Magneto-elastic resonance in randomly inhomogeneous ferromagnets with zero-mean magnetostriction

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It is shown that the magneto-elastic resonance in disordered ferromagnets with fluctuating magnetostriction parameter can be used to observe the disorder-induced crossing resonance predicted in a recent paper by the authors.[11] The possibilities of measuring the rms fluctuation and the correlation radius of the magnetostriction inhomogeneity parameter in amorphous ferromagnetic alloys with zero-mean magnetostriction by observing the dispersion laws and the damping of elastic and spin waves are investigated.

1. INTRODUCTION

Amorphous ferromagnets are an example of a wide class of randomly inhomogeneous condensed media. For a theoretical description of such systems a long list of approaches have been developed, differing in the degree of detail of the disorder model as well as the degree of complexity of the implemented mathematical apparatus (see, e.g., Refs. 1 and 2). Some peculiarities of the physical phenomena caused by disorder can be quite completely described already on the phenomenological level. In such an approach all of the main characteristics of the material, such as, for example, the exchange parameter, anisotropy, magnetostriction, etc. are modeled by random (continuous or discrete) functions of the coordinates. The statistical characteristics of these random functions (mainly, the rms fluctuation and the correlation radius) figure in the theory as phenomenological parameters awaiting subsequent experimental determination. Taking this approach, we have studied the influence of inhomogeneities of various parameters on the dispersion laws and the attenuation of spin,[3,4] elastic,[5] and electromagnetic[6,7] waves; and also on the state[8] and low-temperature dependence of the magnetization[9] of ferromagnets.

In recent years amorphous magnetic materials with zero-mean magnetostriction constant have attracted special attention.[10] The present paper is dedicated to a study of the peculiarities of the magneto-elastic resonance in such systems. Interest in this problem is connected with two circumstances. First, for these materials, spatial inhomogeneities of the magnetostriction parameter play a fundamental role, where the main quantities characterizing the magnetostrictive properties of the material are the rms fluctuation and the correlation radius of these inhomogeneities. However, the methods that are available today to study such materials do not allow one to directly determine these quantities. We will show that a study of the dispersion laws and damping of magneto-elastic excitations can provide a direct method of measuring both of these characteristics.

Second, magneto-elastic oscillations in ferromagnets with almost zero-mean magnetostriction constant are a concrete physical realization of the model of disorder-induced crossing resonances that we considered in a recent paper.[11] In that paper we considered a simple model of linear-coupled scalar waves with random interaction parameters and showed that degeneracy removal in such a situation possesses a number of important properties in comparison with the well-known phenomenon of level repulsion in ordered systems. In the present paper we investigate to what extent the magneto-elastic resonance in randomly inhomogeneous ferromagnets can be described within the framework of the proposed model and consider under what conditions disorder-induced crossing-resonance effects, predicted in Ref. 11, can be experimentally detected in experiments studying the magneto-elastic resonance in randomly inhomogeneous ferromagnets.

2. FORMULATION OF THE MODEL AND DERIVATION OF THE DISPERSION RELATIONS

In order to emphasize the most important features of the phenomenon under study, let us consider the simple model of an isotropic ferromagnet whose energy density is

$$U=\frac{1}{2}\,\alpha(\nabla\mathbf{M})^2-\mathbf{HM}+\frac{1}{2}\,d_{ijkl}\,\frac{\partial u_i}{\partial x_j}\,\frac{\partial u_k}{\partial x_l}+\frac{1}{2}\,P(\mathbf{x})M_i(\mathbf{x})M_j(\mathbf{x})\,\frac{\partial u_i}{\partial x_j}. \quad (1)$$

Here the first and second terms are the exchange and Zeeman energies, the third term describes the elastic energy, and finally, the last term accounts for the magneto-elastic coupling. The magneto-elastic interaction parameter $P(\mathbf{x})$ is assumed to be an inhomogeneous random function of the coordinates, whereas the remaining parameters of the system are the exchange parameter α, the magnetization M_0, the density of the medium G, and the elastic force constants d_{ijkl} are assumed to be uniform. In addition, we neglect the magnetic-dipole interaction and the ponderomotive forces. Writing the equations of motion for the magnetic and elastic subsystems in the standard way, we obtain in the linear approximation

$$\frac{\partial m_x}{\partial t}=\alpha g M_0\,\frac{\partial^2 m_y}{\partial \mathbf{x}^2}-gHm_y-\frac{1}{2}\,gPM_0^2\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right),$$

$$\frac{\partial m_y}{\partial t}=-\alpha g M_0\,\frac{\partial^2 m_x}{\partial \mathbf{x}^2}+gHm_x+\frac{1}{2}\,gPM_0^2\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right),$$

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$$G\frac{\partial^2 u_x}{\partial t^2}=(d_1+d_2)\frac{\partial}{\partial x}\left(\frac{\partial u_k}{\partial x_k}\right)+d_1\frac{\partial^2 u_x}{\partial x_k \partial x_k}+\frac{1}{2}M_0\frac{\partial}{\partial z}(Pm_x), \tag{2}$$

$$G\frac{\partial^2 u_y}{\partial t^2}=(d_1+d_2)\frac{\partial}{\partial y}\left(\frac{\partial u_k}{\partial x_k}\right)+d_1\frac{\partial^2 u_y}{\partial x_k \partial x_k}+\frac{1}{2}M_0\frac{\partial}{\partial z}(Pm_y),$$

$$G\frac{\partial^2 u_z}{\partial t^2}=(d_1+d_2)\frac{\partial}{\partial z}\left(\frac{\partial u_k}{\partial x_k}\right)+d_1\frac{\partial^2 u_z}{\partial x_k \partial x_k}+\frac{1}{2}M_0\left[\frac{\partial}{\partial x}(Pm_x)+\frac{\partial}{\partial y}(Pm_y)\right].$$

Equations (2) are written in the coordinate system whose z axis is oriented in the equilibrium direction of the magnetization, parallel to the external magnetic field H. Here we have omitted terms describing the static deformation, taking the displacements $\mathbf{u}$ to be reckoned from the true equilibrium positions.

Our problem now is to obtain a closed system of equations for the displacements $\mathbf{u}$ averaged over the realizations of the random function $P(\mathbf{x})$. The condition for the existence of nontrivial solutions of these equations determines the dispersion relations for the various modes of the system. It is convenient to represent the magneto-elastic interaction parameter here in the form

$$P(\mathbf{x})=\langle P\rangle+\gamma\rho(\mathbf{x}), \tag{3}$$

where $\langle P\rangle$ is the mean value of this parameter, which we will assume to be equal to zero, γ is its rms fluctuation, and $\rho(\mathbf{x})$ is a normalized random function whose stochastic properties are characterized by its correlation function

$$K(\mathbf{r})=\langle\rho(\mathbf{x})\rho(\mathbf{x}+\mathbf{r})\rangle. \tag{4}$$

Carrying out the spatiotemporal Fourier transformation of Eqs. (2) and taking into account definition (3), we obtain the following system of integral equations:

$$(\omega-\varepsilon_k)m_+=\frac{i\gamma g M_0^2}{2}\left[\int q_z u_+(\mathbf{q})\rho(\mathbf{k}-\mathbf{q})d^3q+\int q_+ u_z(\mathbf{q})\rho(\mathbf{k}-\mathbf{q})d^3q\right],$$

$$(\omega+\varepsilon_k)m_-=-\frac{i\gamma g M_0^2}{2}\left[\int q_z u_-(\mathbf{q})\rho(\mathbf{k}-\mathbf{q})d^3q+\int q_- u_z(\mathbf{q})\rho(\mathbf{k}-\mathbf{q})d^3q\right],$$

$$[\omega^2-\omega_t^2(k)]u_+(k)=-\frac{\gamma M_0 k_z}{2G}\left[\cos^2\theta_k+\sin^2\theta_k\frac{\omega^2-\omega_t^2(k)}{\omega^2-\omega_l^2(k)}\right]\times\int m_+(q)\rho(\mathbf{k}-\mathbf{q})d^3q+\frac{\gamma M_0 k_z}{2G}\sin^2\theta_k\exp(2i\phi_k)\left[1-\frac{\omega^2-\omega_t^2(k)}{\omega^2-\omega_l^2(k)}\right]\times\int m_-(q)\rho(\mathbf{k}-\mathbf{q})d^3q, \tag{5}$$

$$[\omega^2-\omega_t^2(k)]u_-(k)=\frac{\gamma M_0 k_z}{2G}\left[\cos^2\theta_k+\sin^2\theta_k\frac{\omega^2-\omega_t^2(k)}{\omega^2-\omega_l^2(k)}\right]\times\int m_-(\mathbf{q})\rho(\mathbf{k}-\mathbf{q})d^3q-\frac{\gamma M_0 k_z}{2G}\sin^2\theta_k\exp(2i\phi_k)\left[1-\frac{\omega^2-\omega_t^2(k)}{\omega^2-\omega_l^2(k)}\right]\times\int m_+(\mathbf{q})\rho(\mathbf{k}-\mathbf{q})d^3q,$$

$$[\omega^2-\omega_l^2(k)]u_z(k)=-\frac{\gamma M_0 k_z}{4G}\left[\sin^2\theta_k\cos\theta_k-\sin\theta_k\cos 2\theta_k\frac{\omega^2-\omega_l^2(k)}{\omega^2-\omega_t^2(k)}\right]\times\left[\exp(-i\phi_k)\int m_+(\mathbf{q})\rho(\mathbf{k}-\mathbf{q})d^3q+\exp(i\phi_k)\int m_-(\mathbf{q})\rho(\mathbf{k}-\mathbf{q})d^3q\right].$$

Here $m_\pm=m_x\pm im_y$ are the right- and left-polarized transverse components of the magnetization, $u_\pm=u_x\pm iu_y$ are the corresponding combinations of the elastic displacements, and $q_\pm=q_x\pm iq_y$, $\rho(\mathbf{q})$ is the Fourier transform of the random function introduced in Eq. (3). The angles θ_k and ϕ_k define the directions of the wave vector $\mathbf{k}$ relative to the equilibrium magnetization $\mathbf{M}$ and in the plane perpendicular to $\mathbf{M}$, respectively. In addition, we have introduced generic dispersion laws for the spin waves $\varepsilon_k=\omega_0+\alpha g M_0 k^2$, $\omega_0=gH$, and for the transverse $\omega_t=v_t k$ and longitudinal $\omega_l=v_l k$ elastic waves, respectively.

In what follows, we limit ourselves to a consideration of the simplest case, namely that of waves propagating in the direction of equilibrium magnetization ($\theta_k=\phi_k=0$). In this case, as in a usual homogeneous magneto-elastic resonance, only transverse elastic waves of the same polarization interact with the spin waves. However, in the situation under consideration this interaction is realized only as a result of the integral terms introduced in Eqs. (5). The physical sense of these terms is that they describe the interaction of a coherent wave of one kind with fluctuation (scattered) waves of another kind. Thus, in the system under consideration the coherent components of the elastic and spin waves do not interact. This situation describes any disorder-induced crossing resonances[11] and is the main reason for the unusual properties of such systems.

After averaging over the realizations of the function $\rho(\mathbf{x})$ out to terms in γ^2 (this corresponds to allowing for the first nonvanishing correction in the expansion of the corresponding mass operators), we arrive at the following relations for the unknown frequency ω:

$$\omega^2-\omega_t^2(k)=\frac{\gamma^2 g M_0^3}{4G}k^2\int\frac{S(\mathbf{k}-\mathbf{q})}{\omega-\varepsilon(q)}d^3q, \tag{6}$$

$$\omega-\varepsilon(k)=\frac{\gamma^2 g M_0^3}{8G}\int q^2\left[\frac{\sin^2 2\theta_q}{\omega^2-\omega_l^2(q)}+\frac{\cos^2 2\theta_q+\cos^2\theta_q}{\omega^2-\omega_t^2(q)}\right]S(\mathbf{k}-\mathbf{q})d^3q, \tag{7}$$

where $S(\mathbf{k}-\mathbf{q})$ is the Fourier transform of the correlation function (4). The first of these equations describes excitations characterized by a nonzero value of the mean amplitude of the elastic displacements, while the mean value of the amplitude of the oscillations of the magnetization in this case is zero. In other words, one can say that the given equation corresponds to a quasi-elastic mode that is a coherent elastic wave accompanied by a "cloud" of scattered spin waves. For Eq. (7) the situation is reversed—the excitations described by that equation are characterized by magnetization fluctuations with nonzero mean and zero-mean elastic displacements. For this reason we can say, in the same sense as above, that this equation describes a quasi-spin mode consisting of a coherent spin wave and fluctuating elastic waves. Occasionally for simplicity we will refer to these modes as a coherent elastic wave or a coherent spin wave, depending on context. Questions associated with the limits of applicability of the current approximation are discussed in our recent paper,[11] where the corresponding inequalities can be found.

Further progress requires us to choose a specific form of the function $S(k)$ and calculate the corresponding integrals. We will stick with the commonly used form of this function

$$S(k)=\frac{k_c}{\pi^2}\frac{1}{(k^2+k_c^2)^2}, \tag{8}$$

which corresponds to an exponential correlation function with correlation radius $r_c=k_c^{-1}$. In order to avoid calculating the cumbersome integrals in Eq. (7), we consideration that equation only in the approximation $\omega_t=\omega_l$ (the scalar phonon approximation), which has no appreciable effect on the results relating to the properties of coherent spin waves. Taking this approximation into account, Eq. (7) can be rewritten in the form

$$\omega-\varepsilon(k)=\frac{\gamma^2 g M_0^3}{8G}\int\frac{q^2(1+\cos^2\theta_q)}{\omega^2-\omega_t^2(q)}S(\mathbf{k}-\mathbf{q})d^3q. \tag{9}$$

Calculating the corresponding integrals in Eqs. (6) and (9) using the form of $S(k)$ given in Eq. (8), we obtain dispersion relations for the coherent elastic and spin waves:

1) the elastic wave

$$\omega^2-\omega_p^2=\zeta\omega_M\frac{\omega_p^2}{\omega-\varepsilon_k-\kappa_m-2i\sqrt{(\omega-\omega_0)\kappa_m}},\quad \mathrm{Re}\,\omega>\omega_0,$$
$$\omega^2-\omega_p^2=\zeta\omega_M\frac{\omega_p^2}{\omega-\varepsilon_k-\kappa_m-2\sqrt{(\omega_0-\omega)\kappa_m}},\quad \mathrm{Re}\,\omega>\omega_0, \tag{10}$$

2) the spin wave

$$\omega-\varepsilon_k=\zeta\omega_M\left\{\frac{\omega_p^2+\kappa_p^2+2i\omega\kappa_p}{\omega^2-\omega_p^2-\kappa_p^2-2i\omega\kappa_p}-\frac{\kappa_p(\kappa_p-i\omega)}{2\omega_p^2}+\frac{\kappa_p}{4\omega_p^3}(\omega^2+\omega_p^2+\kappa_p^2)\left[\mathrm{arctg}\frac{2\omega_p\kappa_p}{\omega^2-\omega_p^2+\kappa_p^2}-\frac{i}{2}\ln\frac{(\omega+\omega_p)^2+\kappa_p^2}{(\omega-\omega_p)^2+\kappa_p^2}\right]\right\}. \tag{11}$$

Here we have introduced the dimensionless parameter

$$\zeta=\frac{\gamma^2 M_0^2}{Gv^2}=16(\Delta\lambda_s)^2\frac{Gv^2}{M_0^2}, \tag{12}$$

characterizing the magnitude of the magneto-elastic interaction. The first equality expresses ζ in terms of the rms fluctuation of the magneto-elastic parameter P; the second, in terms of the rms fluctuation of the magnetostriction parameter λ_s. We have also introduced the notation

$$\kappa_m=\omega_M\alpha k_c^2,\quad \kappa_p=vk_c \tag{13}$$

and $\omega_M=gM_0$. The parameters κ_m and κ_p characterize the relaxation properties of the fluctuating spin and elastic waves, respectively.

Following Ref. 11, we rewrite the dispersion relation (10) for the elastic waves in the form

$$(\omega-vk)(\omega-\varepsilon_k-i\Gamma_m)=\Lambda^2/4, \tag{14}$$

where

$$\Lambda^2=\zeta\omega_M\frac{4\omega_p^2}{\omega+\omega_p},\quad \Gamma_m=2\sqrt{(\omega-\omega_0)\kappa_m}. \tag{15}$$

Here we have introduced the crossing-resonance frequency ω_r, defined by the equation

$$\omega_0+\omega_M\alpha k^2+\kappa_p=vk. \tag{16}$$

Equation (14) has two solutions, the physical meaning and behavior of which are determined, according to Ref. 11, by a relation between the parameters Λ and Γ_m at the resonance point. If $\Lambda_r<\Gamma_{m,r}$, then one of the solutions is the dispersion law of the weakly damped elastic wave, modified in the vicinity of the point defined by Eq. (16). Here the magnitude of the modification is of the order of ζ. In this case, the second solution does not correspond to any propa-

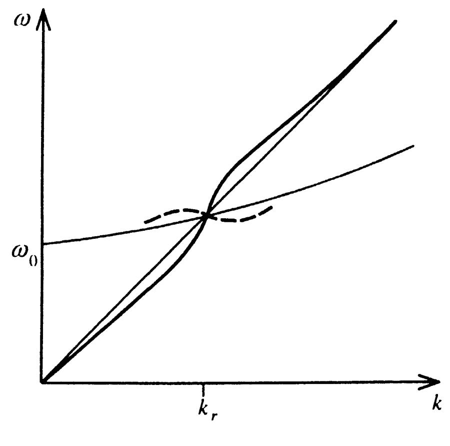

FIG. 1. Qualitative behavior of the solutions of the dispersion relation for the coherent elastic mode in the case of a closed gap.

gating mode, but is a short-lived oscillation whose wave vector is poorly defined, and, consequently, it cannot be characterized by a dispersion law. The behavior of the solutions considered in the opposite case $\Lambda_r > \Gamma_{r,m}$ is illustrated in Fig. 1. The solid curve depicts the solution that far from resonance corresponds to the dispersion law of the elastic waves. It was shown in Ref. 11 that in this case in the vicinity of the resonance the second solution of Eq. (15) can be considered within the framework of the current approximation, and also corresponds to some propagating mode (the dashed curve in Fig. 2). As one moves away from the resonance point, the rate of damping of this mode increases, so that finally it becomes poorly defined. These two solutions describe a new—stochastic—type of magneto-elastic oscillation. Its distinguishing feature is that on both branches of the foregoing dispersion curve the amplitude of only the elastic oscillations is different from zero, i.e., both of these branches correspond to the same quasi-elastic type of oscillation, as explained above.

The gap Δ arising in the case under consideration between the solutions of Eq. (11) at the resonance point is equal to

$$\Delta = \sqrt{\Lambda_r^2 - \Gamma_{m,r}^2}, \tag{17}$$

where

$$\Lambda_r = 2\zeta\omega_M\omega_r, \quad \Gamma_{m,r} = 2\omega_M\alpha k_r k_c. \tag{18}$$

It should be noted, however, that in the case under consideration the term κ_p in the definition of the resonance frequency [Eq. (16)] should be omitted, since taking it into account exceeds the accuracy of the method.

The frequencies of coupled oscillations in the vicinity of the resonance are given by the same expression as the frequencies of ordinary magneto-elastic oscillations in a homogeneous medium[12] with allowance for the fact that the gap Δ now is defined by expression (17), which takes account of the presence of an effective relaxation in the system. This circumstance leads to the result that in the case under consideration the oscillations possess finite lifetimes, which are characterized by the corresponding relaxation parameters $\xi_\pm$, where the "+" sign corresponds to the upper branch of the dispersion curve and the "−" sign corresponds to the lower. In the vicinity of the resonance, this parameter is given by

$$\xi_\pm \simeq \frac{\Gamma_{m,r}}{2}\left(1 \pm \frac{|\omega_p - \varepsilon|}{\Delta}\right). \tag{19}$$

The rate of relaxation of $\xi_\pm$ depends both on the wave number k and the magnitude of the external magnetic field H. These two dependences, with their second argument fixed, are similar, and in the vicinity of the resonance have the form of a decaying (for the lower branch) and a growing (for the upper branch) linear function with separation from the resonance point in either direction. If we vary the magnetic field and the wave number in such a way that the system remains at the crossing resonance, then the resonance value of the relaxation parameter (identical for the two solutions of the dispersion relation) depends on the magnetic field in the following way:

$$\xi_r \simeq \omega_M\alpha k_c gH/v. \tag{20}$$

To conclude this section, let us discuss the conditions under which it is possible to observe the appearance of the gap (17) in the dispersion curve of the coherent elastic waves. The condition for the existence of a gap follows from Eq. (17):

$$\omega_r < \zeta v^2 r_c^2 / 2\omega_m\alpha^2, \tag{21}$$

which bounds the maximum value of ω_r (and, consequently, the magnitude of the magnetic field H) at which the gap exists. Relation (21) contains both parameters that have roughly the same values for most materials investigated (such as $G \sim 10$ g/cm^3, $v \sim 3\cdot10^5$ cm/s, $\alpha \sim 10^{-12}$ cm^2) and parameters that can very significantly, depending on chemical composition, conditions of preparation, etc. (such as $M_0 \sim 10^2$–10^3 G, $\Delta\lambda_s \sim 10^{-6}$–$10^{-5}$, $r_c \sim 10^{-7}$–10^{-5} cm). Estimates show that for cobalt-rich ferromagnetic alloys with almost zero-mean magnetostriction, for which $\Delta\lambda_s \sim 10^{-6}$ (Ref. 9), condition (21) can be satisfied only for $r_c \gtrsim 10^{-5}$ cm. The actual value of r_c in these materials is unknown;

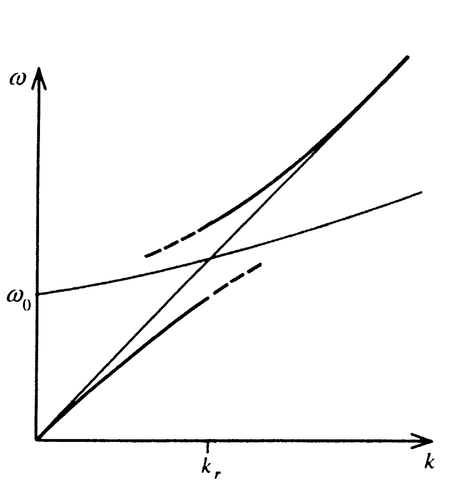

FIG. 2. Two branches of the solution of the dispersion relation of the quasi-elastic mode for a closed gap.

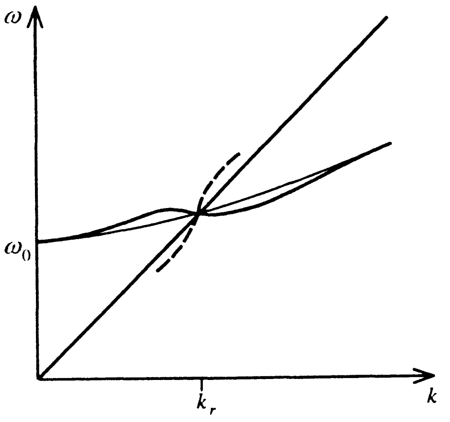

FIG. 3. Qualitative form of the modification of the spin-wave dispersion law.

however, it seems unlikely that these materials have such high values of this parameter, and we are led to the conclusion that such materials cannot be used to observe the effect under discussion. To this end, it is necessary to create special alloys with large values of the magnetostriction ($\sim 10^{-5}$) and specially create large-scale inhomogeneities in them. Such values of the magnetostriction parameter are typical of pure ferromagnets, so creation of materials with the required properties is a completely solvable problem.

In obtaining the derived estimates, we did not take account of inhomogeneities of the density of the material or the elastic moduli, the exchange parameter, or the magnetization, all of which are internal parameters of the elastic and spin subsystems, respectively. The influence of these inhomogeneities on the size of the gap and the possibility of its existence reduce, for the most part, to the appearance of an additional contribution to the effective relaxation parameters Γ, which leads to a decrease in the size of the gap (of course given sufficiently weak inhomogeneities, which do not lead to the appearance in the vicinity of the resonance of fundamentally new localization effects). The remaining relaxation mechanism in the system will play a similar role, and the most important contribution will come, as a rule, from the relaxation of the spin subsystem. Simple estimates show that the quantity Γ_m, which determines the efficiency of the relaxation mechanism considered in this paper, is of an order of magnitude typical of spin relaxation, so that taking it into account does not have a significant effect on the character of the derived estimates.

As for the dispersion law of the coherent spin waves, a similar analysis shows that in this case the appearance of a gap is hardly possible for any real materials, and the dispersion law of the spin waves will undergo only a weak modification of the type depicted in Fig. 3.

Thus, it follows from the derived estimates that in specially created ferromagnetic alloys possessing large enough values of the local magnetostriction parameter, it is possible, in principle, to observe one of the most important effects predicted in Ref. 11, namely, the situation in which in the dispersion law of one of the interacting waves the degeneracy removal effect is manifested due to random interaction (Fig. 2) at the same time that the dispersion curve of the second wave undergoes only an insignificant modification in the vicinity of the resonance (Fig. 3).

3. DISPERSION CURVES AND ATTENUATION OF ELASTIC AND SPIN WAVES IN THE CASE OF A CLOSED GAP

In this section we consider in more detail the properties of elastic and spin waves in the case $\Lambda_r < \Gamma_r$, which, as follows from the foregoing estimates, is realized in standard amorphous ferromagnets with zero-mean magnetostriction. The main task before us is to single out those aspects of the dispersion and relaxation properties of elastic and spin waves whose experimental observation should allow us to determine the main stochastic characteristics of the inhomogeneities of the magnetostriction parameter in such materials, i.e., the rms fluctuation and the correlation radius.

In the situation under consideration, as was already pointed out, only those solutions of the dispersion relations (10) and (11) have physical meaning which transform into generic elastic- and spin-wave dispersion laws as the interaction parameter tends to zero. Solving Eq. (10) with the help of perturbation theory, we find a dispersion law and damping of the elastic waves, respectively, in the form

$$\omega_p = vk\left[1 + \frac{1}{2}\zeta\omega_M \times \frac{vk - \varepsilon_k - \kappa_m + 2\theta(\omega_0 - vk)\sqrt{(\omega_0 - vk)\kappa_m}}{(vk - \varepsilon_k - \kappa_m)^2 + 4(vk - \omega_0)\kappa_m}\right], \tag{22}$$

$$\xi_p(k) = \zeta\omega_M vk\,\frac{\theta(vk - \omega_0)\sqrt{(vk - \omega_0)\kappa_m}}{(vk - \varepsilon_k - \kappa_m)^2 + 4(vk - \omega_0)\kappa_m}, \tag{23}$$

where $\theta(x)$ is the unit step function.

The most interesting effects to emerge from Eqs. (22) and (23) are connected with the dependence of the velocity and damping of the elastic waves on the magnetic field, which enters into these equations in the form of the combination $vk - gH_0$. Therefore, if we neglect the weak k-dependence of the generic spin-wave dispersion law for sufficiently long waves, it can be seen that the functions $\omega_p(k)$ and $\omega_p(H)$ for constant H and k, respectively, are identical except for one difference, namely that an increase in k corresponds to a decrease in H, and vice versa. Thus, the first and simplest conclusion that we obtain is that in this situation it is possible to measure the modified dispersion law of the elastic waves by examining the dependence of their frequency on the magnetic field at some fixed wave number. A more detailed discussion of the behavior of the dispersion curve for various k and H makes sense only when considering a specific experimental situation. Therefore, we limit ourselves in this paper to an analysis of the simplest case of the longest waves, and consider the relative variation of the speed of sound δ_v in comparison with its generic value $\delta_v = (v_p - v)/v$ in the limit $k \to 0$. From Eq. (22) we find that

$$\delta_v=-\frac{1}{2}\zeta\frac{M_0}{[\sqrt{H}+\sqrt{H_c}]^2}, \qquad (24)$$

where $H_c=\alpha M_0 k_c^2$. From this expression it is clear that the dependence $\delta_v(H)$ changes over from the law $\delta_v \propto$const to the law $\delta_v \propto H^{-1}$ at $H \sim H_c$, which can be called the correlation field of the fluctuations of the magnetostriction parameter (see the analogous expression for the fluctuations of the magnetic anisotropy in Ref. 7). By measuring the value of H_c we can find the correlation radius of the inhomogeneities of the magnetostriction parameter $r_c \sim \sqrt{\alpha M_0/H_c}$. Taking as our estimates of the parameters α and M_0 the values $\alpha \sim 10^{-12}$ cm^2 and $M_0 \sim 10^2$ Hz, we find that to measure values of r_c of the order of $\sim 10^{-7}$ cm requires fields $H \sim 10^2$ Oe. The magnitude of the effect can be estimated by using the value of the parameter $\Delta\lambda_s$ given in Ref. 9. For the same values of the parameters as above, we obtain $\delta_v \sim 10^{-4}-10^{-5}$.

It should also be noted that despite that fact that expression (24) was obtained using a particular form of $S(k)$, the character of the asymptotic behavior of δ_v for $H \ll H_c$ and $H \gg H_c$ does not depend on the details of the behavior of this function. Moreover, in the latter case even the value of the coefficient of the term $1/H$ does not depend on the form of the spectral density, which may allow us, in principle, to use this asymptotic form to find the absolute value of the parameter ζ. In order to demonstrate the truth of the latter assertion, we write down the expression for δ_v for an arbitrary spectral density $S(k)$:

$$\delta_v=-\frac{1}{2}\zeta M_0\int\frac{S(q)}{H+\alpha M_0 q^2}\,d^3q.$$

It is easy to see that the integral in this expression does not depend on the magnetic field for small values of the field and behaves like $1/H$ for large fields, independent of the form of the function $S(q)$ if, of course, the latter does not fall off too slowly at infinity and does not have any singularities at zero. In this case it is obvious that the integral remaining for the case $H \gg H_c$ is simply equal to unity for any normalized spectral density.

It goes without saying, even apart from what we have said so far, that there are other mechanisms for modifying the dispersion law of the elastic waves and, especially, of renormalizing their velocity. Such mechanisms are associated, for example, with scattering off inhomogeneities in the density or the elastic moduli.[5] If these inhomogeneities are assumed to be uncorrelated with the fluctuations of the magnetoelastic parameter, then in the current approximation their contribution enters into the dispersion law in the form of an additive term. By virtue of the fact that the dispersion law does not contain any kind of dependence on the magnetic field, expression (24) will not change if by the generic quantity v we understand the speed of sound renormalized to this correction. A more substantial role is played in the behavior of the damping of the elastic waves by the additional sources of modification of the dispersion laws and the relaxation. We take account of their influence in a purely phenomenological way by introducing the corresponding relaxation parameters $\Gamma_{p,i}$ and $\Gamma_{m,i}$. It can be taken for granted that these parameters include the relaxation mechanisms associated with scattering off the inhomogeneities of the remaining parameters of the material as well as the remaining relaxation mechanisms which are internal for each subsystem. Here the first of these parameters corresponds to relaxation of the elastic subsystem, and the second takes into account internal relaxation of the spin excitations. Taking the parameters $\Gamma_{p,i}$ and $\Gamma_{m,i}$ into account, expression (23) for the damping can be rewritten in the form

$$\xi_p(k)=\Gamma_{p,i}+\zeta\omega_M vk\,\frac{\Gamma_{m,i}+\theta(vk-\omega_0)\sqrt{(vk-\omega_0)\kappa_m}}{(vk-\varepsilon_k-\kappa_m)^2+4(vk-\omega_0)\kappa_m}. \qquad (25)$$

The intrinsic elastic relaxation enters into this expression as an additive term and does not depend on the magnetic field. In addition, it is quite small as a rule, and its dependence on the wave number differs significantly from that of the second term in expression (25). Thanks to all these circumstances, its presence does not hinder us in any significant way from separating out the interesting contribution associated with scattering off the inhomogeneities of the magnetostriction parameter. A more important role is played by the contribution associated with the intrinsic magnetic relaxation. It is transferred into the elastic subsystem, thanks to the interaction and leads to the same characteristic resonance dependence on the wave number as the contribution associated with the fluctuations of the interaction constant. Nevertheless, separating it out from the total damping is possible, thanks to the characteristic behavior

$$\xi_p \sim \sqrt{vk-\omega_0}\,\theta(vk-\omega_0)$$

as $k \rightarrow \omega_0/v$. Such behavior leads to a singularity in the overall damping at $k=\omega_0/v$, the observation of which can allow us, in principle, to isolate this interesting contribution. An additional circumstance facilitating this task is the dependence of the position of this singularity on the magnetic field. In our subsequent analysis we will be interested only in that part of the damping due to scattering of the elastic waves off the fluctuations of the magneto-elastic coupling parameter. This contribution to the damping takes its largest value at the resonance point, defined by Eq. (16), and is given by the following expression:

$$\xi_p^{res}=\frac{1}{4}\zeta\frac{vk}{\alpha k_c\sqrt{k_r^2+k_c^2}}. \qquad (26)$$

The resonant damping depends on the magnitude of the magnetic field since the resonant wave number k_r is a function of H. This dependence experiences a characteristic changeover at $k_r \sim k_c$. Therefore, measuring it also provides us with the possibility, in principle, of determining the magnitude of r_c. Realization of such a possibility depends on the wavenumber region in which the unknown value k_c lies, and is limited by the attainable values of the magnetic field. In reality, such a method can be used to measure correlation radii of the order of 10^{-6} cm or greater.

We begin our consideration of the properties of spin waves with a study of their relaxation characteristics. The corresponding damping parameter ξ_m can be found from Eq. (11) by perturbation theory and has the form

$$\xi_m=\frac{1}{2}\zeta\omega_M\kappa_p\varepsilon_k\left\{\frac{\varepsilon_k^2}{(\varepsilon_k^2-\omega_k^2-\kappa^2)^2+4\varepsilon_k^2\kappa_p^2}+\frac{1}{4\omega_k^2}\right.$$
$$\left.\times\left[1-\frac{\varepsilon_k^2+\omega_k^2+\kappa_p^2}{4\omega_k\varepsilon_k}\ln\frac{(\varepsilon_k+\omega_k)^2+\kappa_p^2}{(\varepsilon_k-\omega_k)^2+\kappa_p^2}\right]\right\}. \quad (27)$$

For $k\ll k_r$ the quantity ξ_m tends to a constant value,

$$\xi_m(0)=\frac{1}{3}\zeta\ \omega_M\frac{\kappa_p\omega_0^3}{(\omega_0^2+\kappa_p^2)^2}. \quad (28)$$

It is interesting to note that this expression can be rewritten in the form

$$\xi_m=\frac{\pi^2}{3}\zeta\ \omega_M\frac{\omega_0^3}{v^3}S\left(\frac{\omega_0}{v}\right), \quad (29)$$

where $S(k)$ is the spectral density of the inhomogeneities of the magneto-elastic parameter, introduced in Eq. (8). It is not hard to show that in the form (29) the expression for the spin-wave damping at $k=0$ is valid for functions $S(k)$ of arbitrary form. An analogous situation arises, for example, when considering the damping of electromagnetic waves due to their random interaction with elastic waves.[13] An expression analogous to expression (29) is also obtained in the limiting case $k\gg k_r$:

$$\xi_m=\frac{\pi^2}{3}\zeta\ \omega_M\frac{\varepsilon_k^3}{v^3}S(k), \quad (30)$$

where, however, the wave number k serves as the argument of the spectral density function, not the quantity ω_0/v. Both expressions, (29) and (30), can be used to measure the correlation radius r_c; however, whereas the first of these is more suitable for investigating inhomogeneities with characteristic dimensions greater than 10^{-5} cm (the main limitation here are connected with the magnitude of the attainable magnetic fields), the second is suitable only in the case $k_c\gg k_r$, since the validity of formula (30 is limited by the condition $vk\gg\varepsilon_k$.

4. CONCLUSION

The present study has pursued two main goals: to investigate the possibility of using magneto-elastic resonance in random ferromagnets with zero-mean magnetostriction to experimentally examine the phenomenon of disorder-induced crossing resonances predicted in Ref. 11, and, in addition, to elucidate the feasibility of experimental observation of the spectral and relaxation characteristics of elastic and spin waves in order to obtain information about the main stochastic characteristics of the magnetostriction parameter in such materials.

Our estimates show that to observe the effects predicted in Ref. 11, it is necessary to create special materials, the main requirements on which are to achieve as large a value as possible of the local magneotostriction and to make its spatial distribution as smooth as possible. For this purpose one can use alloys of high-magnetostrictive components, which are characterized by identical signs of the magnetostriction parameter so that complete compensation should take place, on average. In such a case, the more it is possible to fulfill the requirements on the local magnetostriction parameter, the easier will be the requirements on its spatial distribution and conversely, the larger-scale the inhomogeneities it is possible to create, the smaller the local value of this parameter can be. From our estimates it follows that the problem as posed can be solved if the local magnetostriction is of the order of magnitude characteristic of pure ferromagnets of iron or nickel type, and the characteristic dimension of the inhomogeneities is of the order of 10^{-6} cm.

To answer the second question posed in this paper, we have carried out a detailed study of the dispersion law and damping of elastic and spin waves for the case in which no gap appears at the crossing resonance, since in standard amorphous ferromagnets with zero-mean magnetostriction we are dealing specifically with such a situation. We showed that an experimental study of the properties of elastic and spin waves in such materials should make it possible in principle to determine both the rms fluctuation and the characteristic dimension of the inhomogeneities of the magnetostriction parameter. One of the most attractive of these possibilities is the potential to investigate the dependence of the speed of ultrasound on the magnetic field, which has a singularity at some characteristic value of H, whose magnitude is determined by the correlation radius of the inhomogeneities. In addition, analogous information can be extracted from the dependence of the damping of the elastic and spin waves on the magnetic field, and from the k-dependence of the damping of the latter in the region of wave numbers much greater than k_r. Here it is important to note that each of these methods is effective over some range in the unknown parameter r_c, so they can be used to complement each other. The main difficulty in the study of relaxation characteristics is isolating the interesting effects from the total relaxation picture, in which other mechanisms participate as well. In this paper we have discussed this question and we conclude that such an isolation is possible in principle.

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