Resonant transition radiation in a magnetic field

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We study the resonant transition radiation emitted by fast particles in magnetized plasma with random variations in the electron density. Transition radiation spectra of the ordinary and extraordinary waves are calculated for different magnetic fields in the weak gyrotropic limit. The peak shape is shown to depend strongly on the magnetic field strength. The polarization of the radiation is shown to correspond to preferential emission of ordinary waves. We also analyze the role of anisotropy of the emitting particles. The transition radiation spectra generated by an ensemble of charges with a power-law momentum distribution are calculated. Finally, we discuss the important role of the results in applications. © *1995 American Institute of Physics.*

1. INTRODUCTION

The transition radiation emitted by fast charged particles of arbitrary energies moving in a plasma without a magnetic field has been studied in Ref. 1. The spectrum of this radiation was found to exhibit a high, narrow peak near the plasma frequency ω_p, thus describing the phenomenon of resonance transition radiation. The origin of this phenomenon is related to the strong increase in the plasma polarizability in the vicinity of resonance frequencies. The total energy emitted by such radiation is high enough to manifest itself in natural conditions. However, in objects with conditions for emission of the resonant transition radiation (the local interstellar medium,[2], the solar atmosphere, the Earth's ionosphere, etc.) there are magnetic fields that have a strong effect on the motion of particles, dispersion of the medium, and emission of electromagnetic waves.

The effect of the curvature of the path of a fast particle on the transition radiation has been studied in Ref. 3, where it was found that for

$$\frac{\omega_p}{\omega_B} \gg 1, \tag{1}$$

where $\omega_B = eB/mc$ is the electron gyration frequency, the path can be considered a straight line when $\omega < \omega_p(\omega_p\gamma/\omega_B)^{1/2}$, where γ is the Lorentz-factor of the particle. This is true, for one thing, near the plasma frequency. In this paper we restrict our discussion to weak fields (Eq. (1)) and ignore the curvature of the particle's path but take into account the effect of the magnetic field on the plasma's dispersion.

2. THE SPECTRAL–ANGULAR DISTRIBUTION OF RESONANT TRANSITION RADIATION IN A MAGNETIC FIELD

When $\omega \sim \omega_p$, the phase velocities of electromagnetic waves are much higher than the velocity of light (and, consequently, the velocity of the fast particle). Hence we can use, as we did in Ref. 1, the nonrelativistic approximation for particles of arbitrary energy. The general formula for the intensity of the resonant transition radiation in a magnetic field must take into account the presence of ordinary ($\sigma = o$) and extraordinary ($\sigma = x$) waves in a magnetized plasma with distinct refractive indices n_σ and the effect of the magnetic field on the longitudinal dielectric constant of the plasma:

$$\varepsilon(\omega,\mathbf{k}-\mathbf{k}') = \varepsilon(\omega) - 3(\mathbf{k}-\mathbf{k}')^2 d^2 - \frac{\omega_p^2\omega_B^2}{\omega^4}\sin^2\theta + i\varepsilon'', \tag{2}$$

where $\delta = v_T/\omega_p$ is the Debye radius, and θ is the angle between the magnetic field vector $\mathbf{B}$ and the vector $\mathbf{k}-\mathbf{k}'$. Thus, the intensity of the resonant transition radiation in one of the normal modes can be written as

$$I^{R,\sigma}_{\mathbf{n},\omega} = \frac{4\pi e^4 q^2 n_\sigma}{m^2c^3}\int d^3k' \frac{[\mathbf{n}\mathbf{k}']^2\delta[\omega-(\mathbf{k}-\mathbf{k}')\mathbf{v}]|\delta N|^2_{\mathbf{k}'}}{(\mathbf{k}-\mathbf{k})^4\{[\varepsilon(\omega)-3(\mathbf{k}-\mathbf{k}')^2d^2-(\omega_p^2\omega_B^2/\omega^4)\sin^2\theta]^2+\varepsilon''^2\}}, \tag{3}$$

where q and $\mathbf{v}$ are the charge and velocity of the emitting particle, $\mathbf{n}$ is the unit vector pointing in the direction of wave propagation, m and e are the electron mass and charge, $|\delta N|^2_{\mathbf{k}'}$ is the spectrum of the density inhomogeneities in the plasma, and ε'' is the imaginary part of the dielectric constant. For an arbitrary mutual orientations of vectors $\mathbf{B}$, $\mathbf{v}$, and $\mathbf{n}$, the integrand in (3) depends on the azimuthal angle $\varphi_{\mathbf{k}'}$. Unfortunately, this fact makes it impossible to complete

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the integration explicitly. Hence we calculate the resonant transition radiation of an ensemble of particles with an anisotropic distribution function of the form

$$f(\mathbf{p})=f(p,\theta_v), \tag{4}$$

which depends on the polar angle θ_v between the magnetic field $\mathbf{B}$ and the velocity $\mathbf{v}$ but not on the azimuthal angle φ_v. Allowing for an anisotropy of this type is sufficient for most astrophysical applications. The resonant transition radiation intensity for an ensemble of particles can be represented as follows:

$$P^{R,\sigma}_{\mathbf{n},\omega}=\frac{4\pi e^4q^2n_\sigma}{m^2c^3}\int d^3p f(p,\theta_v)\int d^3k' \times\frac{[\mathbf{n}\mathbf{k}']^2\delta\,[\omega-(\mathbf{k}-\mathbf{k}')\mathbf{v}]|\delta N|^2_{\mathbf{k}'}}{(\mathbf{k}-\mathbf{k}')^4\{[\varepsilon(\omega)-3(\mathbf{k}-\mathbf{k}')^2d^2-(\omega_p^2\omega_B^2/\omega^4)\sin^2\theta]^2+\varepsilon''^2\}} \tag{5}$$

We now write the distribution function of the particles in the form of a series in Legendre polynomials:

$$f(p,\theta_v)=\frac{1}{4\pi}\sum_{l=0}^{\infty}F_l(p)P_l(\cos\theta_v). \tag{6}$$

First we evaluate the integral over the angles of the velocity (momentum) vector. To this end we select the direction of vector $\mathbf{k}'$ as the z axis and express $P_l(\cos\theta_v)$ in terms of the angles θ and $\theta_{\mathbf{k}'\mathbf{v}}$ (the latter is the angle between the vectors $\mathbf{v}$ and $\mathbf{k}'$) according to the well-known addition formula:[4]

$$P_l(\cos\theta_v)=\sum_{m=-l}^{l}P_l^{|m|}(\cos\theta_{\mathbf{k}'\mathbf{v}})P_l^{|m|}(\cos\theta)e^{im\varphi_v}. \tag{7}$$

The integrals with respect to the azimuthal angle φ_v are nonzero only for terms with $m=0$, and the integration with respect to $\cos\theta_v$ can be done trivially using the delta function. As a result we obtain

$$P^{R,\sigma}_{\mathbf{n},\omega}=\frac{2\pi e^4q^2n_\sigma}{m^2c^3}\sum_{l=0}^{\infty}\int\frac{p^2F_l(p)dp}{v}\times\int_{\omega/v}^{\infty}\frac{dk'}{k'^3}|\delta N|^2_{\mathbf{k}'}P_l\left(\frac{\omega}{k'v}\right)\int d\varphi_{k'}d\cos\theta \times\frac{[\mathbf{n}\mathbf{k}']^2P_l(\cos\theta)}{[\varepsilon(\omega)-3(k')^2d^2-(\omega_p^2\omega_B^2/\omega^4)+(\omega_p^2\omega_B^2/\omega^4)\cos^2\theta]^2+\varepsilon''^2}. \tag{8}$$

Note that only terms with even values of l contribute to the radiation intensity. Integrating with respect to $d\varphi_{k'}$ and introducing the notation $x=\cos\theta$ yields

$$P^{R,\sigma}_{\mathbf{n},\omega}=\frac{2\pi e^4q^2n_\sigma}{m^2c^3}\sum_{l=0}^{\infty}\int\frac{p^2F_{2l}(p)dp}{v}\times\int_{\omega/v}^{\infty}\frac{dk'}{k'}|\delta N|^2_{\mathbf{k}'}P_{2l}\left(\frac{\omega}{k'v}\right)\frac{\omega^8}{\omega_p^4\omega_B^4}\times\int_{-1}^{1}\frac{[1+\cos^2\theta_n+(1-3\cos^2\theta_n)x^2]P_{2l}(x)dx}{(x^2-a)^2+b^2}, \tag{9}$$

where

$$a=-\frac{\varepsilon(\omega)\omega^4}{\omega_p^2\omega_B^2}+\frac{3k'^2d^2\omega^4}{\omega_p^2\omega_B^2}+1,\quad b=\frac{\varepsilon''\omega^4}{\omega_p^2\omega_B^2}.$$

The limit $b\to 0$ corresponds to the case of a nonabsorbing medium. If in addition we have $0<a<1$, then the integrals with respect to x in Eq. (9) are divergent. The physical reason for this fact and the way to remove the divergence are discussed in Ref. 1. Essentially, when $0<a<1$ holds, Cherenkov emission of plasma waves sets in, and the scattering of these waves in an infinite medium (which formally is taken into account by Eq. (9)) causes the intensity of the resulting transverse waves to be infinite. The emission of "new" photons corresponds to the regions $a<0$, $a>1$, where the integrals are finite and ε'' can be dropped. A study of the scattering of Cherenkov plasmons and an estimate of the respective contribution to the radiation can be found in Ref. 1. In accordance with Ref. 1, at $b=0$ the result of integration with respect to x must be multiplied by $\Theta(3k'^2d^2-\varepsilon(\omega)-6kk'd^2)$ when $a>1$ and by $\Theta(\varepsilon(\omega)-3k'^2d^2-(\omega_p^2\omega_B^2/\omega^4)-6kk'd^2)$ when $a<0$ (here $\Theta(z)$ is the Heaviside theta function):

$$P^{R,\sigma}_{\mathbf{n},\omega}=\frac{4\pi^2e^4q^2n_\sigma}{m^2c^3}\sum_{l=0}^{\infty}\int_{p_0}^{\infty}\frac{p^2F_{2l}(p)dp}{v}\times\int_{\omega/v}^{\infty}\frac{dk'}{k'}|\delta N|^2_{\mathbf{k}'}P_{2l}\left(\frac{\omega}{k'v}\right)\frac{\omega^8}{\omega_p^4\omega_B^4}J_{2l}(a), \tag{10}$$

where

$$J_{2l}(a)=\int_0^1\frac{[1+\cos^2\theta_n+(1-3\cos^2\theta_n)x^2]P_{2l}(x)dx}{(x^2-a)^2}\times\Theta(3k'^2d^2-\varepsilon(\omega)-6kk'd^2)+\int_0^1\frac{[1+\cos^2\theta_n+(1-3\cos^2\theta_n)x^2]P_{2l}(x)dx}{(x^2-a)^2}\times\Theta\left(\varepsilon(\omega)-3k'^2d^2-\frac{\omega_p^2\omega_B^2}{\omega^4}-6kk'd^2\right). \tag{11}$$

Thus, the evaluation of (11) reduces to calculating integrals of the type

$$\int_0^1 \frac{x^{2n}dx}{(x^2 \pm x_0^2)^2} \tag{12}$$

for $n=0,1,2,\ldots$, which can be expressed in terms of elementary functions, i.e., for a known anisotropy, the resonant transition radiation emitted by an ensemble of fast particles can be calculated in final form.

3. RESONANT TRANSITION RADIATION EMITTED BY ISOTROPICALLY DISTRIBUTED PARTICLES

Let us discuss in greater detail the case of a monoenergetic, isotropic distribution of fast particles (normalized to one particle):

$$F_0=\frac{1}{p_0^2}\delta(p-p_0), \quad F_l=0 \text{ at } l\neq 0. \tag{13}$$

By denoting the resonant transition radiation emitted by particles with such a distribution by $I^{R,\sigma}_{\mathbf{n},\omega}$ and evaluating the integrals (12) for $n=0,1$ we obtain

$$I^{R,\sigma}_{\mathbf{n},\omega}=\frac{2\pi^2 e^4 q^2 n_\sigma}{vm^2c^3}\frac{\omega^8}{\omega_p^4\omega_B^4}\int_{\omega/v}^{\infty}\frac{dk'}{k'}|\delta N|^2_{\mathbf{k}'}$$
$$\times\left\{\Theta(3k'^2d^2-\varepsilon(\omega)-6kk'd^2)\left[(1+\cos^2\Theta_n)\right.\right.$$
$$\times\left(\frac{1}{a(a-1)}+\frac{1}{2a^{3/2}}\ln\frac{a^{1/2}+1}{a^{1/2}-1}\right)+(1-3\cos^2\theta_n)\left(\frac{1}{a-1}\right.$$
$$\left.\left.+\frac{1}{2a^{1/2}}\ln\frac{a^{1/2}-1}{a^{1/2}+1}\right)\right]+\Theta\left(\varepsilon(\omega)-3k'^2d^2-\frac{\omega_p^2\omega_B^2}{\omega^4}\right.$$
$$\left.-6kk'd^2\right)$$
$$\times\left[(1+\cos^2\theta_n)\left(\frac{1}{|a|(|a|+1)}+\frac{1}{a^{3/2}}\arctan(|a|^{-1/2})\right)\right.$$
$$\left.\left.+(1-3\cos^2\theta_n)\left(\frac{\arctan(|a|^{-1/2})}{|a|^{1/2}}-\frac{1}{|a|+1}\right)\right]\right\}. \tag{14}$$

Usually the spectrum of inhomogeneities in the plasma density can be approximated within fairly broad limits by a power dependence:

$$|\delta N|^2_{\mathbf{k}'}=\frac{\nu-1}{4\pi}\frac{k_0^{\nu-1}\langle\Delta N^2\rangle}{k'^{\nu+2}}, \tag{15}$$

where ν is the index of the inhomogeneity spectrum, $k_0=2\pi/L_0$, and L_0 and $\langle\Delta N^2\rangle$ are the main scale and mean square of inhomogeneities.

For an arbitrary value of ν, the integrals in (14) can be expressed in terms of special functions. The resonant transition radiation is generated within a small frequency range with $\omega\sim\omega_p$ in which the results depend only weakly on the slope of the inhomogeneity spectrum. Hence we do our calculations for the particular case $\nu=2$ for which the radiation spectrum can be expressed in terms of elementary functions. We assume $\omega\sim\omega_p$ everywhere except in $\varepsilon(\omega)$, and in integrating the terms that contain logarithms or arc tangents we expand these functions in power series and keep only the first terms in the expansions, which ensures an accuracy of about 20%.

The intensity of the resonant transition radiation assumes the form

$$I^{R,\sigma}_{\mathbf{n},\omega}=\frac{\pi e^4 q^2 n_\sigma}{3vm^2c^3}\langle\Delta N^2\rangle k_0\left(\frac{v}{\omega}\right)^4\Phi, \tag{16}$$

where the function Φ is given by the following expression:

$$\Phi=\frac{3}{4}\left(\frac{\omega}{v}\right)^4\frac{\omega_p^4}{\omega_B^4}\int_{(\omega/v)^2}^{\infty}\frac{dx}{x^3}\left\{\Theta(3xd^2-\varepsilon(\omega)-6kx^{1/2}d^2)\right.$$
$$\times\left[2\sin^2\theta_n\left(\frac{1}{a-1}-\frac{1}{a}+\frac{1}{a^2}\right)+\frac{4(3\cos^2\theta_n-1)}{3a^2}\right]$$
$$+\Theta\left(\varepsilon(\omega)-3xd^2-\frac{\omega_B^2}{\omega_p^2}-6kx^{1/2}d^2\right)$$
$$\times\left[\Theta(1-|a|)\frac{\pi(1+\cos^2\theta_n)}{2|a|^{3/2}}+\Theta(|a|-1)\left(2\sin^2\theta_n\left(\frac{1}{|a|}\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{1}{|a|+1}+\frac{1}{a^2}\right)+\frac{4(3\cos^2\theta_n-1)}{3a^2}\right)\right]\right\}. \tag{17}$$

The presence of the theta functions in the integrand in (17) imposes certain restrictions on the range of variation of x as a function of the frequency. Taking this into account in integration (17), we arrive at

$$\Phi\equiv\Phi(\alpha,\beta)=\frac{3}{4}\frac{\omega_p^4}{\omega_B^4}(\Phi_1+\Phi_2+\Phi_3+\Phi_4), \tag{18}$$

where (in what follows we drop the subscript n in the emission angle θ_n to simplify the notation)

$$\Phi_1=\Theta(\omega_1-\omega)\left\{2\beta\sin^2\theta\left[\frac{\beta}{2\alpha(\alpha-\beta)}+\frac{(2\alpha-\beta)\beta}{\alpha^2(\alpha-\beta)^2}\right.\right.$$
$$\left.+\frac{\ln(1-\alpha+\beta)}{(\alpha-\beta)^3}-\frac{\ln(1-\alpha)}{\alpha^3}\right]+\frac{2(3\cos^2\theta+1)}{3}$$
$$\times\beta^2\left[\frac{1}{(\alpha-\beta)^3(1-\alpha+\beta)}+\frac{2}{(\alpha-\beta)^3}+\frac{1}{2(\alpha-\beta)^2}\right.$$
$$\left.\left.+\frac{3\ln(1-\alpha+\beta)}{(\alpha-\beta)^4}\right]\right\}, \tag{19}$$

$$\Phi_2=\Theta(\omega-\omega_1)$$

$$\times\left\{2\beta\sin^2\theta\left[\frac{1}{\alpha^3}\left(\ln\frac{c}{2\sqrt{3}v_T}-\frac{3}{2}\right)-\frac{1}{(\alpha-\beta)^3}\right.\right.$$

$$\left.\times\ln\frac{1+2\sqrt{3}v_T/c}{\beta/\alpha+2\sqrt{3}v_T/c}+\frac{3\alpha-\beta}{2\alpha^2(\alpha-\beta)^2}\right]$$

$$+\frac{2(3\cos^2\theta+1)}{3}\beta^2\left[\frac{1}{\alpha(\alpha-\beta)^3(\beta/\alpha+2\sqrt{3}v_T/c)}\right.$$

$$+\frac{2}{\alpha(\alpha-\beta)^3}+\frac{1}{2\alpha^2(\alpha-\beta)^2}$$

$$\left.\left.+\frac{3}{(\alpha-\beta)^4}\ln\frac{\beta/\alpha+2\sqrt{3}v_T/c}{1+2\sqrt{3}v_T/c}\right]\right\}, \tag{20}$$

$$\Phi_3=\Theta(\omega-\omega_2)\Theta(\omega_3-\omega)\frac{\pi(1+\cos^2\theta)}{2}\beta^{3/2}\left\{\frac{2}{(\alpha-\beta)^3}\right.$$

$$\times\left(\frac{c}{2\sqrt{3}v_T(\alpha(\alpha-\beta))^{1/2}}\right)^{1/2}+\left(\frac{1}{2(\alpha-\beta)}+\frac{5}{4(\alpha-\beta)^2}\right.$$

$$\left.\left.-\frac{15}{4(\alpha-\beta)^3}\right)\frac{1}{(\alpha-\beta-1)^{1/2}}\right\}, \tag{21}$$

$$\Phi_4=\Theta(\omega-\omega_3)\left\{\left(\frac{\pi(1+\cos^2\theta)}{2}\right)\beta^{3/2}\left[\frac{2}{(\alpha-\beta)^3}\right.\right.$$

$$\times\left(\frac{c}{2\sqrt{3}v_T(\alpha(\alpha-\beta))^{1/2}}\right)^{1/2}-\frac{8\alpha^2-41\alpha\beta+48\beta^2}{4(\alpha-\beta)^3(\alpha-2\beta)^2}$$

$$\left.\times\frac{1}{(\beta+2\sqrt{3}v_T/c(\alpha(\alpha-2\beta))^{1/2})^{1/2}}\right]$$

$$+2\beta\sin^2\theta\left[\frac{1}{(\alpha-\beta)^3}\ln\frac{(\alpha-\beta-1)(\alpha-2\beta)}{\beta+2\sqrt{3}(v_T/c)(\alpha(\alpha-2\beta))^{1/2}}\right.$$

$$-\frac{1}{\alpha^3}\ln\frac{(\alpha-1)(\alpha-2\beta)}{2\beta+2\sqrt{3}(v_T/c)(\alpha(\alpha-2\beta))^{1/2}}$$

$$+\frac{(2\alpha-\beta)\beta(\alpha-2\beta-1)}{\alpha^2(\alpha-\beta)^2(\alpha-2\beta)}$$

$$\left.+\frac{\beta(\alpha-2\beta-1)(\alpha-2\beta+1)}{2\alpha(\alpha-\beta)(\alpha-2\beta)^2}\right]+\frac{2(3\cos^2\theta)}{3}\beta^2$$

$$\times\left[\frac{1}{(\alpha-\beta)^3[\beta+2\sqrt{3}(v_T/c)(\alpha(\alpha-2\beta))^{1/2}]}\right.$$

$$+\frac{3}{(\alpha-\beta)^4}\ln\frac{(\alpha-\beta-1)(\alpha-2\beta)}{\beta+2\sqrt{3}(v_T/c)(\alpha(\alpha-2\beta))^{1/2}}$$

$$-\frac{1}{(\alpha-\beta)^3(\alpha-\beta-1)}+\frac{2(\alpha-2\beta-1)}{(\alpha-\beta)^3(\alpha-2\beta)}$$

$$\left.\left.+\frac{(\alpha-2\beta-1)(\alpha-2\beta+1)}{2(\alpha-\beta)^2(\alpha-2\beta)^2}\right]\right\}, \tag{22}$$

with

$$\alpha=(\varepsilon/3)(v/v_T)^2,\quad \beta=(\omega_B^2/3\omega_p^2)(v/v_T)^2, \tag{23}$$

v_T is the thermal velocity of the plasma electron, and

$$\omega_1=\omega_p\left[1+\frac{3}{2}\left(\frac{v_T}{v}\right)^2\left(1-\frac{2\sqrt{3}v_T}{c}\right)\right], \tag{24}$$

$$\omega_2=\omega_p\left[1+\frac{\omega_B^2}{2\omega_p^2}+\frac{3}{2}\left(\frac{v_T}{v}\right)^2\left(1+\frac{2\sqrt{3}v_T}{2}(1+\beta)\right)^{1/2}\right], \tag{25}$$

$$\omega_3=\omega_p\left[1+\frac{\omega_B^2}{\omega_p^2}+\frac{3}{2}\left(\frac{v_T}{v}\right)^2\left(1+\frac{2\sqrt{3}v_T}{c}\right)(1+2\beta)^{1/2}\right]. \tag{26}$$

The limiting value of the function $\Phi(\alpha,\beta)$ at high frequencies ($\omega\gg\omega_p$) is

$$\Phi(\alpha,\beta)\approx\varepsilon^{-2}(\omega), \tag{27}$$

which makes it possible to "match" the given expression with the transition radiation spectrum at high frequencies (where spatial dispersion is insignificant) for arbitrary values of ν (see Ref. 1):

$$I^{\sigma}_{\mathbf{n},\omega}=\frac{\pi(\nu-1)}{2\nu^2(\nu+1)}\frac{e^4q^2n_\sigma}{m^2c^3}\langle\Delta N^2\rangle k_0^{\nu-1}\frac{v}{k^\nu\omega^2}\Phi(\alpha,\beta)$$

$$\times\left\{\left(\frac{\omega}{kv}-1\right)^{-\nu}+\frac{8\nu^3+8\nu^2-3\nu-6}{3(\nu+2)}\left(\frac{kv}{\omega}\right)^{\nu}\right.$$

$$\left.-\frac{400(1.18\nu^2-2.17\nu_1 1.18)}{3(\nu+2)}\left(\frac{kv}{\omega}\right)^{3.03\nu+1.14}\right\}. \tag{28}$$

The spectrum (28) is valid for frequencies lying within the range $\omega_p\leqslant\omega\ll\omega_p\sqrt{\omega_p\gamma/\omega_B})^{1/2}$, and at higher frequencies the curvature of the path of a fast particle moving in the magnetic field has an effect on the transition radiation spectrum.

Near the plasma frequency the formula (28) for the intensity simplifies:

$$I^{R,\sigma}_{\mathbf{n},\omega}=\frac{4\pi(\nu-1)}{3(\nu+2)}\frac{e^4q^2n_\sigma}{vm^2c^3}\langle\Delta N^2\rangle k_0^{\nu-1}\left(\frac{v}{\omega}\right)^{\nu+2}\Phi(\alpha,\beta). \tag{29}$$

Let us analyze in greater detail the properties of the function $\Phi(\alpha,\beta)$, which describes the resonant transition radiation. When there is no magnetic field,

$$\lim_{\beta\to 0}(\mathbf{n}_\sigma\Phi(\alpha,\beta))=F(\alpha), \tag{30}$$

where the function $F(\alpha)$ is defined in Eq. (22) of Ref. 1. Equations (19)–(26) show that the magnetic field begins to affect the resonant transition radiation spectrum for $\beta>2\sqrt{3}v_T/c$, or

$$\frac{\omega_B^2}{\omega_p^2}>\frac{6\sqrt{3}v_T^3}{v^2c}, \tag{31}$$

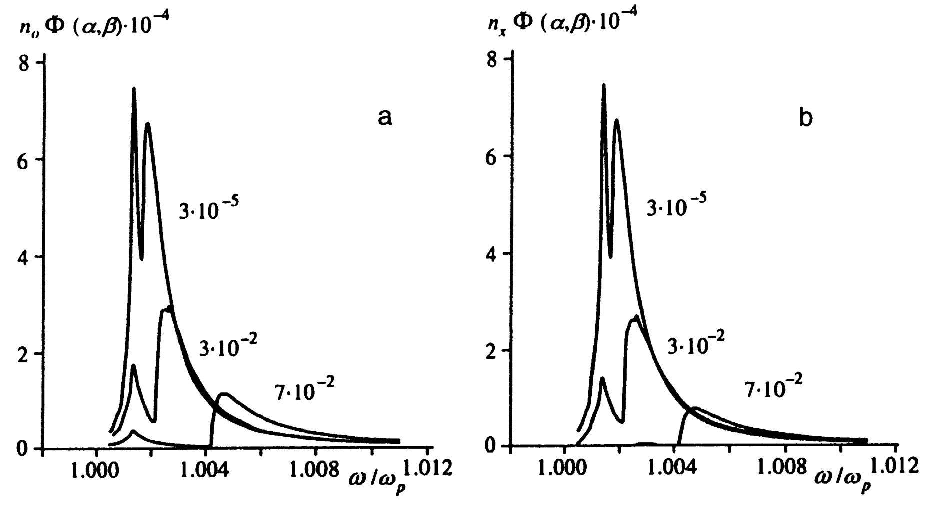

FIG. 1. The spectrum of the resonant transition radiation near the plasma frequency (the functions $n_\sigma\Phi(\alpha,\beta)$) for $x=10$, $v_T/c=0.032$, and different values of β (the figures at the curves): (a) ordinary waves; (b) extraordinary waves.

with the effect becoming especially evident for $\beta \gg 1$. The physical reason for this effect is the variation in the plasma dispersion in an external magnetic field satisfying condition (31).

Let us now examine the behavior of the function $\Phi(\alpha,\beta)$ as $\omega \to \omega_p$. Expanding the function Φ_1 of Eq. (19) in a power series for small values of α, we obtain

$$\Phi_1=\beta \sin^2\theta\left[\frac{1}{3}-\frac{1}{2\beta}+\frac{1}{\beta^2}-\frac{\ln(1+\beta)}{\beta^3}\right]+\frac{2(3\cos^2\theta+1)}{3}\beta^2\left[\frac{1}{2\beta^2}+\frac{3\ln(1+\beta)}{\beta^4}-\frac{1}{\beta^3(1+\beta)}-\frac{2}{\beta^3}\right]. \quad (32)$$

Then, in accordance with the results of Ref. 1, for $\beta \ll 1$ we have

$$\Phi(\alpha,\beta)\to\frac{1}{18}\frac{v^4}{v_T^4}. \quad (33)$$

For $\beta \gg 1$ the corresponding expression depends on the angle. If $\sin^2\theta$ is not too close to zero, then

$$\Phi(\alpha,\beta)\to\frac{1}{18\beta}\frac{v^4}{v_T^4}\sin^2\theta, \quad (34)$$

which is smaller by a factor of β than (33), while if $\sin^2\theta \approx 0$, then

$$\Phi(\alpha,\beta)\to\frac{v^4}{9\beta^2}\frac{v^4}{v_T^4}, \quad (35)$$

which is smaller by a factor of β^2 than (33).

In the absence of a magnetic field, the value of the resonant transition radiation at the peak of the spectrum is[1]

$$(n_\sigma\Phi)_{\max}\sim\frac{v^3c}{v_T^4}. \quad (36)$$

In the presence of a magnetic field, for $\beta \ll 1$ we have

$$(n_\sigma\Phi)_{\max}\sim\frac{v^2}{v_T^2}\frac{\omega_p}{\omega_B}\left(\frac{c}{v_T}\right)^{1/2}, \quad (37)$$

while for $\beta \gg 1$ (for an ordinary wave) we have

$$(n_\sigma\Phi)_{\max}\sim\left(\frac{v}{v_T}\right)^{5/2}\left(\frac{\omega_p}{\omega_B}\right)^{1/2}\left(\frac{c}{v_T}\right)^{1/2}. \quad (38)$$

Thus, the presence of a magnetic field decreases the intensity of the resonant transition radiation, with I^R first decreasing in inverse proportion to the magnetic field strength, $I^R_{\max} \propto B^{-1}$ (for $\beta \ll 1$), and then (for $\beta \gg 1$) more slowly, $I^R_{\max} \propto B^{-1/2}$. The slower decrease is caused by the fact that for $\beta \gg 1$ the peak in the resonant transition radiation spectrum shifts to the right, where the refractive index n_o is greater. The functions $n_\sigma\Phi(\alpha,\beta)$ are depicted in Figs. 1a and b for $\sigma=o,x$ and different values of the parameter β.

Now let us discuss the problem of the total (i.e., integrated over frequencies) energy emitted by the resonant transition mechanism. For $\beta \ll 2\sqrt{3}v_T/c$, the magnetic field has an insignificant effect, so that

$$I_{\rm n}^\sigma=\frac{8\pi(\nu-1)}{45(\nu+2)}\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{c^3m^2}\left(\frac{v}{\omega_p}\right)^{\nu+1}\frac{vc}{v_T^2} \quad (39)$$

for each normal mode ($\sigma=o,x$). Naturally, in this case $I_{\rm n}^\sigma$ is independent of the direction in which the radiation propagates.

But if $\beta \gg 2\sqrt{3}v_T/c$, the main contribution to the total intensity of the resonant transition radiation is proved by the integrals of the terms in Φ_3 and Φ_4 containing the factor $(c/v_T)^{1/2}$:

$$I_{\rm n}^\sigma=\frac{\pi^2(\nu-1)}{(\nu+2)}\frac{(1+\cos^2\theta)}{2^{3/2}\times 3^{1/4}}\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{c^3m^2}\times\left(\frac{v}{\omega_p}\right)^{\nu+1}\left(\frac{c}{v_T}\right)^{1/2}\frac{\omega_p}{\omega_B}J_\sigma(\beta), \quad (40)$$

where

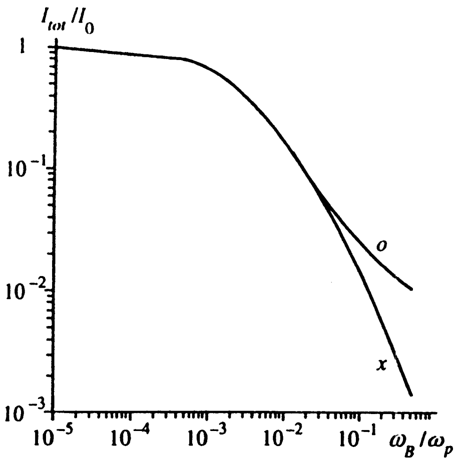

FIG. 2. The total energy of the resonant transition radiation as a function of the magnetic field strength (the parameter ω_B/ω_p) for ordinary and extraordinary waves.

$$J_\sigma(\beta)=\begin{cases}\dfrac{4}{\beta^2}\left[\dfrac{(1+\beta)^{9/4}}{9}-\dfrac{(1+\beta)^{5/4}}{5}+\dfrac{4}{45}\right], & \sigma=o,\\[2ex] \dfrac{4}{\beta^2}\left[\dfrac{(1+\beta)^{7/4}}{7}-\dfrac{(1+\beta)^{3/4}}{3}+\dfrac{4}{21}\right], & \sigma=x.\end{cases} \tag{41}$$

The function $J_\sigma(\beta)$ simplifies in the limiting cases. For one thing, for $\beta \ll 1$ we have

$$J_{o,x}(\beta)=\frac{1}{2}\left(1\pm\frac{\beta}{6}\right). \tag{42}$$

Accordingly, the degree of polarization of the resonant transition radiation in this case is

$$P=\frac{I_{\mathrm{n}}^o-I_{\mathrm{n}}^x}{I_{\mathrm{n}}^o+I_{\mathrm{n}}^x}=\frac{\beta}{6}=\frac{1}{8}\,\frac{\omega_B^2}{\omega_p^2}\,\frac{v^2}{v_T^2}\ll 1. \tag{43}$$

In the opposite limiting case, $\beta \gg 1$, we have

$$J_o(\beta)=4\beta^{1/4}/9,\quad J_x(\beta)=4/7\beta^{1/4}, \tag{44}$$

and the degree of polarization of the resonant transition radiation is

$$P=1-\frac{18}{7\beta^{1/2}}=1-\frac{18\sqrt{3}}{7}\,\frac{\omega_p v_T}{\omega_B v}. \tag{45}$$

Thus, for $\beta \gg 1$ the resonant transition radiation can be highly polarized, with preferential emission of ordinary waves.

If we allow for the explicit form of the functions $J_\sigma(\beta)$ (Eqs. (42) and (44), the total energy of the resonant transition radiation in the specified limiting cases can be represented as

$$I_{\mathrm{n}}^\sigma=\frac{\pi^2(\nu-1)}{(\nu+2)}\,\frac{(1+\cos^2\theta)}{2^{5/2}\times 3^{1/4}}\,\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{c^3m^2}\times\left(\frac{v}{\omega_p}\right)^{\nu+1}\left(\frac{c}{v_T}\right)^{1/2}\frac{\omega_p}{\omega_B} \tag{46}$$

for $\omega_B/\omega_p<\sqrt{3}v_T/v$,

$$I_{\mathrm{n}}^o=\frac{\sqrt{2}\pi^2(\nu-1)}{(\nu+2)}\,\frac{(1+\cos^2\theta)}{9\sqrt{3}}\,\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{c^3m^2}\times\left(\frac{v}{\omega_p}\right)^{\nu+1}\left(\frac{\omega_p}{\omega_B}\right)^{1/2}\left(\frac{cv}{v_T^2}\right)^{1/2} \tag{47}$$

for $\omega_B/\omega_p>\sqrt{3}v_T/v$, and

$$I_{\mathrm{n}}^x=\frac{\sqrt{2}\pi^2(\nu-1)}{(\nu+2)}\,\frac{(1+\cos^2\theta)}{7}\,\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{c^3m^2}\times\left(\frac{v}{\omega_p}\right)^{\nu+1}\left(\frac{\omega_p}{\omega_B}\right)^{3/2}\left(\frac{c}{v}\right)^{1/2} \tag{48}$$

for $\omega_B/\omega_p>\sqrt{3}v_T/v$.

The dependence of the total energy of the resonant transition radiation on the magnetic field strength (the ratio ω_B/ω_p)) is depicted in Fig. 2.

In natural conditions plasma is usually inhomogeneous, so that if the corresponding distribution function over the plasma frequencies is broader than the peak in the spectrum of the resonant transition radiation, the latter can be approximately represented as

$$I_{\mathrm{n},\omega}^{R,\sigma}=I_{\mathrm{n}}^\sigma\delta(\omega-\omega_*), \tag{49}$$

where the frequency ω_* corresponds to the peak in the spectrum of the resonant transition radiation and is close to the local plasma frequency.

4. RESONANT TRANSITION RADIATION EMITTED BY ANISOTROPICALLY DISTRIBUTED PARTICLES

Now let us establish what effect the presence of an anisotropy in the ensemble of the emitting particles has on the emission of resonant transition radiation. We assume that this anisotropy can be described by the second Legendre polynomial, so that

$$F(p,\mu)=\frac{1}{p_0^2}\delta(p-p_0)[1+AP_2(\mu)], \tag{50}$$

where $\mu=\cos\theta_v$.

In this case all the calculations are similar to those done above, although the expressions for the resonant transition radiation spectrum prove to be fairly cumbersome. For this reason below we give only the results for the resonant transition radiation integrated over the frequencies.

For $v_T/c\ll\beta\ll 1$ we have

$$I_{\mathrm{n}}^{o,x}=I_{\mathrm{n}}^{o,x(0)}\left[1-\frac{A}{4}\left(1\pm\frac{3\beta}{16}\right)\right], \tag{51}$$

where $I_{\mathrm{n}}^{o,x(0)}$ is the value of the energy of the resonant transition radiation for isotropically distributed particles. Since the condition of positive definiteness of the distribution function (50) implies that

$$-1<A<2,$$

we obtain

$$I_{\mathrm{n}}^{\sigma(0)}/2\leq I_{\mathrm{n}}^\sigma\leq 5I_{\mathrm{n}}^{\sigma(0)}/4. \tag{52}$$

TABLE I.

		$\xi<\nu+\frac{3}{2}$	$\nu+\frac{3}{2}<\xi<\nu+2$	$\nu+2<\xi<\nu+\frac{5}{2}$	$\nu+\frac{5}{2}<\xi<\nu+3$	$\nu+3<\xi$
$\frac{\omega_B}{\omega_p}<\left(\frac{v_T}{c}\right)^{3/2}$	o	P_1	P_1	P_1	P_1	P_2
	x	P_1	P_1	P_1	P_1	P_2
$\left(\frac{v_T}{c}\right)^{3/2}<\frac{\omega_B}{\omega_p}<\frac{v_T}{c}$	o	P_4	P_4	P_3	P_3	P_2
	x	P_4	P_4	P_3	P_3	P_2
$\frac{v_T}{c}<\frac{\omega_B}{\omega_p}<\left(\frac{v_T}{c}\right)^{1/2}$	o	P_5	P_5	P_3+P_5	P_3	P_2
	x	P_7	P_6	P_3	P_3	P_2
$\left(\frac{v_T}{c}\right)^{1/2}<\frac{\omega_B}{\omega_p}<1$	o	P_5	P_5	P_3+P_8	P_8	P_8
	x	P_7	P_6	P_8	P_8	P_8

Thus, for distributions that are "flattened" with respect to the magnetic field ($A<0$) the intensity of the resonant transition radiation increases in relation to that for an isotropic distribution, while for "elongated" distributions ($A>0$) the intensity decreases.

This conclusion remains valid for the case where $\beta\gg1$, although the explicit expressions are somewhat modified. For instance, for ordinary waves we have

$$I_{\mathbf{n}}^{o}=I_{\mathbf{n}}^{o(0)}\left(1-\frac{7A}{26}\right), \tag{53}$$

while for extraordinary waves we have

$$I_{\mathbf{n}}^{x}=I_{\mathbf{n}}^{x(0)}\left(1-\frac{5A}{22}\right). \tag{54}$$

Similar calculations can be done in final form for more complicated particle distribution functions containing higher-order Legendre polynomials

5. RESONANT TRANSITION RADIATION EMITTED BY PARTICLES WITH A POWER-LAW ENERGY DISTRIBUTION

Now let us investigate the properties of the resonant transition radiation generated by isotropically distributed charged particles with a power-law momentum distribution:

$$dN_e=(\xi-1)N_e(x>x_0)\frac{x_0^{\xi-1}dx}{x^{\xi}},\quad x_0<x<x_1, \tag{55}$$

where $x=p/mc$ is the dimensionless particle momentum. Since the parameter β depends on the particle velocity and the range of integration with respect to x providing the main contribution to the integrals is determined by the ratio of the spectral indices ν and ξ, there exists a fairly large number of different asymptotic limits depending on the parameters of the problem. The integration of expressions (39) and (46)–(48) with the spectrum (55) can easily be carried out, so that we list the results of the calculations below:

$$P_1=\frac{4\pi(\nu-1)(\xi-1)}{45(\nu+2)}\frac{\Gamma\left(\frac{\xi-1}{2}\right)\Gamma\left(\frac{\nu-\xi+3}{2}\right)}{\Gamma\left(\frac{\nu+2}{2}\right)}\times\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{m^2c^3}x_0^{\xi-1}N_e\left(\frac{c}{\omega_p}\right)^{\nu+1}\frac{c^2}{v_T^2}, \tag{56}$$

$$P_2=\frac{8\pi(\nu-1)(\xi-1)}{45(\nu+2)(\xi-\nu-3)}\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{m^2c^3}\times N_e\left(\frac{v_0}{\omega_p}\right)^{\nu+1}\frac{cv_0}{v_T^2},\quad v_0=x_0c, \tag{57}$$

$$P_3=\frac{\pi(\nu-1)(\xi-1)}{3(\nu+2)}\left[\frac{8}{15(\nu+3-\xi)}+\frac{\pi(1+\cos^2\theta)}{8(\xi-\nu-2)}\right]\times\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{m^2c^3}x_0^{\xi-1}\times N_e\left(\frac{c}{\omega_p}\right)^{\nu+1}\frac{c^2}{v_T^2}\left[\frac{\omega_p}{\omega_B}\left(\frac{6\sqrt{3}v_T^3}{c^3}\right)^{1/2}\right]^{\nu+3-\xi}, \tag{58}$$

$$P_4=\frac{\pi^2(\nu-1)(\xi-1)\Gamma\left(\frac{\xi-1}{2}\right)\Gamma\left(\frac{\nu-\xi+2}{2}\right)}{8\sqrt{2}3^{1/4}(\nu+2)\Gamma\left(\frac{\nu+1}{2}\right)}\times(1+\cos^2\theta)\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{m^2c^3}x_0^{\xi-1}\times N_e\left(\frac{c}{\omega_p}\right)^{\nu+1}\left(\frac{c}{v_T}\right)^{1/2}\frac{\omega_p}{\omega_B}, \tag{59}$$

$$P_5=\frac{\pi^2(\nu-1)(\xi-1)\Gamma\left(\frac{\xi-1}{2}\right)\Gamma\left(\frac{\nu-\xi+2.5}{2}\right)}{9\sqrt{6}(\nu+2)\Gamma\left(\frac{\nu}{2}+\frac{3}{4}\right)}$$
$$\times(1+\cos^2\theta)\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{m^2c^3}x_0^{\xi-1}$$
$$\times N_e\left(\frac{c}{\omega_p}\right)^{\nu+1}\left(\frac{\omega_p}{\omega_B}\right)^{1/2}\frac{c}{v_T}, \tag{60}$$
$$P_6=\frac{\pi^2(\nu-1)(\xi-1)(1+\cos^2\theta)}{2^{1/2}3^{1/4}(\nu+2)}$$
$$\times\left(\frac{1}{4(\nu+2-\xi)}+\frac{2}{7(\xi-\nu-1.5)}\right)\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{m^2c^3}x_0^{\xi-1}$$
$$\times N_e\left(\frac{c}{\omega_p}\right)^{\nu+1}\left(\frac{c}{v_T}\right)^{1/2}\frac{\omega_p}{\omega_B}\left(\sqrt{3}\frac{v_T}{c}\frac{\omega_p}{\omega_B}\right)^{\nu+2-\xi}, \tag{61}$$
$$P_7=\frac{\pi^2(\nu-1)(\xi-1)(1+\cos^2\theta)}{7\sqrt{2}(\nu+2)}\frac{\Gamma\left(\frac{\nu}{2}-\frac{\xi}{2}+\frac{3}{4}\right)\Gamma\left(\frac{\xi-1}{2}\right)}{\Gamma\left(\frac{\nu}{2}+\frac{1}{4}\right)}$$
$$\times\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{m^2c^3}x_0^{\xi-1}N_e\left(\frac{c}{\omega_p}\right)^{\nu+1}\left(\frac{\omega_p}{\omega_B}\right)^{3/2}, \tag{62}$$
$$P_8=\frac{\pi^2(\nu-1)(\xi-1)(1+\cos^2\theta)}{4\sqrt{2}3^{1/4}(\nu+2)(\xi-\nu-2)}\frac{e^4q^2\langle\Delta N^2\rangle k_0^{\nu-1}}{m^2c^3}$$
$$\times N_e\left(\frac{v_0}{\omega_p}\right)^{\nu+1}\left(\frac{c}{v_T}\right)^{1/2}\frac{\omega_p}{\omega_B}. \tag{63}$$

The different ratios of the parameters determine the dominant asymptotic behavior; this is illustrated by the data listed in Table I. Analysis of these asymptotic data shows that for a momentum power–law distribution of the particles the resonant transition radiation proves to be weakly polarized for $\xi>\nu+5/2$ irrespective of the value of ω_B/ω_p, and for $\omega_B/\omega_p<v_T/c$, irrespective of the ξ–to–ν ratio. Noticeable polarization emerges for $\omega_B/\omega_p>v_T/c$ and $\xi<\nu+5/2$, i.e., for fairly rigid electron distributions in a fairly strong magnetic field. The factor by which the power of resonant transition radiation in a magnetic field exceeds that of ordinary transition radiation varies from c^2/v_T^2 in a weak field to $(c/v_T)(\omega_p/\omega_B)^{1/2}$ for ordinary waves and to $(\omega_p/\omega_B)^{3/2}80$ state for extraordinary waves in a strong field.

6. CONCLUSION

Taking into account the effect of a magnetic field on the dispersion of the plasma leads to a radical change in the properties of the resonant transition radiation. Indeed, the peak value of the resonant transition radiation intensity decreases, the peak is shifted and becomes narrower, and the total intensity of the resonant transition radiation decreases. The radiation emitted by an isotropic ensemble of particles acquires an angular dependence and a finite polarization with preferential emission of ordinary waves. The degree of polarization can reach a value of several tenths, and in some cases it can become practically 100%.

The formulas obtained in this paper can be used to analyze ionospheric noise, solar radio bursts, and laboratory conditions, since they allow for all the factors relevant in the given problem (averaging the obtained expressions in a gently varying plasma is a trivial procedure). Note that the high intensity of the radiation makes it competitive with other, previously known, mechanisms of nonthermal radiation.

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