Non-Compton thermal emission of a spherically accreting plasma

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Compton cooling is treated from the standpoint of the electrons for optically thick transport of thermal bremsstrahlung in a fully ionized plasma falling freely into a supermassive black hole. Here $\dot{m}=\dot{M}c^2/L_E \gg 1$ is assumed, where $\dot{M}$ is the mass flux of the plasma and L_E is the Eddington emissivity. The cooling gives rise to an exponential falloff in the spectrum for $h\nu > m_e c^2/\dot{m}$. The Compton temperature of the resulting spectrum is also determined.

1. INTRODUCTION

Spherically symmetric accretion of interstellar material on a supermassive ($M_{BH}>10^6 M_\odot$, $r_g>3\cdot 10^{11}$ cm) black hole is the basis of the simplest models of the emission from active galactic nuclei and quasars[1,2] (for a general review see Ref. 3). In this connection it becomes important to study the nature of the thermal emission spectrum of the accreting plasma, which should make a considerable contribution to the total observed spectra. In fact, it may dominate in certain ranges. For the x-ray part of the continuous spectrum (0.2–100 keV), which is the subject of the present work, the principal factors determining the thermal emission are the bremsstrahlung of the fully ionized plasma and Compton scattering.

In the optically thin case the emission spectrum is determined by spatial superposition of the original spectra of the sources. For plasma falling freely (i.e., with a density $n_e \propto r^{-3/2}$) with an adiabatic temperature profile as a function of radius r,

$$T_e(r)=T_{max}\frac{r_{min}}{r}, \tag{1}$$

Meszaros[4] found the total (integrated over the volume $r>r_{min}$ in Newtonian geometry) bremsstrahlung spectrum. The low-frequency asymptotic form of the spectral emissivity took the form of a power law:

$$L_\nu \propto \nu^{-\alpha} \tag{2}$$

with index $\alpha=0.5$ determined as noted above by the density dependence of n_e and T_e, while the high-frequency part dropped off exponentially for $h\nu>T_{max}$, due to the absence of radiation sources for $r<r_{min}$. As Shapiro[5] has shown, the spectrum of optically thin radiation resulting from the accretion of interstellar gas on a black hole for $T_e(r_g)=1.5\cdot 10^9$ K has the same asymptotic behavior.[1)]

The spectrum of the radiation diffusing in a spherically accreting plasma was determined in the optically dense case[6] neglecting the Compton exchange of energy with electrons. The low-frequency asymptotic behavior of this spectrum is also dictated by the great radial variation of the source and hence agrees with the optically thin case. However, at high frequencies, due to adiabatic heating associated with the compression of the radiation itself the behavior (2) is again found with index $\alpha=2.0$. Because of the logarithmic divergence, this spectrum does not permit the Compton radiation temperature

$$T_{ph}=\int_0^\infty d\nu (h\nu)^2 L_\nu / 4\int_0^\infty d\nu h\nu L_\nu \tag{3}$$

to be determined with great accuracy; this temperature determines the heating of the accreting gas[7] and hence the possibility of self-consistent closure of the accretion flow model.

The aim of the present work is to improve on the radiation spectrum in the optically dense case, which allows for the necessary treatment of Compton cooling of the radiation coming from the electron component of the accreting plasma.

2. RADIATION TRANSPORT EQUATION

Consider spherically symmetric accretion of a fully ionized plasma, i.e., with local velocity $\mathbf{u}=-cv\mathbf{r}/r$ at a point $\mathbf{r}$ measured from the accretion center. Here c is the speed of light, so that v is a dimensionless variable. The radiation in the plasma can be described by the photon occupation number $n_{ph}(r,p,\mu)$ in momentum space $[pc=h\nu;\mu=\cos(\widehat{\mathbf{p},\mathbf{r}})]$,

$$\mathbf{p}=p(\sqrt{1-\mu^2}\cos\psi, \sqrt{1-\mu^2}\sin\psi, \mu).$$

The radiation propagation ψ depends on the local optical thickness τ with respect to Thompson scattering (with cross section σ_T) on the electron component of the plasma with density n_e. It is determined by

$$\tau=\int_r^\infty \sigma_T n_e dr .$$

Following Ref. 8 (see also Ref. 9), for $\tau \gg 1$ and $v \ll 1$ the spectral density

$$n(r,p)=\frac{1}{2}\int_{-1}^1 d\mu n_{ph}(r,p,\mu)$$

satisfies the diffusion equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}[r^2 H_r]+\frac{1}{p^2}\frac{\partial}{\partial p}[p^2 H_p]=k_{BS}(n_0-n). \tag{4}$$

The flux in configuration space

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$$H_r = -\frac{1}{3n_e\sigma_T}\frac{\partial n}{\partial r} - vn$$

describes photon diffusion from the accretion center and photons carried by the incoming material toward the accretion center. The flux

$$H_p = \left[\frac{p}{3r^2}\frac{\partial}{\partial r}(r^2 v)\right] n - \sigma_T n_e p^2\left(\frac{n}{m_e c} + \frac{T_e}{m_e c^2}\frac{\partial n}{\partial p}\right)$$

in momentum space includes adiabatic compression of the photons propagating in a medium with convergent flow (i.e., div $\mathbf{u} = -(c/r^2)(\partial/\partial r)(r^2 v) < 0$), and also includes terms associated with Compton cooling and heating. The first term in (4) describes bremsstrahlung and inverse bremsstrahlung. Here $n_0 = 1/(e^q - 1)$ is the Planck occupation number and the inverse bremsstrahlung coefficient[10] can be written in the form

$$k_{BS} = \kappa\frac{1-e^{-q}}{q^3}e^{q/2}K_0(q/2),$$

where $q = pc/T_e$ and $K_0(s)$ is a modified Bessel function of the second kind. Here we have written

$$\kappa = (1 + z^2\eta)n_p n_e \sigma_T\frac{(hc)^3(T_e E_H)^{1/2}}{2\pi^{5/2}T_e^4},$$

where E_H is the ionization energy of the hydrogen atom. In this expression it is assumed that the plasma consists of protons with density n_p, an admixture of heavy ions (charge state z, atomic weight A) with density ηn_p, and electrons with density $n_e = (1 + z\eta)n_p$. For interstellar gas of the composition usually assumed the main impurity is helium ($z = 2$, $A = 4$, $\eta = 0.1$). The proton density is determined by the continuity equation for steady flow with a given total mass flux $\dot{M}$:

$$4\pi(1 + A\eta)m_p n_p c v r^2 = \dot{M}.$$

In the case of free fall we have for the dimensionless quantity $\zeta = \sigma_T n_e r_g$ and optical depth τ

$$v^2 = \frac{r_g}{r}, \quad \zeta = \frac{\dot{m}}{2}\sqrt{\left(\frac{r_g}{r}\right)^3}, \quad \tau = \dot{m}\sqrt{\frac{r_g}{r}}. \tag{5}$$

Here we have introduced the dimensionless form of the mass flux $\dot{m} = \dot{M}c^2/L_E$ usually employed in the theory of accretion radiation in connection with the Eddington emissivity (the maximum value determined by Thompson scattering)

$$L_E = \frac{4\pi c r_g}{\sigma_T}\frac{(1 + A\eta)m_p c^2}{2(1 + z\eta)} = 1.49\cdot 10^{38}\frac{M_{BH}}{M_\odot}\ \text{erg/s}.$$

When the mass flux is determined by hydrodynamic accretion of interstellar gas from a region of photoionization equilibrium with density $n_\infty \sim 1$ cm and temperature $T_\infty \sim 1$ eV, then (see, e.g., Ref. 1)

$$\dot{m} = 0.56(1 + z\eta)n_\infty\sigma_T r_g\left\{\frac{(1 + A\eta)m_p c^2}{[2 + (z+1)\eta]T_\infty}\right\}^{3/2}$$
$$= 1.8\frac{M_{BH}}{10^6 M_\odot}\frac{n_\infty}{1\ \text{cm}^{-3}}\left(\frac{T_\infty}{1\ \text{eV}}\right)^{-3/2}.$$

The boundary conditions for Eq. (4) are naturally expressed by requiring that there be no extrinsic sources:

$$r^2 H_r = O(1) \quad \text{for } r \to \infty, \quad r^2 H_r = o(1) \quad \text{for } r \to 0,$$
$$p^2 H_p = o(1) \quad \text{for } p \to \infty \quad \text{and } p \to 0. \tag{6}$$

The spatial variation of $n(r,p)$ in the limit $h\nu \ll m_e c^2/\dot{m}$ is determined by the competition between diffusion and convection. Comparison of the corresponding flux terms in coordinate space in Eqs. (5) determines the characteristic scale of the problem, the so-called radiation trapping radius:

$$r_0 \sim r_t \equiv \frac{3}{2}\dot{m}r_g.$$

In the limit $h\nu \gg m_e c^2/\dot{m}$ it follows from a comparison of the diffusion term and the term describing Compton cooling in Eq. (4) that the characteristic scale depends on the frequency

$$r_0 \sim \frac{h\nu}{m_e c^2}\dot{m}^2 r_g.$$

When (1) is satisfied, we find by comparing the orders of magnitude of the adiabatic and Compton heating of photons in (4) that the latter is turned on at radii $r_C \sim r_t \Upsilon$, where the Compton parameter

$$\Upsilon = \frac{\dot{m}}{2}\frac{T_e(r_t)}{m_e c^2} \tag{7}$$

is on the order of the ratio of the Compton heating to the diffusive term in (4) at the trapping radius. Note that this parameter is related by $\Upsilon = 3/16\ Y_C(r_t)$ to the usual local nonrelativistic Compton parameter

$$Y_C = \frac{4T_e}{m_e c^2}\tau^2,$$

which is the fraction of the photon energy associated with scattering from the region with optical depth τ. In the present work we treat the case $\Upsilon \ll 1$, i.e., un-Comptonized radiation. Hence the Compton heating in the region $r \gg r_C$ corresponds to $O(\Upsilon)$ corrections for the spectral moments. As will be clear below in the solution we obtain [cf. Eq. (13)], the radiation sources within the trapping radius make only an exponentially small contribution to the spectrum for $r \gg r_t$. Consequently, including the Compton heating in the region $r \ll r_t$ yields corrections of order $O[\Upsilon^{-s}\exp(-1/\Upsilon)]$, where $s > 0$. This means that to lowest order in $\Upsilon \ll 1$ the Compton heating can be dropped. In contrast, the relative role of the Compton cooling does not depend on radius, and it dominates by a wide margin in the limit $h\nu \gg m_e c^2/\dot{m}$, so they cannot be omitted. Neglecting in addition the inverse bremsstrahlung, which is not important for this part of the spectrum, we find in the dimensionless variables

$$x = \frac{r_t}{r}, \quad y = \frac{pc}{T_e(r_t)}$$

a final equation for the spectral density of the un-Comptonized thermal emission of the accreting plasma:

$$x\frac{\partial^2 n}{\partial x^2}-(\gamma-1+x)\frac{\partial n}{\partial x}-\frac{\gamma-1}{3}y\frac{\partial n}{\partial y}+\frac{\Upsilon}{y^2}\frac{\partial}{\partial y}(y^4 n)=-f(x,y). \quad (8)$$

Here the adiabatic index is $\gamma=5/2$ and the source is

$$f(x,y)=\frac{Bx}{y}\exp\left(-\frac{y}{2x}\right)K_0\left(\frac{y}{2x}\right), \quad (9)$$

where we have introduced the constant

$$B=\left(\frac{2}{3}\right)^{3/2}\frac{\dot{m}^{1/2}(1+z^2\eta)}{8\pi^{5/2}(1+z\eta)}\left[\frac{m_e c^2}{T_e(r_t)}\right]^{7/2}\times\left(\frac{E_H}{m_e c^2}\right)^{1/2}\frac{(h/m_e c)^3}{\sigma_T r_g}.$$

3. SOLUTION OF THE RADIATION TRANSPORT EQUATION

Equation (8) can be solved by separation of variables. As was found in Ref. 6 (see also Ref. 11) the eigenfunctions of the operator in x on the left-hand side of (8) which satisfy the boundary condition (6) and the corresponding eigenvalues $\gamma+k$ ($k=0$, 1, 2, ...) are $x^\gamma L_k^\gamma(x)$, where L_k^γ are the generalized Laguerre polynomials. Consequently, expanding the solution of (8) as a power series

$$n(x,y)=x^\gamma\sum_{k=0}^{\infty}L_k^\gamma(x)u_k(y) \quad (10)$$

we find immediately from the orthogonality properties of the Laguerre polynomials that

$$(k+\gamma)u_k=\frac{\gamma-1}{3}y\frac{du_k}{dy}-\frac{\Upsilon}{y^2}\frac{d(y^4u_k)}{dy}=f_k, \quad (11)$$

where

$$f_k(y)=\frac{k!}{\Gamma(k+\gamma+1)}\int_0^\infty dx e^{-x}f(x,y)L_k^\gamma(x).$$

Equation (11) has a singular point (a node):

$$y_c=\frac{\gamma-1}{3\Upsilon}\equiv a,\quad u_c=\frac{f_k(a)}{\beta},\quad \beta=k+1+\frac{1-\gamma}{3}.$$

The presence of the singularity enables us to find a solution satisfying the two boundary conditions (6), even though the equation is first-order. This solution takes the form

$$u_k(y)=\int_0^\infty d\eta t^\beta \chi f_k(\eta), \quad (12)$$

where

$$t\equiv t(y,\eta)=\left|\frac{\eta}{y}\frac{y-a}{\eta-a}\right|^{3/(\gamma-1)}$$

$$\chi\equiv\chi(y,\eta,a)=\left(\frac{\eta}{y}\right)^4\frac{\theta(y-a)\theta(\eta-y)-\theta(a-y)\theta(y-\eta)}{\Upsilon\eta(\eta-a)}$$

$$\theta(s)=\begin{cases}1, & s\geqslant 0\\ 0, & s<0\end{cases}.$$

Substituting (12) in the series (10) and performing the summation in analogy to Ref. 6, we can obtain an integral representation for the complete solution (8). The principal term in the limit $x\to 0$ (i.e., for $r\to\infty$) takes the form

$$n(x,y)\approx\frac{x^\gamma}{\Gamma(\gamma+1)}\int_0^\infty\frac{d\eta\chi t^{(4-\gamma)/3}}{(1-t)^{\gamma+1}}\int_0^\infty d\xi\times\exp\left(-\frac{\xi}{1-t}\right)f(\xi,\eta). \quad (13)$$

This expression can also be obtained by using the Green's function of the original equation (4) for isothermal flow, which was found[11] by expanding in confluent hypergeometric functions. For this purpose it suffices to go to the limit $T_e\to 0$.

Using (9) and integrating with respect to ξ in (13), we obtain a final representation for the spectral density of the radiation at large radii:

$$n(x,y)\approx\frac{4Bx^{5/2}}{y^4\Gamma(7/2)}\lambda(y,a), \quad (14)$$

where

$$\lambda(y,a)=\int_{\varphi(1-a/y)}^{1}\frac{d\tau}{\sqrt{1-\tau^2}}g\left(\frac{\eta(\tau)}{1-\tau^2}\right).$$

Here we have also changed variables according to

$$\eta(\tau)=\frac{a\tau}{\tau+a/y-1}$$

and have introduced the functions

$$\varphi(s)=s\theta(s),\quad g(s)=s^3\frac{d^2}{ds^2}K_0^2(\sqrt{s}).$$

The dependence of (14) on y and $\Upsilon\equiv 1/2a$ is shown in Fig. 1.

Using the expression for B at the end of the previous section, we can write the spectral emissivity

$$L_\nu=4\pi r^2\cdot 4\pi p^2\cdot 2npc/h^3$$

in the form

$$L_\nu=\frac{1+z^2\eta}{1+A\eta}\frac{L_E}{m_p c^2}\left[\frac{E_H}{T_e(r_t)}\right]^{1/2}\dot{m}^2\lambda(y,a).$$

We give some asymptotic expressions for the function $\lambda(y,a)$ in the limit $a\gg 1$, which have the explicit form:

$$\begin{aligned}
&\lambda\approx\frac{3\pi^2}{16}\sqrt{y},\quad y\ll 1,\\
&\lambda\approx\frac{4(a-y)}{ay},\quad 1\ll y\lesssim a,\\
&\lambda=\frac{\pi}{4\sqrt{2}}\exp(-2\sqrt{a}),\quad y=a,\\
&\lambda\approx\frac{1}{12}\left(\frac{2\pi y^2}{a}\right)^{3/2}\exp\left(-2\sqrt{\frac{2}{a}y}\right),\quad y\gg a.
\end{aligned} \quad (15)$$

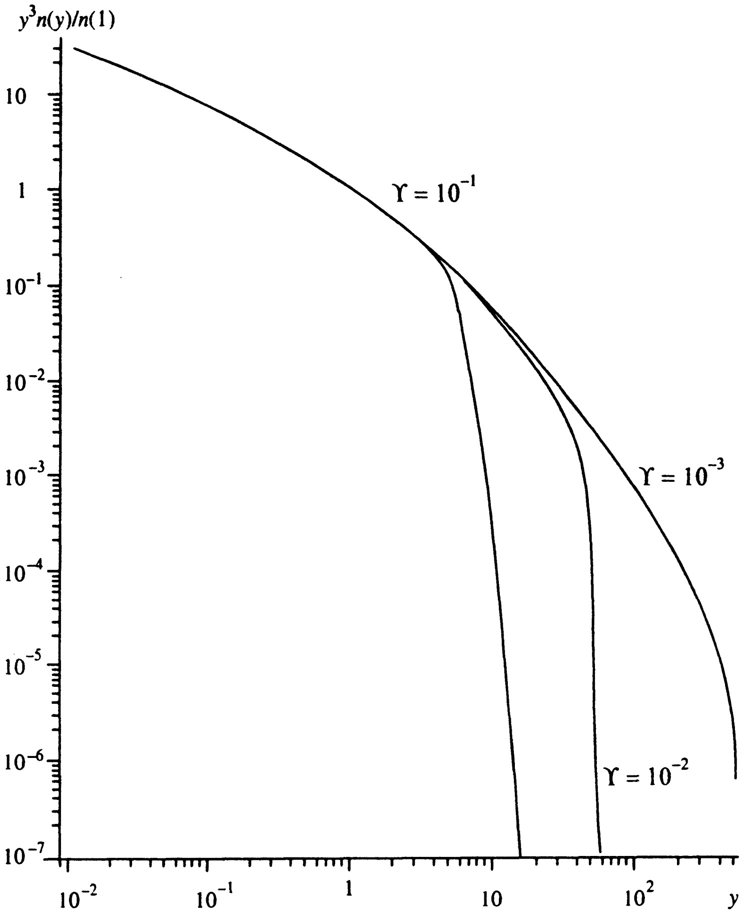

FIG. 1. Energy spectrum of thermal bremsstrahlung from a spherically accreting optically dense plasma. Here $n(y)$ is the photon spectral density (14), and we have written $y=h\nu/T_e(r_t)$, where $h\nu$ is the photon energy, $T_e(r_t)$ is the plasma temperature at the trapping radius r_t, and Υ is the Compton parameter.

The first asymptotic expression in (15) does not depend on the situation at the characteristic radius r_0. It arises because at a given photon energy $h\nu$ the effective contribution to the emissivity comes from sources at radius r satisfying the relation

$$T_e(r)\lesssim h\nu.$$

Hence for power-law profiles $n_e(r)$ and $T_e(r)$ the total power of the sources is an algebraic function of ν as well.[4] In the same way the second asymptotic form does not depend on the source and is determined by balancing the adiabatic heating and the Compton cooling at r_t. The third and fourth asymptotic forms are again consequences of the spatial variation of $T_e(r)$. They are determined by the radius at which the photon yield of the exponential tail of the source is largest, which follows from the relation

$$\frac{h\nu}{T_e(r)}\sim\frac{r_0(h\nu)}{r},$$

corresponding to the maximum contribution of the integrand in the integral with respect to ξ in Eq. (13), and also to the maximum of the function $g(s)$ in (14). Consequently, limiting ourselves to exponential accuracy we can write uniformly with respect to y in the region $y\geqslant a$ that

$$\lambda\sim\exp\left\{-\left[2a\cdot\min_{\tau}\left(\frac{\eta(\tau,y,a)}{1-\tau^2}\right)\right]^{1/2}\right\}.$$

For a comparison we give the asymptotic form for $q=h\nu/T_e(r_g)\gg 1$ of the spectrum found in Ref. 5 for optically thin radiation associated with accretion on a black hole:

$$L_\nu=0.09\,\frac{1+z^2\eta}{1+A\eta}\,\frac{L_E}{m_p c^2}\sqrt{\frac{E_H}{T_e(r_g)}}\,\frac{\dot m^2}{q^2}\times\exp\left(-\frac{3\sqrt{3}}{2}\,q\right).$$

It is immediately evident that this asymptotic form is determined by the impact parameter $3\sqrt{3}/2r_g$ for relativistic trapping of photons.[12]

The spectrum (14) enables us to determine the normalized moments

$$w_k(x)=\frac{\Gamma(7/2)}{4Bx^{5/2}}\int_0^\infty dy y^{2+k} n(x,y).$$

After some manipulations we can find the following representation:

$$w_k=\frac{a^{k+2}}{2}\,\frac{\partial^2}{\partial a^2}\int_0^\infty du u^{k-1}K_0^2(\sqrt{au})I_k(u),$$

where

$$I_k(u)=\int_1^\infty\frac{ds s^{(k-1)/2}}{[u\sqrt{s}-(u-s)\sqrt{s-1}]^k}.$$

The main terms in the asymptotic expansion in the limit $a \gg 1$ take the form

$$w_1 = \pi + \frac{4}{a}\left(\frac{\pi}{4} + \frac{2}{3} - 2C - \ln a\right) + O\left(\frac{\ln a}{a^2}\right),$$

$$w_2 = 4\left(\ln\frac{a}{2} + 2C - \frac{7}{3}\right) + O\left(\frac{\ln a}{a}\right),$$

where $C = 0.5772...$ is Euler's constant. From these expressions we can find to the same accuracy the total emissivity

$$L = \int_0^\infty d(h\nu) L_\nu$$

and the Compton temperature (3):

$$L = \frac{32}{3\pi}\frac{1+z^2\eta}{1+A\eta}\frac{L_E\sqrt{E_H T_e(r_t)}}{m_p c^2} \times \dot{m}^2\{1 + 2.55Y \ln(1760Y)\},$$

$$T_{ph} = \frac{w_2}{4w_1} T_e(r_t) \approx \left[0.32 \ln\frac{0.077}{Y}\right] T_e(r_t).$$

In the first expression we have also included the contribution $\sim Y$, which is required for the specified accuracy and is associated with Compton heating. It was calculated by ordinary perturbation theory, using the solution of the equation for the total emissivity at $Y=0$ given in Ref. 13. Both expressions can usefully be compared with the corresponding quantities for the optically thin spectrum found in Ref. 5:

$$L = 0.41\frac{1+z^2\eta}{1+A\eta}\frac{L_E\sqrt{T_e(r_g)E_H}}{m_p c^2}\dot{m}^2,$$

$$T_{ph} = 3.5\cdot 10^{-2} T_e(r_g).$$

4. DISCUSSION

The spectrum found in the previous section contains a free parameter Y and does not describe the accretion radiation self-consistently. However, it enables us to infer something about the spectral properties of this radiation. The most important thing to point out is that the spectral emissivity determined from Eq. (14) in the optically dense region (i.e., $r \ll \dot{m}^2 r_g$) is also found in the optically thin region to terms of order $O(1/\dot{m})$, since this is just the relative role of the adiabatic heating and the Compton cooling of photons (in the limit $h\nu \ll m_e c^2/\dot{m}$) in the transition region $r \sim \dot{m}^2 r_g$. Consequently, this spectral emissivity is observable. In order to apply the spectrum (14) to x radiation with characteristic energy $h\nu_X \simeq 10$ keV, the latter quantity should not exceed the exponential drop. Then the condition $\dot{m} \gg 1$ for the applicability of the diffusion approximation also determines the lower limit of the applicable values of Y. Including (15) we have

$$Y \gg \left[2\frac{h\nu_X}{m_e c^2}\right]^2 \approx 1.5\cdot 10^{-4}.$$

For magnitudes $Y \sim 10^{-3}$–10^{-2} the spectrum (14) in the region $h\nu \ll m_e c^2/\dot{m}$ has a slope

$$\alpha = -\frac{\partial \log L_\nu}{\partial \log \nu} \sim 0.7\text{–}2.0.$$

Including corrections associated with Compton heating and increasing as a function of Y and $\dot{m}$ [for fixed $T_e(r_t)$] should further flatten the spectrum.[11] The departure from the diffusive regime of radiation propagation that results when $\dot{m}$ and Y decrease [for fixed $T_e(r_t)$] should have the opposite effect. However, in the optically thin spectrum[5] a transition region with $\alpha \sim 0.7$–1.2 also exists over two to three decades. Hence it seems safe to conclude that these spectral indices are typical for thermal accretion radiation regardless of the mass flux $\dot{m}$.

In concluding this section it should be noted that spectral measurements of active galactic nuclei and quasars in the range 2–10 keV, which are usually approximated by functions of the form (2), give rise to spectral indices close to those obtained above. For example, observations by the satellite EXOSAT yield $\alpha = 0.89 \pm 0.06$ (Ref. 14), observations of "Ginga" yield $\alpha = 0.81 \pm 0.19$ (Ref. 15) for groups of objects. Similar data were obtained by ROSAT.[16] The spectra of Seyfert galaxies are closest to those treated in the present work. For example, the ROSAT data[17] yield a spectral index $\alpha = 1.38 \pm 0.25$. Spectra of a similar nature are discussed in Refs. 18 and 19. Another argument that the spectra of these objects have a thermal origin is the observation of an exponential dropoff.[20,21]

[1)]The spectrum is given graphically in Ref. 5. In Sec. 3 of the present work some analytical properties of this spectrum are presented without derivation under conditions formulated in Sec. 2 for the radiation source.

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