behavior at $H = \mathrm{const}$, we can derive the frequency dependence of the impedance for a given field. The results are shown in Fig. 4. The experimental points were approximated by curves of the form $\mathrm{Re}\, Z \propto (f - f_c)^n$, and we solved for f_c and n. It turned out that to within the accuracy of our approximation, f_c is essentially independent of the surface state of the sample and the value of the external magnetic field for which the behavior was plotted, while the exponent n changes appreciably (Fig. 4).

Using an array of such curves, i.e., finding the frequency dependence for each value of the external magnetic field, we can get the field dependence of the exponent n. These curves are displayed in Fig. 5. Apart from an increase in n with growing film thickness in fields $H > 0.5$ Oe,

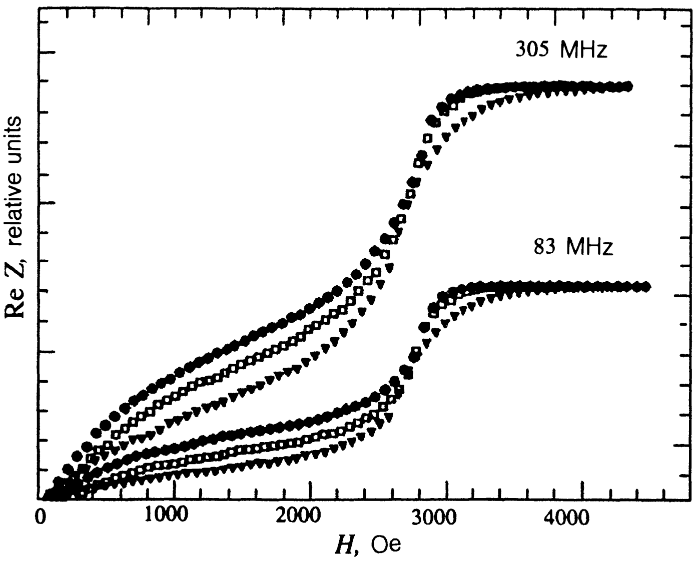

FIG. 3. Field dependence of the real part of the impedance at two frequencies for different samples (○—sample with a mirror surface; □—sample with a deposited film of thickness 300 Å; ∇—sample with a deposited film of thickness 1000 Å).

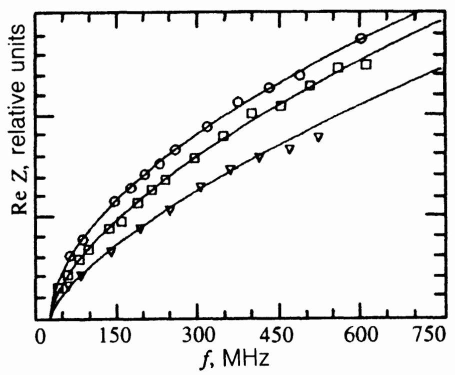

FIG. 4. Frequency dependence of the real part of the impedance approximated by the formula Re $Z=\gamma_i(f-f_{c(i)})^{n(i)}$, where γ_i is the scale factor, $f_{c(i)}=25$ MHz, and $n(i)$ is the fitting parameter (○—$i=1$, sample with a mirror surface; □—$i=2$, sample with a deposited film of thickness 300 Å; ∇—$i=3$, sample with a deposited film of thickness 1000 Å).

it is seen that n grows appreciably for film-coated surfaces in fields $H<0.5$ Oe.

5. DISCUSSION

1. Field dependence of Re Z

To extrapolate the experimental data for external fields weaker than 0.4 kOe, we used the expression Re $Z = \alpha(H - H^*_{c1})$ (see Fig. 1, curve *1*). Here α is the scale factor and $H^*_{c1} = 45$ Oe was determined by the point where the impedance started to grow. The first critical field can be found from the expression $H_{c1} = H^*_{c1}/(1 - D)$,[8] where D is the demagnetization factor. For a disc $D=1-\pi d/2b$, where d is the disc thickness and b is its radius. Hence, $H_{c1}\cong 230$ Oe, which agrees well with the value $H_{c1}=210$ Oe reported in Ref. 6.

For fields 0.4 kOe$<H<$2 kOe, we extrapolated the data with the expression Re $Z=\beta(H-H_x)^{1/2}$ (Fig. 1, curve *2*). Apart from the scale factor β, we used another fitting parameter H_x. The theory gives $H_x=H_{c1}$ (see Ref. 4), while fitting leads to $H_x=240$ Oe, which agrees with the value of the first critical field given above.

2. Frequency dependence of Re Z

To extrapolate the frequency dependences of Re Z, we used the expression Re $Z=\gamma_i(f-f_{c(i)})^{n(i)}$, where the subscript $i=1$, 2, and 3 denotes the corresponding curve (Fig. 4). In addition to the scale factor γ_i, we determined two more parameters: the frequency $f_{c(i)}$ and the exponent $n(i)$. As noted above, the parameter $f_{c(i)}$ is insensitive to the state of the superconductor surface. For all extrapolated curves $f_{c(i)}$ is within the range 25 ± 10 MHz. Figure 4 shows the curves with $f_{c(i)}=25$ MHz for $i=1$, 2, and 3. This can probably be regarded as the frequency of volume depinning. To order of magnitude, this value coincides with those found earlier (see Ref. 1, $f_c\cong 5$ MHz); however, the frequency range used in the present study (>40 MHz) does not make it possible to find $f_{c(i)}$ more accurately.

Another parameter grows appreciably with the coating thickness the exponent n (Fig. 5). According to the predictions of Ref. 4, in the case of "ideal" surface pinning, the exponent should reach 3/2. In our experiment, the number of pinned vortices probably grows with the film thickness. However, free vortices remain as well. The frequency dependence of Re Z will then be determined by the dynamics of pinned and free vortices, which results in the growth of n. These considerations are also confirmed by the field dependence of n (Fig. 5). For coated samples the exponent grows markedly in small external fields. In this case the vortices which penetrate the sample likely become pinned, and they are relatively common. As the field increases, the number of free vortices grows, since the effective pinning centers are already occupied. Note that the field dependence of n for irradiated samples is qualitatively the same.[7]

Thus, the field dependence of the real part of the surface impedance is given quantitatively by the theory developed in Ref. 4. As for the frequency dependence of Re Z, our experiments agree only qualitatively with the theory.

In conclusion, we would like to thank E. B. Sonin, A. K. Tagantsev, and K. B. Traito for helpful discussions.

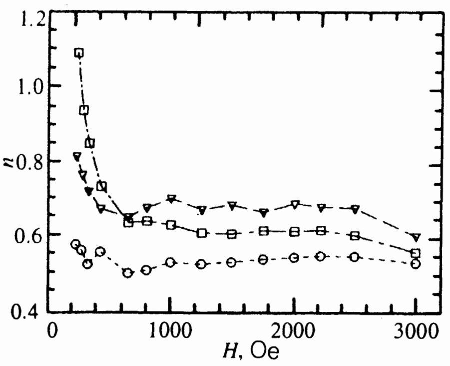

FIG. 5. Magnetic field dependence of the exponent in the frequency dependence of the impedance for three samples (○—sample with a mirror surface; □—sample with a deposited film of thickness 300 Å; ∇—sample with a deposited film of thickness 1000 Å).

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