# The dynamics of charges induced by a fast particle traversing a conducting or insulating layer

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We study the dynamics of surface and space charges induced by a fast charged particle traversing a layer of insulator or conductor. We show that the polarization of the layer is of an oscillatory nature, and the net induced charge in an insulating layer and in a layer of a conductor thinner than $u/2\pi\sigma$, where $u$ is the particle velocity and $\sigma$ is the DC conductivity, remains zero at all stages of the motion. In a conducting layer thicker than $u/2\pi\sigma$ the net induced charge is also time independent but, as with a half-space, is equal to the charge of the particle with the opposite sign. © *1996 American Institute of Physics.* © *1996 American Institute of Physics.* [S1063-7761(96)01401-2]

The study of charges induced by a charged point particle traveling in a material medium has a long history. On the one hand, a point charge at rest is known to induce on the surface of a conductor an "image charge," and various methods have been developed for calculating such charges.[1,2] Fairly recently, in connection with problems of emission electronics and optoelectronics, the image charge induced by a moving particle has also been investigated. To describe the process of formation of the image charge, various approaches (the quantum mechanical, the hydrodynamic, etc.) and various models of the medium have been employed (see Ref. 3 and the literature cited therein). On the other hand, many researchers have studied the induced charge for the case where the particle is inside the medium and boundary effects can be ignored. A fast-moving particle leaves behind it a trace in the form of an induced oscillating charge,[4,5] which is known as the wake charge. It is the excitation of wake waves that causes particles traveling through the medium to lose energy to polarization.[6–9]

In a previous paper[10] we studied a process in which a fast particle crossed the boundary of a semi-infinite conducting medium (plasma). There we showed that the transformation of the image charge into a wake charge is accompanied by oscillations of the surface charge at the plasma frequency $\omega_p$ and at the frequency of surface oscillations, $\omega_0=\omega_p/\sqrt{2}$.

In the present work we study the passage of a fast charged particle through an insulating or conducting layer. We analyze the dynamics of reversal of the sign of the charges induced at the layer's boundary (repolarization of the layer) as the particle crosses the interface. The process is found to be of a nonmonotonic, oscillatory nature. Here, in the case of a conductor, the value of the net induced charge depends on the layer's thickness and conductivity. If the conducting layer is thinner than $u/2\pi\sigma$, with $u$ the particle velocity and $\sigma$ the DC conductivity, it acts as an insulator, and the net charge induced in it remains zero at all stages of the particle's motion, which are the approach stage, the passage through the layer, and the recession stage. When the conducting layer is thicker than $u/2\pi\sigma$, the net charge induced in it is equal in absolute value to the particle's charge but has the opposite sign. In this sense the situation is similar to the one with the image charge of a semi-infinite conductor.

## 1. GENERAL RELATIONSHIPS

Let us examine the behavior of a test charge $Q$ moving with a velocity $u$ along the $z$ axis perpendicular to the flat interfaces between three media. We assume that the particle is in the first medium ($z<0$) when the time is in the interval $-\infty<t<0$, in the second medium ($0\leqslant z\leqslant a$) when $0<t<a/u$, and in the third medium ($z>a$) when $a/u<t<+\infty$. We also assume that the particle is nonrelativistic and has a velocity much lower than the velocity of light ($u\ll c$). The potential of the particle is then determined by the Poisson equation

$$\hat{\varepsilon}\nabla^2\varphi=-4\pi Q\,\delta(z-ut)\,\delta(\mathbf{r}), \tag{1}$$

where $\mathbf{r}=(x,y)$, and $\hat{\varepsilon}$ is the dielectric constant operator.[2]

To find $\varphi$, we go over to the Fourier representation in time $t$ and position $\mathbf{r}$ in Eq. (1):

$$\varphi(\mathbf{r},z,t)=\int_{-\infty}^{+\infty}d^2\mathbf{k}\int_{-\infty}^{+\infty}d\omega\,\varphi(\mathbf{k},\omega,z)\exp(i\mathbf{kr}-i\omega t), \tag{2}$$

$$\delta(z-ut)\,\delta(\mathbf{r})=\frac{1}{u(2\pi)^3}\int_{-\infty}^{+\infty}d^2\mathbf{k}\int_{-\infty}^{+\infty}d\omega\times\exp[i\mathbf{kr}-i\omega(t-z/u)]. \tag{3}$$

The action of the linear operator $\hat{\varepsilon}$ on an arbitrary function $f(\mathbf{r},z,t)$ reduces to multiplying the Fourier transform $f(\mathbf{k},\omega,z)$ by the medium's dielectric constant $\varepsilon(\omega)$ (see Ref. 2). Substituting the expansions (2) and (3) into Eq. (1), we arrive at the following equation for $\varphi(\mathbf{k},\omega,z)$:

$$\frac{\partial^2}{\partial z^2}(\mathbf{k},\omega,z)-k^2\varphi(\mathbf{k},\omega,z)=-\frac{2Q}{u(2\pi)^2\varepsilon(\omega)}\exp\left(i\frac{\omega}{u}z\right). \tag{4}$$

The solution of this equation is the sum of the particular solution of the inhomogeneous equation and the general solution of the corresponding homogeneous equation. Bearing in mind that the boundary effects diminish as the distance from the interface grows, we can write the solution in the form

$$\varphi(\mathbf{k},\omega,z)=\frac{G_0}{\varepsilon_1(\omega)}\left[\exp\left(i\frac{\omega}{u}z\right)+\alpha(\mathbf{k},\omega)e^{kz}\right],\quad z<0, \tag{5}$$

$$\varphi(\mathbf{k},\omega,z)=\frac{G_0}{\varepsilon_2(\omega)}\times\left[\exp\left(i\frac{\omega}{u}z\right)+\beta(\mathbf{k},\omega)e^{-kz}+\gamma(\mathbf{k},\omega)\exp[-k(a-z)]\right\},\quad 0\leqslant z\leqslant a, \tag{6}$$

$$\varphi(\mathbf{k},\omega,z)=\frac{G_0}{\varepsilon_3(\omega)}\left\{\exp\left(i\frac{\omega}{u}z\right)+\delta(\mathbf{k},\omega)\exp[-k(z-a)]\right\},\quad z>a, \tag{7}$$

where

$$G_0(k^2,\omega)=\frac{Qu}{2\pi^2(\omega^2+k^2u^2)}. \tag{8}$$

The constants $\alpha$, $\beta$, $\gamma$, and $\delta$ can be found from a system of equations following from the boundary conditions that the normal component of the induction and the tangential component of the electric field at $z=0$ and $z=a$ are continuous:

$$\frac{1}{\varepsilon_1}(1+\alpha)=\frac{1}{\varepsilon_2}(1+\beta+\gamma e^{-k\alpha}), \tag{9}$$

$$\frac{1}{\varepsilon_2}\exp\left(i\frac{\omega}{u}a\right)+\beta e^{-ka}+\gamma=\frac{1}{\varepsilon_3}\left(\exp\left(i\frac{\omega}{u}a\right)+\delta\right), \tag{10}$$

$$\alpha+\beta=\gamma e^{-ka}, \tag{11}$$

$$\gamma+\delta=\beta e^{-ka}. \tag{12}$$

Solving the system of equations (9)–(12) yields

$$\alpha=\frac{2}{D(k,\omega)}\left\{(\varepsilon_1-\varepsilon_2)[\varepsilon_3\cosh(ka)+\varepsilon_2\sinh(ka)]+\varepsilon_1(\varepsilon_2-\varepsilon_3)\exp\left(i\frac{\omega}{u}a\right)\right\}, \tag{13}$$

$$\beta=\frac{\varepsilon_2-\varepsilon_1}{D(k,\omega)}\left\{(\varepsilon_3+\varepsilon_2)e^{ka}-(\varepsilon_3-\varepsilon_2)\exp\left(i\frac{\omega}{u}a\right)\right\}, \tag{14}$$

$$\gamma=\frac{\varepsilon_2-\varepsilon_3}{D(k,\omega)}\left\{\varepsilon_2-\varepsilon_1+(\varepsilon_2+\varepsilon_1)e^{ka}\exp\left(i\frac{\omega}{u}a\right)\right\}, \tag{15}$$

$$\delta=\frac{2}{D(k,\omega)}\left\{\varepsilon_3(\varepsilon_2-\varepsilon_1)+(\varepsilon_3-\varepsilon_2)[\varepsilon_1\cosh(ka)+\varepsilon_2\sinh(ka)]\exp\left(i\frac{\omega}{u}a\right)\right\}, \tag{16}$$

The first terms on the right-hand sides of Eqs. (5)–(7) determine the potentials that the particle generates in infinite media. The other terms appear in Eqs. (5)–(7) because of the interfaces.

The equation $D(k,\omega)=0$ gives the law of dispersion for surface oscillations in a layer,[11,12] where

$$D(k,\omega)=(\varepsilon_1+\varepsilon_2)(\varepsilon_3+\varepsilon_2)e^{ka}+(\varepsilon_2-\varepsilon_1)(\varepsilon_3-\varepsilon_2)e^{-ka}. \tag{17}$$

The surface charge density induced at the interfaces is related to the jump in the normal component of the electric field. Equations (5)–(7) and (13)–(16) yield

$$\sigma_i(\mathbf{r},t)=\int_{-\infty}^{+\infty}d^2\mathbf{k}\int_{-\infty}^{+\infty}d\omega\sigma_i(\mathbf{k},\omega)\exp[i(\mathbf{kr}-\omega t)], \tag{18}$$

where the subscript $i=0$, $a$ refers to the front and rear boundaries of the layer, respectively:

$$\sigma_0(\mathbf{k},\omega)=\frac{G_0}{4\pi}\left(\frac{1}{\varepsilon_1}-\frac{1}{\varepsilon_2}\right)\times\left\{i\frac{\omega}{u}+\frac{2k}{D(k,\omega)}\left([\varepsilon_1-\varepsilon_2)(\varepsilon_3\cosh(ka)+\varepsilon_2\sinh(ka)]+\varepsilon_1(\varepsilon_2-\varepsilon_3)\exp\left(i\frac{\omega}{a}a\right)\right)\right\}, \tag{19}$$

$$\sigma_a(\mathbf{k},\omega)=\frac{G_0}{4\pi}\left(\frac{1}{\varepsilon_2}-\frac{1}{\varepsilon_3}\right)\times\left\{i\frac{\omega}{u}\exp\left(i\frac{\omega}{u}a\right)+\frac{2k}{D(k,\omega)}\left(\varepsilon_3(\varepsilon_1-\varepsilon_2)+(\varepsilon_2-\varepsilon_3)[\varepsilon_1\cosh(ka)+\varepsilon_2\sinh(ka)]\exp\left(i\frac{\omega}{a}a\right)\right)\right\}. \tag{20}$$

Equations (18)–(20) make it possible to find the net induced surface charge

$$Q_{is}(t)=\int_{-\infty}^{+\infty}d^2\mathbf{r}\sigma_i(\mathbf{r},t)=(2\pi)^2\int_{-\infty}^{+\infty}d\omega\sigma_i(\omega)\exp(-i\omega t), \tag{21}$$

where $\sigma_i(\omega)=\sigma_i(\mathbf{k}=0,\omega)$. Thus, the net charge induced at the surface of the layer is determined by the values of the functions (19) and (20) in the limit of $\mathbf{k}\to 0$.

## 2. THE DYNAMICS OF CHARGES IN AN INSULATING LAYER

In the case of an insulating layer, the well-known Sochozki–Plemelj formula[2] for the $\sigma_i(\omega)$ can be applied. As a result, at $\varepsilon_1=\varepsilon_3=1$ and $\varepsilon_2=\varepsilon(\omega)$ (the layer is in a vacuum) Eqs. (19) and (20) yield

$$(2\pi)^2\sigma_0(\omega)=\frac{Q}{2\pi i}\frac{1-\varepsilon(\omega)}{\varepsilon(\omega)}P\frac{1}{\omega}$$

$$-\frac{Q}{4}\delta(\omega)\left(\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)}\right)^2\left[1-\exp\left(i\frac{\omega}{u}a\right)\right], \qquad (22)$$

$$(2\pi)^2\sigma_a(\omega)=-\frac{Q}{2\pi i}\frac{1-\varepsilon(\omega)}{\varepsilon(\omega)}\exp\left(i\frac{\omega}{u}a\right)P\frac{1}{\omega}$$

$$+\frac{Q}{4}\delta(\omega)\left(\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)}\right)^2\left[1-\exp\left(i\frac{\omega}{u}a\right)\right], \qquad (23)$$

where $P$ stands for the principal value of the integral. Equations (22) and (23) were obtained on the assumption that the layer's dielectric constant $\varepsilon(\omega)$ acquires no singularities as $\omega\rightarrow 0$, which is not the case for conductors.

The first terms on the right–hand sides of Eqs. (19), (20), (22), and (23) emerge because of the jump in the electric fields that the particle induces in infinite media. The other terms are caused by the surface fields. We see that because of the delta functions the contribution of the latter to the surface charge (21) is zero. Hence the net charge at the surface of a dielectric layer is determined solely by the fields generated by the particle in infinite media:

$$Q_{0s}(t)=Q\Phi(t), \quad Q_{as}(t)=-Q\Phi(\tau), \qquad (24)$$

where $\tau=t-a/u$, and

$$\Phi(t)=\frac{1}{2\pi i}P\int_{-\infty}^{+\infty}\frac{d\omega}{\omega}\exp(-i\omega t)\frac{1-\varepsilon(\omega)}{\varepsilon(\omega)}. \qquad (25)$$

To calculate the function $\Phi(t)$ we examine the following contour integral:

$$\Phi_0(t)=\frac{1}{2\pi i}\int_C\frac{d\omega}{\omega}\exp(-i\omega t)\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)}, \qquad (26)$$

where the contour $C$ in the complex $\omega$ plane consists of the real axis, a semicircle of infinitesimal radius that encompasses from below the singularity at $\omega=0$, and a semicircle with an infinitely large circle in the upper or lower half-plane, depending on the sign of $t$ (the contour is closed from above if $t<0$ and from below if $t>0$). Since $\varepsilon(\omega)$ is analytic in the complex $\omega$ plane, has zeros (symmetric with respect to the imaginary $\omega$ axis; see Ref. 2) only in the lower half-plane, and tends to unity as $|\omega|\rightarrow\infty$, the residue theorem[13] can be applied to $\Phi_0(t)$:

$$\Phi_0(t)=\left(1-\frac{1}{\varepsilon_0}\right)\theta(-t)$$

$$+\theta(t)\sum_j \exp(-\nu_j t)[A_j\cos(\omega_j t)+B_j\sin(\omega_j t)], \qquad (27)$$

where $\varepsilon_0=\varepsilon(\omega=0)$ is the DC dielectric constant of the substance, $\theta(t)$ is the Heaviside step function ($\theta(0)=\frac{1}{2}$), $\pm\omega_j-i\nu_j$ the solutions of the equation $\varepsilon(\varepsilon)=0$, and

$$A_j=\frac{2}{\omega_j^2+\nu_j^2}\,\mathrm{Re}\left\{\frac{\omega_j+i\nu_j}{\varepsilon'(\omega_j-i\nu_j)}\right\},$$

$$B_j=\frac{2}{\omega_j^2+\nu_j^2}\,\mathrm{Im}\left\{\frac{\omega_j+i\nu_j}{\varepsilon'(\omega_j-i\nu_j)}\right\}. \qquad (28)$$

Here the prime stands for the derivative with respect to the argument, or $\varepsilon'(\omega)=d\varepsilon(\omega)/d\omega$. Summation is over all the roots of the equation $\varepsilon(\omega)=0$.

According to the definitions (25) and (26), the functions $\Phi_0(t)$ and $\Phi(t)$ are related thus:

$$\Phi_0(t)=-\Phi(t)+\frac{1}{2}\left(1-\frac{1}{\varepsilon_0}\right). \qquad (29)$$

Hence the function $\Phi(t)$ in (24) has the form

$$\Phi(t)=-\frac{1}{2}\left(1-\frac{1}{\varepsilon_0}\right)+\theta(t)\left\{1-\frac{1}{\varepsilon_0}-\sum_j\exp(-\nu_j t)\right.$$

$$\left.\times[A_j\cos(\omega_j t)+B_j\sin(\omega_j t)]\right\}. \qquad (30)$$

Note that the expressions (27)–(30) lead to the following sum rule:

$$\sum_j A_j=1-\frac{1}{\varepsilon_0}. \qquad (31)$$

Thus, Eqs. (24) and (30) make it possible to find the net surface charge if the solutions of the equation $\varepsilon(\omega)=0$ are known.

Now let us turn to the problem of calculating the space (wake) charge in the layer. Integrating the respective charge density[12] over the volume, we get

$$Q_v(t)=-Q[\theta(t)-\theta(\tau)]\left\{1-\frac{1}{\varepsilon_0}-\sum_j\exp(-\nu_j t)\right.$$

$$\left.\times[A_j\cos(\omega_j t)+B_j\sin(\omega_j t)]\right\}$$

$$-Q\theta(\tau)\sum_j\exp(-\nu_j\tau)\left\{A_j\cos(\omega_j\tau)+B_j\sin(\omega_j\tau)\right.$$

$$\left.-\exp\left(-\nu_j\frac{a}{u}\right)[A_j\cos(\omega_j t)+B_j\sin(\omega_j t)]\right\}. \qquad (32)$$

Let us analyze the formulas (24), (25), and (32). As the particle approaches the surface of the layer from the vacuum ($t<0$), we have

$$Q_{0s}=-\frac{Q}{2}(1-1/\varepsilon_0), \quad Q_{as}=\frac{Q}{2}(1-1/\varepsilon_0), \quad Q_v=0.$$

Since both boundaries of the layer have the same net surface charges of opposite signs, the net charge induced in the layer is also zero.

As the particle moves inside the layer ($0<t<a/u$), the corresponding quantities are

$$Q_{0s}(t)=Q\left\{\frac{1}{2}\left(1-\frac{1}{\varepsilon_0}\right)\right.$$
$$\left.-\sum_j \exp(-\nu_j t)[A_j\cos(\omega_j t)+B_j\sin(\omega_j t)]\right\}, \tag{33}$$

$$Q_{as}(t)=\frac{Q}{2}\left(1-\frac{1}{\varepsilon_0}\right), \tag{34}$$

$$Q_v(t)=-Q\left\{1-\frac{1}{\varepsilon_0}\right.$$
$$\left.-\sum_j \exp(-\nu_j t)[A_j\cos(\omega_j t)+B_j\sin(\omega_j t)]\right\}. \tag{35}$$

Equations (33)–(35) show that the charge at the front boundary of the layer oscillates and decreases, while in the volume the charge increases correspondingly. The charge at the rear boundary remains the same.

After the particle crosses the rear boundary and leaves the layer ($t>a/u$) we have

$$Q_{as}(t)=-Q\left\{\frac{1}{2}\left(1-\frac{1}{\varepsilon_0}\right)\right.$$
$$\left.-\sum_j \exp(-\nu_j \tau)[A_j\cos(\omega_j \tau)+B_j\sin(\omega_j \tau)]\right\}, \tag{36}$$

$$Q_v(t)=-Q\sum_j \exp(-\nu_j\tau)\left\{A_j\cos(\omega_j\tau)+B_j\sin(\omega_j\tau)\right.$$
$$\left.-\exp\left(-\nu_j\frac{a}{u}\right)[A_j\cos(\omega_j\tau)+B_j\sin(\omega_j\tau)]\right\}. \tag{37}$$

Equations (33), (36), and (37) imply that the charge at the front boundary decreases and tends to the limit $\frac{1}{2}Q(1-1/\varepsilon_0)$. At the rear edge the charge increases and for

$$t\gg\frac{a}{u}+\max(\nu_j^{-1})$$

assumes the maximum value of

$$-\frac{Q}{2}(1-\varepsilon_0^{-1}).$$

Here the wake charge in the volume of the layer vanishes. It can easily be shown that at each moment of time the net induced charge in the layer is zero.

The above analysis suggests that after the particle crosses the first boundary the surface charge transforms into a wake charge, whereas after it crosses the second boundary the wake charge is transformed back to a surface charge.

## 3. THE MODEL OF SINGLE-FREQUENCY OSCILLATORS

Let us now analyze the above formulas for the induced charges in the simplest model of one-frequency oscillators,[15] where

$$\varepsilon(\omega)=1+\frac{\tilde{\omega}_0^2}{\omega_q^2-\omega(\omega+i\nu)}, \tag{38}$$

$\omega_q$ is an averaged atomic transition frequency, $\tilde{\omega}_0^2=4\pi n_0 e^2/m$, $n_0$ is the electron number density in the substance, and $\nu$ is an atom's reciprocal lifetime in an excited state, which we assume to be small ($\nu\ll\min(\omega_q,\tilde{\omega}_0)$). Equation (38) implies that its root $\varepsilon(\omega)=0$ has the following form:

$$\omega_j-i\nu_j=-\frac{i\nu}{2}+\left(\Omega^2-\frac{\nu^2}{4}\right)^{1/2}, \tag{39}$$

where $\Omega^2=\omega_q^2+\tilde{\omega}_0^2$. Substituting (38) and (39) into Eq. (29), we find the coefficients determining the net induced charges:

$$A_j\equiv\mu=\frac{\varepsilon_0-1}{\varepsilon_0}=\frac{\tilde{\omega}_0^2}{\tilde{\omega}_0^2+\omega_q^2},\quad B_j=\frac{\mu\nu/2}{(\Omega^2-\nu^2/4)^{1/2}}. \tag{40}$$

When the particle traveling in the vacuum approaches the surface of the layer ($t<0$), we have $Q_{0s}=-\mu Q/2, Q_{as}=\mu Q/2$, and $Q_v=0$. For solid insulators, $\varepsilon_0\gg1$ and $\mu=\approx1$ (see Ref. 15). Hence as the particle approaches the boundary of such an insulator, it induces a surface charge whose absolute value is half the particle's charge. For other moments in time in the limit of $\nu\ll\Omega$, Eqs. (33)–(37) yield

$$Q_{as}/Q=\mu/2,$$
$$Q_{0s}/Q=\mu[1/2-\exp(-\nu t/2)\cos(\Omega t)], \tag{41}$$
$$Q_v/Q=-\mu[1-\exp(-\nu t/2)\cos(\Omega t)]$$

for $0<t<a/u$, and

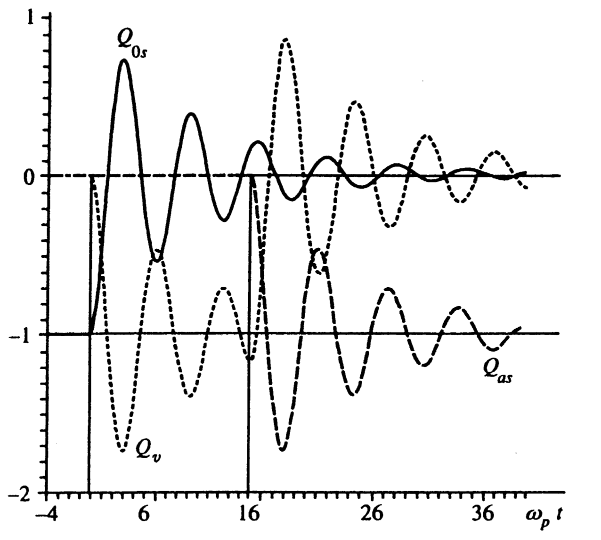


FIG. 1. The dynamics of the net induced charge at the front boundary (solid curve), the rear boundary (dashed curve), and in the volume of an insulator layer (dotted curve): $\mu=0.9$, $\tilde{\omega}_0/\omega_q=3$, $\nu/\Omega=0.2$, and $\Omega a/u=13$.

$$Q_{as}/Q=-\mu[1/2-\exp(-\nu\tau/2)\cos(\Omega\tau)],$$
$$Q_v/Q=-\mu[\exp(-\nu\tau/2)\cos(\Omega\tau)-\exp(-\nu t/2)\cos(\Omega t)] \tag{42}$$

for $ta/u$.

Figure 1 depicts the time dependence of $Q_{0s}$, $Q_{as}$, and $Q_v$. The following values of the parameters were taken for numerical calculations: $\mu=0.9$ ($\tilde{\omega}_0/\omega_q=3$), $\nu/\Omega=0.2$, and $\Omega a/u=13$. We see that as the particle crosses the boundary, the surface and space charges oscillate with a frequency $\Omega$, although the net induced charge remains equal to zero.

## 4. THE DYNAMICS OF CHARGES IN A CONDUCTING LAYER

Up to this point we have considered insulators. Now we examine a conducting layer with a dielectric constant

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega(\omega+i\nu)}. \tag{43}$$

The above formulas for the net surface charge do not work in this case since the dielectric constant of a plasma has a singularity at $\omega\to 0$.

As shown above, the net surface charge (21) is determined by the value of $\sigma_i(\omega)$. Here the terms in Eqs. (19) and (20) proportional to the hyperbolic sine sinh $ka$ (we assume that $\varepsilon_1=\varepsilon_3=1$ and $\varepsilon_2=\varepsilon(\omega)$) tend to finite limits. Hence, in contrast to insulators, in conductors the net surface charge is determined also by the surface fields generated by the particle.

To allow for this effect, let us examine the terms in Eqs. (13) and (20) that do not vanish as $k\to 0$:

$$\Delta\sigma_0(k,t)=-\frac{Quk}{(2\pi)^3}\sinh(ka)\int_{-\infty}^{+\infty}\frac{d\omega\exp(-i\omega t)}{\omega^2+k^2u^2}\times\frac{(\varepsilon-1)^2}{(\varepsilon^2+1)\sinh(ka)+2\varepsilon\cosh(ka)}, \tag{44}$$

$$\Delta\sigma_a(k,t)=\Delta\sigma_0(k,\tau), \tag{45}$$

where $\varepsilon(\omega)$ is determined by (43).

The contributions of the terms (44) and (45) to the net surface charge are, respectively,

$$\Delta Q_0(t)=(2\pi)^2\Delta\sigma_0(k=0,t), \tag{46}$$

$$\Delta Q_a(t)=(2\pi)^2\Delta\sigma_0(k=0,\tau). \tag{47}$$

Let us first find the integral in (44). To this end we write the expression (44) in the following form:

$$(2\pi)^2\Delta\sigma_0(k,t)=-\frac{Q}{2\pi}\int_{-\infty}^{+\infty}ds\,\frac{\exp(-ikust)}{(s^2+1)}\times\frac{[1-1/\varepsilon(kus)]^2}{[1+1/\varepsilon^2(kus)+[2/\varepsilon(kus)]\coth(ka)}. \tag{48}$$

Substituting (43) into this expression, we go to the limit $k\to 0$. As a result, for $\Delta Q_0$ and $\Delta Q_a$ we find that

$$\Delta Q_0=\Delta Q_a=-\frac{Q}{2\pi}\left(\frac{\omega_p^2 a}{2\nu u}\right)^2\times\int_{-\infty}^{+\infty}\frac{ds}{(s^2+1)(s^2+(\omega_p^2 a/2\nu u)^2)}=-\frac{Q}{2}\frac{k_p a}{k_p a+2\nu/\omega_p}, \tag{49}$$

where $k_p=\omega_p/u$.

With allowance for Eqs. (24), (28), (32), and (49), the expressions for the charges induced at the surface and in the volume of the conducting layer are

$$Q_{0s}(t)=-Q\frac{k_p a+\nu/\omega_p}{k_p a+2\nu/\omega_p}+Q\theta(t)\left[1-\exp\left(-\frac{\nu t}{2}\right)\cos(\omega_p t)\right], \tag{50}$$

$$Q_{as}(t)=Q\frac{\nu/\omega_p}{k_p a+2\nu/\omega_p}-Q\theta(\tau)\left[1-\exp\left(-\frac{\nu\tau}{2}\right)\cos(\omega_p\tau)\right], \tag{51}$$

$$Q_v(t)=-Q[\theta(t)-\theta(\tau)]\left[1-\exp\left(-\frac{\nu t}{2}\right)\cos(\omega_p t)\right]-Q\theta(\tau)\exp\left(-\frac{\nu\tau}{2}\right)\left[\cos(\omega_p\tau)-\exp\left(-\frac{\nu a}{2u}\right)\cos(\omega_p t)\right]. \tag{52}$$

The net charge induced in the conductor is

$$Q'=Q_{0s}(t)+Q_{as}(t)+Q_v(t)=-Q\frac{k_p a}{k_p a+2\nu/\omega_p}. \tag{53}$$

Note that in deriving (50)–(53) we employed the condition that $\nu\ll\omega_p$.

Equation (53) implies that for thick layers ($a\gg u/2\pi\sigma$, where $\sigma=\omega_p^2/4\pi\nu$ is the conductivity of the plasma) the net induced charge is equal to $-Q$, which corresponds to the results obtained for a semi-infinite conductor.[10] For a thin layer ($a\ll u/2\pi\sigma$) the induced charge $Q'$ is equal to $-Q\omega_p^2 a/2\nu u$, and tends to zero as $a\to 0$. Figure 2 depicts the time dependence of the charges induced in a plasma layer for $a\gg u/2\pi\sigma$ at $\nu/\omega_p=0.2$ and $\omega_p a/u=15$.

## 5. CONCLUSION

Let us briefly discuss the conditions in which the processes taking place at the boundaries of the layer can be considered independent and the boundary can be interpreted as that of a half-space, as was done in Ref. 10. To this end we examine the net induced charges at the layer boundaries at the moment when the particle crosses the second boundary ($t=a/u$):

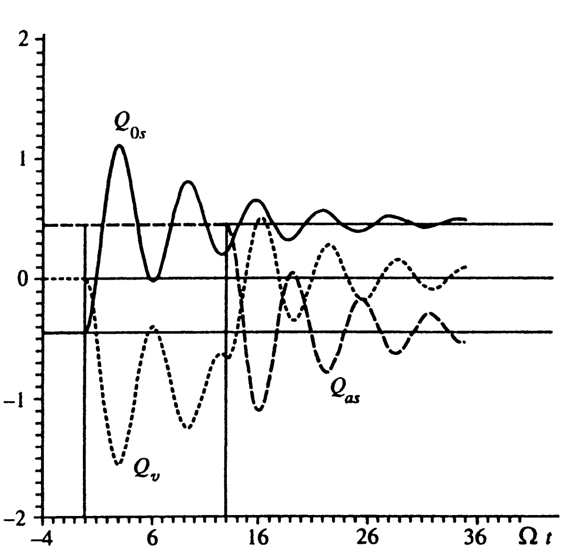


FIG. 2. The dynamics of the induced charge at the front boundary (solid curve), the rear boundary (dashed curve), and in the volume (dotted curve) of a thick plasma layer: $\alpha \gg u/2\pi\sigma$, $\nu/\omega_p = 0.2$, and $\omega_p\, a/u = 15$.

$$\frac{Q_{0s}}{Q} = \frac{1}{2}\left(1 - \frac{1}{\varepsilon_0}\right) - \sum_j \exp\left(-\nu_j \frac{a}{u}\right) \times [A_j \cos(k_j a) + B_j \sin(k_j a)],$$
$$\frac{Q_{as}}{Q} = \frac{1}{2}\left(1 - \frac{1}{\varepsilon_0}\right), \quad \frac{Q_v}{Q} = -\left(1 + \frac{1}{\varepsilon_0}\right) + \sum_j \exp\left(-\nu_j \frac{a}{u}\right)[A_j \cos(k_j a) + B_j \sin(k_j a)], \tag{54}$$

where $k_j = \omega_j/u$. We see that if the conditions $u/\omega_j < a < u/\nu_j$ are met, the $-Q(1 - 1/\varepsilon_0)$ has no time to transform into the wake charge before the particle reaches the second boundary. For this reason the transformation of a surface charge into a wake charge and the transformation of the latter into a surface charge at the second boundary are interrelated. When the particle crosses the second boundary of the layer, near the boundary it excites electric field oscillations[10] whose phase is related to that of the oscillations of the electric field near the first boundary. For the fields at the boundaries to be completely independent, $a$ must exceed $u/\nu_j$ (for a conductor this condition has the form $a > 2u/\nu$). In this case not only the amplitudes but also the phases of oscillations of the electric fields at the boundaries are independent.

As Eqs. (24), (30), and (32) imply, the net charge induced in an insulator layer at each moment of time is zero. In contrast, the net charge induced in a conducting layer is determined by the relationship between the quantities $a$ and $u/2\pi\sigma$. When $a \ll u/2\pi\sigma$, a conducting layer behaves like an insulator. When the layer is thick ($a \gg u/2\pi\sigma$), the net induced charge in the layer is equal to the particle charge with the opposite sign. In this limiting case, from the standpoint of the induced charges, the conducting layer behaves like a semi-infinite medium.

To explain the physical reason for this effect, let us first take a semi-infinite conductor. A charged particle at rest induces at the surface of the conductor a surface charge density whose field completely screens the field of the particle inside the conductor. If the particle is moving, the surface charge density necessary for screening its field has no time to set in. As a result the field penetrates the conductor. According to Ref. 10, a particle that is at a distance greater than $u/\nu$ from the conducting surface of the plasma and is moving toward the boundary of the plasma generates inside the plasma a potential of the dipole form

$$\varphi = \frac{p\xi}{(\xi^2 + r^2)^{3/2}}, \tag{55}$$

where $\xi = z - ut$ is the distance from the particle to the observation point with the coordinates $z$ and $r$, and $p = Qd$ is the dipole moment, with $d = u/2\pi\sigma$.

The field penetrating the conducting layer generates a surface charge at the rear boundary, with the magnitude of this charge depending on the layer's thickness. Equation (7) implies that the potential at the rear boundary is

$$\varphi(r,a,t) = Q \int_0^\infty dk\, J_0(kr) \times \frac{k(k+g)[k(k+g) + k_p^2]\exp[-k(a-ut)]}{[k(k+g) + k_0^2\alpha_-^2][k(k+g) + k_0^2\alpha_+^2]},$$

where $J_0(x)$ is the zeroth-order Bessel function, $\alpha_\pm^2 = 1 \pm \exp\{-ka\}$, $g = \nu/u$, and $k_0 = \omega_0/u$. If $a \ll d$, the electric fields at the front and rear boundaries of the layer are of the same order of magnitude, and the charges they induce are practically equal. But if $a \gg d$, the field at the rear boundary of the layer is much weaker than that at the front boundary, where most of the induced charge is concentrated.

The quantity $a = u/2\pi\sigma = (2u/\omega_p)(\nu/\omega_p)$ characterizes the thickness of the conducting layer at which its properties change in relation to the magnitude of the charge induced in the layer by the moving particle. For the sake of an example we take a layer of the semiconductor $n$-InSb with a carrier concentration $3 \times 10^{15}\,\mathrm{cm}^{-3}$, a plasma frequency $6.4 \times 10^{12}\,\mathrm{s}^{-1}$, and collision rate $10^{12}\,\mathrm{s}^{-1}$ (see, e.g., Ref. 16). For a 10-MeV proton we have $a \simeq 1.4 \times 10^{-4}\,\mathrm{cm}$. Obviously, the requirements on the quality of the layer's surface in this case are moderate and the surface roughness must be less than one micrometer.


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