A generalized Bell's theorem

A. V. Belinskii

M. V. Lomonosov State University, 119899 Moscow, Russia
(Submitted 5 November 1993)
Zh. Eksp. Teor. Fiz. **105**, 818–827 (April 1994)

One of the assumptions made by Bell in deriving his inequalities was that of *locality*, i.e. that two measuring devices cannot affect one another when they are far apart. For this reason, the violations of these inequalities observed in experiments are often regarded as manifestations of the *nonlocality of quantum theory* or as a disproof of *local realism*. In this paper the Bell inequality is proved for two observers in its traditional form, but without using the locality hypothesis, starting only from the condition that the probability distribution function be nonnegative. This distribution function is calculated and applied to a specific optical experiment within the framework of quantum theory, and it is shown that it can take on negative values. This rigorously proves the irrelevance of the locality assumptions with regard to violations of Bell's theorem. In addition, nonlocal Bell's theorems are formulated and proved, along with the Greenberger–Horn–Zeilinger paradox for an arbitrary number of observers N, again without using the assumption of locality. The physical meaning of these results is analyzed in detail.

1. INTRODUCTION

Despite what would appear to be a thorough study of questions connected with the Einstein–Podolsky–Rosen paradox[1] and Bell's theorem,[2] recently there has been a marked increase in the number of publications on this theme (see, e.g., Ref. 3–5 and the references cited therein). Experimental violations of Bell's inequality, which are predicted by quantum theory and have been repeatedly verified in experiment, are treated by the overwhelming majority of authors as manifestations of the *nonlocality of quantum theory*. The fact is that the original Bell inequalities[2] were derived on the basis of concepts from the theory of hidden variables,[1] one of whose assumptions is that of *locality*, i.e., the inability of two distant measuring devices to influence one another. Therefore, it would appear that the results of such experiments could formally be described within the framework of a *nonlocal theory of hidden variables*, i.e., one that contains no hypothesis about locality. A recent review[3] has demonstrated that it is logically inconsistent to require that the concept of nonlocality be enlisted in order to explain violations of Bell's inequality (see also Refs. 4–7). The goal of this paper is to formulate rigorous proofs of various forms of Bell paradoxes, including the beautiful Greenberger–Horn–Zeilinger paradox, without using the hypothesis of locality.

2. NONLOCAL BELL INEQUALITY FOR TWO OBSERVERS

Let us prove the Bell inequality in its traditional form

$$\Pi=(1/2)\,|\langle AB\rangle+\langle A'B\rangle+\langle AB'\rangle-\langle A'B'\rangle|\leqslant 1 \quad (1)$$

without the assumption of locality. Another form of this inequality was treated in Ref. 6 (see also Refs. 7, 10.) Here A,A', B,B' are dichotomous variables that take on unit values:

$$A=\pm 1,\quad A'=\pm 1,\quad B=\pm 1,\quad B'=\pm 1. \quad (2)$$

The averaging is carried out over realizations. In order to prove (1) we require *only* that the normalized probability distribution function be nonnegative:

$$P(A,A',B,B')\geqslant 0, \quad (3)$$

$$\sum_A\sum_{A'}\sum_B\sum_{B'} W(A,A',B,B')=1. \quad (4)$$

It goes without saying that we must fulfill consistency conditions of the type

$$P(A,A',B,B')+P(-A,A',B,B')=P(A',B,B'), \quad (5)$$

along with analogous relations for the other variables and lower-dimensional distributions.

The discrete probability distribution function (3) consists of 2^4 joint probabilities:

$$P_{AA'BB'}(++++)$$
$$\equiv P(A=+1,A'=+1,B=+1,B'=+1),$$
$$P_{AA'BB'}(+++-)$$
$$\equiv P(A=+1,A'=+1,B=+1,B'=-1),\ \text{etc.} \quad (6)$$

We express the average entering into (1) in terms of these joint probabilities, for example,

$$\langle AB\rangle=P_{AB}(++)+P_{AB}(--)-P_{AB}(+-)$$
$$-P_{AB}(-+), \quad (7)$$

where, for instance,

$$P_{AB}(++)=P_{AA'BB'}(++++)+P_{AA'BB'}(+++-)$$
$$+P_{AA'BB'}(+-++)$$
$$+P_{AA'BB'}(+-+-).$$

We substitute these expansions into the left side of (1). As a result we obtain (omitting the lower labels on P)

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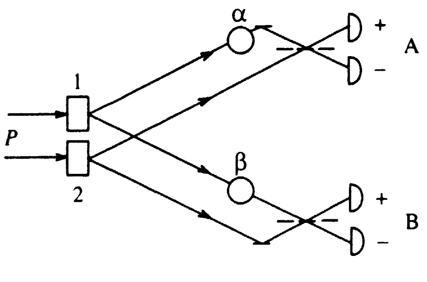

FIG. 1. Scheme for an intensity interferometer with parametric sources of light for two observers A and B. The correlated photons are created simultaneously in nonlinear elements 1 or 2 under the action of a pump P, and are directed towards A and B in two modes, one of which undergoes a phase shift (circles). The modes are mixed at a 50% light divider (dashed segment) and detected.

$$\Pi = (1/2)(\langle AB\rangle + \langle A'B\rangle + \langle AB'\rangle - \langle A'B'\rangle)$$
$$= P(++++) + P(+++-) - P(++-+)$$
$$- P(++--) + P(+-++) - P(+-+-)$$
$$+ P(+--+) - P(+---) - P(-+++)$$
$$+ P(-++-) - P(-+-+) + P(-+--)$$
$$- P(--++) - P(--+-) + P(---+)$$
$$+ P(----). \qquad (8)$$

If all the terms on the right side of (8) were to enter into the sum with a plus sign, this would be an partition of unity as in (4). Since half the terms appear with a minus, by virtue of (3), (4) the sum (8) lies within the interval $[-1,+1]$. Thus, Eq. (1) is proved.

3. EXAMPLE OF A VIOLATION OF BELL'S INEQUALITY (1) AND THE REASON WHY IT IS VIOLATED

Why should inequality (1), which is based on very general assumptions [Eqs. (2)–(5)], be violated in practice?

Let us investigate the setup for the simplest experiment used to verify (1).[5,11,12] Two observers A and B simultaneously record photons one by one at detectors "+" or "−"; we will assign the values $A,B = +1$ or -1 to these events. Changing the phase delays, i.e., α to α' and (or) β to β', constitutes a transformation from variables A and B to A' and (or) B'. Multiple repetition of these measurements allows us to compute the average entering into (1).

The quantum state of the photons arriving at the detectors can be described by the Hilbert-space vector[5]

$$|\Psi\rangle = (1/2)^{1/2}(a_1^+ b_1^+ + a_2^+ b_2^+)|0\rangle$$
$$\equiv (1/2)^{1/2}(|10\rangle_a |10\rangle_b + |01\rangle_a |01\rangle_b), \qquad (9)$$

where a_j^+ and b_j^+ are operators for creating photons in the two signal (arriving at detector A) and dummy (arriving at B) modes; $j=1,2$ labels the crystal that radiates the corresponding mode (see Fig. 1), while $|0\rangle$ denotes the vacuum.

The number operators for photons received at the detectors "+" and "−" in channel A have the form

$$n^a_\pm \equiv a^+_\pm a_\pm = (1/2)[n^a_1 + n^a_2 \pm (\sigma^a_- e^{i\alpha} + \sigma^a_+ e^{-i\alpha})], \qquad (10)$$

where $n^a_j \equiv a^+_j a_j$, $\sigma^a_- \equiv a_1 a^+_2$, $\sigma^a_+ \equiv (\sigma^a_-)^+$. Analogous relations define $n^b_\pm$ in channel B.

Let us find $P(A,A',B,B')$ by calculating the joint probabilities

$$P_{AA'BB'}(++++) = \langle\Psi| n^a_+ n^{a'}_+ n^b_+ n^{b'}_+ |\Psi\rangle,$$
$$P_{AA'BB'}(+++-) = \langle\Psi| n^a_+ n^{a'}_+ n^b_+ n^{b'}_- |\Psi\rangle, \text{ etc.} \qquad (11)$$

Here the primes denote the interchange of α and α' in (10) and (or) β with β' in channel B. As a result we obtain

$$P_\Psi(A,A',B,B')$$
$$= (1/16)[1 + AA' \cos(\alpha-\alpha') + BB' \cos(\beta-\beta')$$
$$+ AA'BB' \cos(\alpha+\beta-\alpha'-\beta')$$
$$+ AB\cos(\alpha+\beta) + A'B\cos(\alpha'+\beta)$$
$$+ AB' \cos(\alpha+\beta') + A'B' \cos(\alpha'+\beta')]. \qquad (12)$$

We establish the following phases in the channels:

$$\alpha=0, \quad \alpha'=\pi/2, \quad \beta=-\pi/4, \quad \beta'=\pi/4, \qquad (13)$$

which corresponds to a violation of (1). In this case several joint probabilities turn out to be negative (the lower indices are omitted):

$$P(++++) = P(+--+) = P(-++-)$$
$$= P(----) = 2^{1/2}/16,$$
$$P(++--) = P(+-+-) = P(-+-+)$$
$$= P(--++) = -2^{1/2}/16,$$
$$P(++-+) = P(+---) = P(-+++)$$
$$= P(--+-) = (2-2^{1/2})/16,$$
$$P(+++-) = P(+-++) = P(-+--)$$
$$= P(---+) = (2+2^{1/2})/16, \qquad (14)$$

while direct substitution of (14) into (8) gives $2^{1/2}$, i.e., the Bell inequality (1) is not satisfied.

According to (12), (14) the normalization (4) and consistency conditions (5) are satisfied; therefore, the only reason for the violation of inequality (1) is the fact that (3) is not satisfied.

Negative probability distributions have been encountered in the literature in connection with the Einstein–Podolsky–Rosen paradox and Bell's theorem.[10–15] However, by defining the probability distribution function in the form $P_\varphi(A,A',B,B')$ we can compare (12), (14) with the original premises (3)–(5) and arrive at an unambiguous conclusion regarding the role of locality, or more precisely the lack of it, in producing the violation of (1). Furthermore, there is no need to introduce "hidden variables."

The probability distribution function $P_\varphi(A,A',B,B')$ is analogous to the Wigner distribution function. Not all of the observables entering into it are described by commuting operators, e.g., A and A'. They cannot be measured in the same realization (for various phase delays α and α', a single photon cannot be recorded at all by observer A). Consequently, direct measurement of $P_\varphi(A,A',B,B')$ is impossible. However, indirect methods of measuring probability distribution functions of this kind are nevertheless admissible. Thus, in Ref. 19 an original method was proposed and experimentally implemented that was intended for measuring two-dimensional Wigner distributions, including negative ones. Should we, perhaps, be resigned to negative probability, treating it, as Dirac did[20], as a well-defined mathematical analogue of negative sums of money (see, also, the interesting discussions on this theme in Ref. 15)?

Here is yet another analogy. Negative temperatures do not exist on the Kelvin scale. However, in quantum electronics a formal description of population inversions using negative temperature is widely used. Although such a temperature cannot be measured with a thermometer, we can identify the state of the working levels and compute its negative value. Thus, once the existence of a distribution function $P_\varphi(A,A',B,B')$ is formally acknowledged, as it must be if the concepts of hidden variable theory are correct, there is no longer room for the requirement that this function be unconditionally non-negative.

4. NONLOCAL BELL INEQUALITIES FOR AN ARBITRARY NUMBER OF OBSERVERS *N*

The appeal of Bell's inequalities for the number of observers $N>2$ is attributable principally to the quantitative increase (by a factor of $2^{(N-1)/2}$) of the disagreement with the predictions of the quantum theory.[5,21,22] We will begin this section, however, by introducing yet another variant of the proof of (1) for two observers ($N=2$). This version allows us to easily extend the proof to any number of observers, N.

Let us sequentially number the combined probabilities (6) which comprise the distribution function of the probabilities (3), for example, in the sequence on the right-hand side of Eq. (8):

$$P_1 \equiv P_{AA'BB'}(++\ ++) \equiv P(A_1=+1,\quad A_1'=+1,\quad B_1=+1,\quad B_1'=+1),$$

$$P_2 \equiv P_{AA'BB'}(+++\ -) \equiv P(A_2=+1,\quad A_2'=+1,\quad B_2=+1,\quad B_2'=-1), \tag{15}$$

make up the components of the vector $|W\rangle$; the normalization condition (4) here is expressed by

$$\sum_M P_M = 1, \quad M=1,2,\ldots,2^{2N}. \tag{16}$$

Let us cast the average of products of two variables, e.g., A and B, in the form of the sum

$$\langle AB\rangle = \sum_M A_M B_M P_M, \tag{17}$$

with analogous expressions for the other averages entering into (1). Then

$$\Pi = (1/2)(\langle AB\rangle + \langle A'B\rangle + \langle AB'\rangle - \langle A'B'\rangle) = \sum_M S_M^{(2)} P_M, \tag{18}$$

where the Bell observable for two observers

$$S_M^{(2)} \equiv (1/2)(A_M B_M + A_M' B_M + A_M B_M' - A_M' B_M') = [(B_M + B_M')A_M + (B_M - B_M')A_M']/2 \tag{19}$$

can take on only the unit values ± 1 by virtue of (2). Consequently, (1) follows from (3), (16), and (18).

In fact, the proof we have given generalizes the derivation of the local Bell's theorem[5,22,23] to the nonlocal case. A further natural generalization is to go from two to an arbitrary number of observers N.

Let us consider a random process described by $2N$ dichotomous variables that take on values of unity:

$$A^{(1)}=\pm 1,\quad A^{(1)\prime}=\pm 1,\quad A^{(2)}=\pm 1,\quad A^{(2)\prime}=\pm 1,\ldots,\quad A^N=\pm 1,\quad A^{(1)\prime}=\pm 1. \tag{20}$$

Assume that there exists a positive-definite normalized probability distribution function

$$P(A^{(1)},A^{(1)\prime},A^{(2)},A^{(2)\prime},\ldots,A^N,A^{N\prime}) \geqslant 0, \tag{21}$$

$$\sum_{A^{(1)}} \sum_{A^{(1)\prime}} \ldots \sum_{A^{(N)\prime}} P(A^{(1)},A^{(1)\prime},A^{(2)},A^{(2)\prime},\ldots,A^{(N)},A^{(N)\prime}) = 1, \tag{22}$$

which satisfies consistency conditions of the form

$$\sum_{A^{(1)}} P(A^{(1)},A^{(1)\prime},A^{(2)},A^{(2)\prime},\ldots,A^{(N)},A^{(N)\prime}) = P(A^{(1)\prime},A^{(2)},A^{(2)\prime},\ldots,A^{(N)},A^{(N)\prime}), \tag{23}$$

along with analogous relations for other variables and lower-dimensional distributions.

Let us also introduce the Bell observable of the form

$$S_M^{(N)} = (1/2)[(A_M^{(N)} \pm A_M^{(N)\prime}) S_M^{(N-1)} \pm (A_M^{(N)} \mp A_M^{(N)\prime}) S_M^{(N-1)\prime}] = \pm 1. \tag{24}$$

which is similar to that used in Refs. 5 and 22 to derive Bell's local inequalities. The variables in (20), however, do not correspond to the same set of unspecified parameters (λ), but are instead numbered sequentially through four corresponding probabilities, as in (15).

The recurrence relation (24) allows us to switch from Bell's observable for $N=2$ to Bell's observable for $N=3$, etc. The prime on the last term in (24) means that the primed variables in Bell's observable for $(N-1)$ observer were changed to unprimed variables, and vice versa. For example,

$$S_M^{(2)}=(1.2)[(A_M^{(2)}\pm A_M^{(2)'})A_M^{(1)}\pm(A_M^{(2)}\mp A_M^{(2)'})A_M^{(1)'}], \tag{25}$$

$$S_M^{(2)'}=(1/2)[(A_M^{(2)'}\pm A_M^{(2)})A_M^{(1)'}\pm(A_M^{(2)'}\mp A_M^{(2)})A_M^{(1)}]. \tag{26}$$

while $S_M^{(1)}=A_M^{(1)}$. The signs in (24)–(26) are arbitrary; however, if there is a sum in the first set of parentheses, there should be a difference in the second, and conversely.

Comparison of (21), (22), and (24) allows us to conclude that

$$\left|\sum_M S_M^{(N)}P_M\right|\leqslant 1. \tag{27}$$

This is in fact the prototype of the nonlocal Bell's inequality for an arbitrary number of observers, N. Its actual form can be calculated from (24). We must accordingly remove the subscripts M in (24), express $S^{(N-1)}$ and $S^{(N-1)'}$ in terms of the variables in (20), remove all parentheses, and average out each term. The resulting expression should not be greater than unity in absolute value. For example, one possible combination of signs in (24) for $N=3$ is

$$(1/2)|\langle A'BC\rangle+\langle AB'C\rangle+\langle ABC'\rangle-\langle A'B'C'\rangle|\leqslant 1. \tag{28}$$

For clarity, we made the replacement $A\equiv A^{(1)}$, $B\equiv A^{(2)}$, $C\equiv A^{(3)}$ and a similar replacement for the primed variables.

Relation (28) is equal to the corresponding Bell's inequality, which was derived under the assumption of localizability.[5,22,23] The same argument applies to Bell's inequalities for arbitrary N. Since Bell's observable (24) is in formal agreement with the corresponding expression derived on the basis of the local theory of hidden parameters,[5,22] the local Bell's inequalities admit a generalization to nonlical inequalities based on the algorithm described above.

As noted above, the quantitative violations of Bell's inequality increase by a factor of $2^{(N-1)/2}$ with increasing N.[5,21,22] On the basis of a quantum-mechanical consideration the left side of (28), for example, takes on the value of 2 under certain conditions. Furthermore, beginning with $N=3$, we can graphically formulate the Greenberger–Horn–Zeilinger paradox, which is the subject discussed in the next section.

5. THE NONLOCAL GREENBERGER–HORN–ZEILINGER PARADOX

This beautiful paradox leads to a contradiction of the type $+1=-1$ (Refs. 5, 22–24). It assumes complete correlation of the results of the measurement, e.g.,

$$\langle A'BC\rangle=A'BC=\langle AB'C\rangle=AB'C=\langle ABC'\rangle$$
$$=ABC'=-\langle A'B'C'\rangle=-A'B'C'=1, \tag{29}$$

which is admissible in quantum theory. This is the case where we encounter the violation of the Bell theorem (28) alluded to at the end of the previous subsection. According to (29), we have

$$(A'BC)(AB'C)(ABC')(A'B'C')=-1. \tag{30}$$

We now show that this is impossible when requirements (20)–(23) are fulfilled.

Under conditions of complete correlation (29), the components of P_M that give $A'_MB_MC_M=-1$ must equal zero. The same is true of the components that give $A_MB'_MC_M=A_MB_MC'_M=-1$. As a result, out of 64 components of $P(A,A',B,B',C,C')$ only eight nonzero ones remain:

$$P(++++++),\quad P(++----),$$
$$P(+-+--+),\quad P(+--++-),$$
$$P(-++-+-),\quad P(-+-+-+),$$
$$P(--++--),\quad P(----++),$$

but only one gives $A'_MB'_MC'_M=+1$! Furthermore, there is not a single component of P_M with the property that any three out of four factors under discussion give the same signs, while the sign of the fourth is opposite. This is because of the following product:

$$(A'_MB_MC_M)(A_MB'_MC_M)(A_MB_MC'_M)(A'_MB'_MC'_M)$$
$$=(A_MA'_MB_MB'_MC_MC'_M)^2=+1. \tag{31}$$

Thus, if we admit the condition of full correlation required under these conditions, then the limitations (20)–(23) can hold only in the case where the product (30) equals $+1$. The "nonlocal" Greenberger–Horn–Zeilinger paradox formulated in this way does not require the hypothesis of locality; hence, admission of this hypothesis does not destroy the paradox. These discussions are easily generalized to arbitrary $N\geqslant 3$.

6. NONLOCAL BELL INEQUALITIES FOR NONDICHOTOMOUS VARIABLES

The algorithm for proving the Bell theorem for an arbitrary N described in Sec. 4 also allows us to generalize these Bell theorems to the case of nondichotomous variables with values on the interval $[-1,+1]$:

$$|A^{(1)}|\leqslant 1,\ |A^{(1)'}|\leqslant 1,\ |A^{(2)}|\leqslant 1,\ |A^{(2)'}|\leqslant 1,...,$$
$$|A^{(N)}|\leqslant 1,\ |A^{(N)'}|\leqslant 1. \tag{32}$$

To begin with let us consider the Bell theorem under these conditions for two observers. Turning to the Bell observable (25), we will show that when (32) is satisfied we have

$$|S_M^{(2)}|\leqslant 1. \tag{33}$$

Taking into account (32), we write

$$|(A_M^{(2)} \pm A_M^{(2)'})A_M^{(1)}| = |A_M^{(2)} \pm A_M^{(2)'}|\,|A_M^{(1)}| = |A_M^{(2)} \pm A_M^{(2)'}| - (1-|A_M^{(1)}|)\,|A_M^{(2)} \pm A_M^{(2)'}| \leqslant |A_M^{(2)} \pm A_M^{(2)'}|. \qquad (34)$$

Analogously,

$$|(A_M^{(2)} \mp A_M^{(2)'})A_M^{(1)'}| \leqslant |A_M^{(2)} \mp A_M^{(2)'}|. \qquad (35)$$

Let us add (34) and (35). As a result, taking (25) into account, we obtain

$$2|S_M^{(2)}| = |(A_M^{(2)} \pm A_M^{(2)'})A_M^{(1)} \pm (A_M^{(2)} \mp A_M^{(2)'})A_M^{(1)'}| \leqslant |(A_M^{(2)} \pm A_M^{(2)'})A_M^{(1)}| + |(A_M^{(2)} \mp A_M^{(2)'})A_M^{(1)'}| \leqslant |A_M^{(2)} \pm A_M^{(2)'}| + |A_M^{(2)} \mp A_M^{(2)'}| \leqslant 2, \qquad (36)$$

from which (33) follows.

The iterative character of the algorithm for generating the Bell observable allows us to conclude based on (32), (33) that $|S_M^{(3)}| \leqslant 1$, etc., so that

$$|S_M^N| \leqslant 1. \qquad (37)$$

Let us now assume that the variables (32) take on a finite number of values K, which is always true in practice if only because of the finite number of experimental realizations. When this is true, the dimension of the vector $|P\rangle$ increases to 2^{2KN}; however, (27) remains valid by virtue of (37). Thus, when conditions (21)–(23) hold the Bell theorems obtained above must be satisfied for the case of nondichotomous variables that satisfy (32) as well.

7. CONCLUSION

Thus, we have verified that dropping the assumption of locality does not "rescue" the theory of hidden variables (including nonlocal ones), nor its many modifications that start from "common sense," from the inconsistency with quantum mechanical results that follow from experiments capable of demonstrating violations of Bell's theorem or the Greenberger–Horn–Zeilinger paradox, experiments that can be implemented in principle and that have already been partially implemented in practice (for $N=2$). The only argument that explains the contradictions that arise is the fact that in these cases there does not exist a positive-definite probability distribution function.

This paper does not claim to cover all the possible quantum effects or to clarify the problem of nonlocality in quantum theory as a whole. For example, we may treat the behavior of a single photon in a Mach–Zender interferometer as nonlocal in the sense that the photon simultaneously belongs to the two spatially separated modes (arms) of the interferometer. (See also discussions on this topic given in Ref. 5.) Nevertheless, the considerations presented here lead us to assert that neither the violations of Bell's theorem for $N=2$ that are already reliably recorded, nor the other Bell paradoxes that as yet exist only on paper in the form of thought experiments, are grounds for introducing inherently unexplainable nonlocalities into quantum mechanics or turning to mysticism to address the problem.

I am grateful to Yu. A. Il'inskii, G. Kh. Kitaeva, S. P. Kulik, A. N. Penin, and M. V. Chekhova for useful discussions of the results presented here, and also to the G. Soros International Scientific Fund and the Russian Fund for Fundamental Research, project code 93-02-14848, for their financial support.

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Translated by Frank J. Crowne