Theory of instabilities emerging in the formation of a quark–gluon plasma

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We study collisions of two degenerate quark Fermi liquids and show that such processes initiate the formation of instabilities manifesting themselves in the propagation of growing oscillations related to the modes that exist in a Fermi liquid at rest. We believe that quark jets can be expected to appear in the direction of propagation of such oscillations. The instabilities we consider are similar to the beam instability in ordinary electron plasma. [S1063-7761(96)00308-3]

1. INTRODUCTION

The quark structure of hadrons and, therefore, of nuclei suggests that the collision of two fast heavy nuclei, i.e., two quark systems, may initiate the formation of a new state of matter, a quark–gluon plasma.[1] Each of the two colliding quark systems can be considered a generalized Fermi liquid whose particles have a color degree of freedom in addition to the spin degree of freedom. In this approach the interaction between the particles of the quark liquid caused by gluon exchange is described by generalized Landau amplitudes, as is done in Landau's classical theory of a Fermi liquid. Thus, in studying the formation of a quark–gluon plasma resulting from the collision of two nuclei we arrive at a simpler problem of the interaction of two droplets of a colored Fermi liquid in relative motion.[1)]

In the present paper we solve the problem in the nonrelativistic setting without taking the boundaries of the droplets into account, i.e., we consider the collision of two unbounded colored Fermi liquids. The main conclusion we draw is that the interaction of the two Fermi liquids initiates the formation of a number of instabilities. These instabilities show up within certain ranges of angles with respect to the direction of the relative velocity as growing oscillations related to waves that can exist in a quark–gluon plasma at rest. Among these waves are the modified Landau zero-point sound, modified spin waves, waves related to the excitation of the color degrees of freedom, and more complicated waves that result from a combination of these simple waves. The instabilities are similar to the familiar instability in an ordinary electron plasma when beams of charged particles are sent through the plasma.[3,4]

We believe that one should expect quark jets in the directions along which the growing oscillations propagate.

In the simplest case where the Landau amplitudes are constant quantities not depending on momentum and the interaction between the plasmas is weak, the instabilities appear in the range of angles ϑ_0 for which $\cos\vartheta_0 > s/v_0$, where ϑ_0 is the angle between the wave vector and the velocity vector $\mathbf{v}_0$ of the moving droplet, and s is the speed of the corresponding zero-point sound (ordinary, spin, or color).

2. THE BASIC EQUATIONS

To extend Landau's theory of a Fermi liquid[5] to the case of a colored Fermi liquid, we assume that the state of the liquid is described by a one-particle distribution function $f_i(\mathbf{x},\mathbf{p},t)$, where i denotes the spin, color, and species of the quark, $\mathbf{x}$ is the quark coordinate, and $\mathbf{p}$ is the quark momentum.

Just as in a macroscopic theory the system Hamiltonian determines the dynamics of the particles, in the Landau theory of a Fermi liquid the energy density functional $\mathscr{E}(f)$ determined by a one-particle distribution function plays a similar role. With this functional we can introduce the energy of quasiparticles,

$$\delta\mathscr{E} = \sum \int d\tau \varepsilon_i \delta f_i, \quad \varepsilon_i(\mathbf{x},\mathbf{p}) = \frac{\delta\mathscr{E}(f)}{\delta f_i(\mathbf{x},\mathbf{p})},$$

where $d\tau = d\mathbf{p}/(2\pi\hbar)^3$, which in turn determines the collisionless ($\omega\tau \gg 1$, where ω is the frequency, and τ is the relaxation time) kinetic equations for the distribution function $f_i(\mathbf{x},\mathbf{p},t)$:

$$\frac{\partial f_i}{\partial t} + \frac{\partial \varepsilon_i}{\partial \mathbf{p}}\frac{\partial f_i}{\partial \mathbf{x}} - \frac{\partial \varepsilon_i}{\partial \mathbf{x}}\frac{\partial f_i}{\partial \mathbf{p}} = 0.$$

Linearizing these equations near the equilibrium state $f_{0i}(\mathbf{p})$, which is determined by the requirement that the entropy

$$S = -\sum_i \int d\tau [f_i \ln f_i + (1-f_i)\ln(1-f_i)]$$

be at its maximum for fixed integrals of motion, and going over to the Fourier transforms of the deviations $g_i = f_i - f_{0i}$ of the distribution functions from their equilibrium values, i.e.,

$$\bar{g}_i(\omega,\mathbf{k},\mathbf{p}) = \int d^3x\, dt\, g_i(\mathbf{x},\mathbf{p})\exp(i\omega t - i\mathbf{k}\mathbf{x}),$$

we arrive at a linearized kinetic equation for $\bar{g}_i(\omega,\mathbf{k},\mathbf{p})$:

$$\left(\omega - \mathbf{k}\frac{\partial \varepsilon_0}{\partial \mathbf{p}}\right)\bar{g}_i(\omega,\mathbf{k},\mathbf{p}) + \mathbf{k}\frac{\partial f_{0i}}{\partial \mathbf{p}}\int d\tau' \sum_j F_{ij}(\mathbf{p},\mathbf{p}')\bar{g}_j(\omega,\mathbf{k},\mathbf{p}') = 0,$$

where the Landau amplitudes $F_{ij}(\mathbf{p},\mathbf{p}')$ are determined by the following formula:

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$$\delta\varepsilon_i(\mathbf{p})=\sum_j \int d\tau' F_{ij}(\mathbf{p},\mathbf{p}')\delta f_j(\mathbf{p}'),$$

$$F_{ij}(\mathbf{p},\mathbf{p}')=\left.\frac{\delta^2\mathscr{E}(f)}{\delta f_i\delta f_j}\right|_{f_i=f_{0i}}$$

These formulas belong to the case where only one Fermi liquid is considered. With two interacting Fermi liquids, which is what we are interested in, two distribution functions must be introduced, f_i and $f_{j'}$ (the quantum numbers j' refer to the second Fermi liquid). In order not to violate the Pauli principle we assume that the Fermi-liquid droplets consist of quarks of different species. But if the droplets consist of identical quarks, the system of two droplets must be described by a single distribution function. The latter case is discussed in Sec. 4.

The energy density functional depends on f_i and $f_{j'}$, i.e., $\mathscr{E}=\mathscr{E}(f_i,f_{j'})$. Then the quasiparticle energies are

$$\varepsilon_j(f_i,f_{i'})=\frac{\delta\mathscr{E}(f_i,f_{i'})}{\delta f_j},\quad \varepsilon_{j'}(f_i,f_{i'})=\frac{\delta\mathscr{E}(f_i,f_{i'})}{\delta f_{j'}}.$$

The kinetics of the two Fermi liquids is described by the following system of equations:

$$\frac{\partial f_i}{\partial t}+\frac{\partial\varepsilon_i}{\partial\mathbf{p}}\frac{\partial f_i}{\partial\mathbf{x}}-\frac{\partial\varepsilon_i}{\partial\mathbf{x}}\frac{\partial f_i}{\partial\mathbf{p}}=0,$$

$$\frac{\partial f_{i'}}{\partial t}+\frac{\partial\varepsilon_{i'}}{\partial\mathbf{p}}\frac{\partial f_{i'}}{\partial\mathbf{x}}-\frac{\partial\varepsilon_{i'}}{\partial\mathbf{x}}\frac{\partial f_{i'}}{\partial\mathbf{p}}=0.$$

We denote the equilibrium distribution functions for the Fermi liquids at rest by $f_{i0}(\mathbf{p})$ and $f_{i'0}(\mathbf{p})$ and assume that prior to a collision the Fermi-liquid droplets were in an equilibrium state. Then in the situation when one Fermi liquid is at rest and the other is moving with a velocity $\mathbf{v}_0$, the equilibrium distribution functions are $f_{i0}(\mathbf{p})$ and $f_{i'0}(\mathbf{p}-m_{i'}\mathbf{v}_0)$, where m_j is the mass of a particle of the jth species.

Thus, the linearized kinetic equations for the colliding liquids assume the form

$$\left(\omega-\mathbf{k}\frac{\partial\varepsilon_{0i}}{\partial\mathbf{p}}\right)\bar{g}_i(\omega,\mathbf{k},\mathbf{p})+\mathbf{k}\frac{\partial f_{0i}}{\partial\mathbf{p}}\int d\tau'\left[\sum_k F_{ik}(\mathbf{p},\mathbf{p}')\bar{g}_k(\omega,\mathbf{k},\mathbf{p}')+\sum_{k'} G_{ik'}(\mathbf{p},\mathbf{p}')\bar{g}_{k'}(\omega,\mathbf{k},\mathbf{p}')\right]=0, \tag{1}$$

$$\left(\omega-\mathbf{k}\frac{\partial\varepsilon_{0i'}}{\partial\mathbf{p}}\right)\bar{g}_{i'}(\omega,\mathbf{k},\mathbf{p})+\mathbf{k}\frac{\partial f_{0i'}}{\partial\mathbf{p}}\int d\tau'\left[\sum_{k'} F_{i'k'}(\mathbf{p},\mathbf{p}')\bar{g}_{k'}(\omega,\mathbf{k},\mathbf{p}')+\sum_{i} G_{i'i}(\mathbf{p},\mathbf{p}')\bar{g}_{i}(\omega,\mathbf{k},\mathbf{p}')\right]=0. \tag{2}$$

Here F_{ik} and $F_{i'k'}$ are the Landau amplitudes inside the first and second liquids, respectively,

$$F_{ik}=\left.\frac{\delta^2\mathscr{E}(f)}{\delta f_i\delta f_k}\right|_{f=f_0},\quad F_{i'k'}=\left.\frac{\delta^2\mathscr{E}(f)}{\delta f_{i'}\delta f_{k'}}\right|_{f=f_0},$$

and

$$G_{ii'}=\left.\frac{\delta^2\mathscr{E}(f)}{\delta f_i\delta f_{i'}}\right|_{f=f_0}$$

determines the Landau amplitude of the interaction between the liquids. (At $G_{ii'}=0$ there is no interaction between the liquids.)

The system of equations (1) and (2) makes it possible to determine the laws of dispersions of the coupled vibrations in the system of interacting Fermi liquids.

3. SOLVING THE KINETIC EQUATIONS

To find the solutions of the kinetic equations (1) and (2) we use the Landau model, according to which the interaction amplitudes F_{ik}, $F_{i'k'}$, and $G_{ii'}$ are independent of the momenta $\mathbf{p}$ and $\mathbf{p}'$. This assumption is equivalent to the requirement that the interaction energy depend on the distribution function $f_i(\mathbf{p})$ only through the number densities ρ_i of the droplet particles:

$$\mathscr{E}=\sum_i \int d\tau\,\varepsilon_i(\mathbf{p})+\sum_{i'}\int d\tau\,\varepsilon_{i'}(\mathbf{p})+\mathscr{E}',$$

where $\varepsilon_i(\mathbf{p})=\mathbf{p}^2/2m$, and

$$\mathscr{E}'=\mathscr{E}'(\rho_i,\rho_{i'}),\quad \rho_i=\int d\tau\, f_i(\mathbf{p}),\quad \rho_{i'}=\int d\tau\, f_{i'}(\mathbf{p}).$$

The Landau amplitudes for this case are

$$F_{ik}=\frac{\partial^2\mathscr{E}'}{\partial\rho_i\partial\rho_k},\quad F_{i'k'}=\frac{\partial^2\mathscr{E}'}{\partial\rho_{i'}\partial\rho_{k'}},\quad G_{ik'}=\frac{\partial^2\mathscr{E}'}{\partial\rho_i\partial\rho_{k'}}.$$

Let us assume that the equilibrium distribution functions are "unsmeared" Fermi steps, so that

$$\frac{\partial f_{0i}}{\partial\mathbf{p}}=-\mathbf{v}_i(\mathbf{p})\delta(\varepsilon_i(\mathbf{p})-\mu_i),\quad \mathbf{v}_i(\mathbf{p})=\frac{\mathbf{p}_i}{m_i},$$

$$\frac{\partial f_{0i'}}{\partial\mathbf{p}}=-[\mathbf{v}_{i'}(\mathbf{p})-\mathbf{v}_0]\delta(\varepsilon_{i'}(\mathbf{p}-m_{i'}\mathbf{v}_0)-\mu_{i'}).$$

Then assuming that

$$\bar{g}_i(\mathbf{p})=q_i(\mathbf{p})\delta(\varepsilon_i(\mathbf{p})-\mu_i)\sqrt{\nu_i},$$

$$\bar{g}_{i'}(\mathbf{p})=q_{i'}(\mathbf{p}-m_{i'}\mathbf{v}_0)\delta(\varepsilon_{i'}(\mathbf{p}-m_{i'}\mathbf{v}_0)-\mu_{i'})\sqrt{\nu_{i'}},$$

where $\nu_i=\int d\tau\delta(\varepsilon_i(\mathbf{p})-\mu_i)$, with μ_i the chemical potential of the particles of the ith species, and substituting $\mathbf{p}+m\mathbf{v}_0$ for $\mathbf{p}$ in Eq. (2) we get

$$[\omega-\mathbf{k}\mathbf{v}_i(\mathbf{p})]q_i(\mathbf{p})$$

$$-\frac{\mathbf{k}\mathbf{v}_i(\mathbf{p})}{\sqrt{\nu_i}}\int d\tau'\left[\sum_k F_{ik}\sqrt{\nu_k}q_k(\mathbf{p}')\delta(\varepsilon_k(\mathbf{p}')-\mu_k)\right.$$

$$\left.+\sum_{k'} G_{ik'}\sqrt{\nu_{k'}}q_{k'}(\mathbf{p}')\delta(\varepsilon_{k'}(\mathbf{p}')-\mu_{k'})\right]=0, \quad (3)$$

$$[\omega-\mathbf{k}(\mathbf{v}_{i'}(\mathbf{p})+\mathbf{v}_0)]q_{i'}(\mathbf{p})$$

$$-\frac{\mathbf{k}\mathbf{v}_{i'}(\mathbf{p})}{\sqrt{\nu_{i'}}}\int d\tau'\left[\sum_{k'} F_{i'k'}\sqrt{\nu_{k'}}q_{k'}(\mathbf{p}')\delta(\varepsilon_{k'}(\mathbf{p}')\right.$$

$$\left.-\mu_{k'})+\sum_k G_{i'k}\sqrt{\nu_k}q_k(\mathbf{p}')\delta(\varepsilon_k(\mathbf{p}')-\mu_k)\right]=0. \quad (4)$$

By introducing the notation

$$x_i=\int d\tau\, q_i(\mathbf{p})\delta(\varepsilon_i(\mathbf{p})-\mu_i),$$

$$x_{i'}=\int d\tau\, q_{i'}(\mathbf{p})\delta(\varepsilon_{i'}(\mathbf{p})-\mu_{i'}),$$

and

$$A_i(\omega,\mathbf{k})=\frac{1}{\nu_i}\int d\tau\ \delta(\varepsilon_i(\mathbf{p})-\mu_i)\frac{\mathbf{k}\mathbf{v}_i}{\omega-\mathbf{k}\mathbf{v}_i(\mathbf{p})}, \quad (5)$$

$$B_i(\omega,\mathbf{k})=\frac{1}{\nu_i}\int d\tau\ \delta(\varepsilon_i(\mathbf{p})-\mu_i)\frac{\mathbf{k}\mathbf{v}_i}{\omega-\mathbf{k}\mathbf{v}_i(\mathbf{p}+m\mathbf{v}_0)}, \quad (6)$$

we transform the integral equations (3) and (4) into the following system of algebraic equations:

$$x_i-A_i(\omega,\mathbf{k})\left(\sum_j \mathscr{F}_{ij}x_j+\sum_{k'}\mathscr{G}_{ik'}x_{k'}\right)=0, \quad (7)$$

$$x_{i'}-B_{i'}(\omega,\mathbf{k})\left(\sum_{k'} \mathscr{F}_{i'k'}x_{k'}+\sum_{i}\mathscr{G}_{i'i}x_{i}\right)=0, \quad (8)$$

where $\mathscr{F}_{ij}=\sqrt{\nu_i\nu_j}F_{ij}$ and $\mathscr{G}_{ik}=\sqrt{\nu_i\nu_k}G_{ik}$ are the dimensionless interaction amplitudes.

The consistency condition for this system of equations leads to the dispersion equation.

If we assume that the interaction between the quarks is invariant under spin and color transformations (the SU(2) and SU(3) groups), then the structure of the Landau amplitude for a quark liquid is

$$\mathscr{F}_{ij}=\mathscr{F}^{(0)}_{ij}+\boldsymbol{\sigma\sigma}\mathscr{F}^{s}_{ij}+\lambda_a\lambda_a\mathscr{F}^{c}_{ij}+\boldsymbol{\sigma\sigma}\lambda_a\lambda_a\mathscr{F}^{sc}_{ij},$$

where $\mathscr{F}^{0}_{ij}$, $\mathscr{F}^{s}_{ij}$, $\mathscr{F}^{c}_{ij}$, and $\mathscr{F}^{sc}_{ij}$ are the generalized Landau amplitudes, $\boldsymbol{\sigma}$ stands for the Pauli matrices, and λ_a stands for the Gell-Mann matrices. Then Eqs. (7) and (8) contain one of the dimensionless amplitudes $\mathscr{F}^{0}_{ij}$, $\mathscr{F}^{s}_{ij}$, $\mathscr{F}^{c}_{ij}$, or $\mathscr{F}^{sc}_{ij}$ depending on whether zero-point vibrations of the liquid density, spin density, color density, or spin–color density are considered.

To simplify these equations we assume that one droplet consists of particles of one species and the other of particles of another species. Then the system of equations (7) and (8) becomes a system of the following two equations:

$$x_1-A(\omega,\mathbf{k})(\mathscr{F}_1x_1+\mathscr{G}x_2)=0, \quad (9)$$

$$x_2-B(\omega,\mathbf{k})(\mathscr{F}_2x_2+\mathscr{G}x_1)=0, \quad (10)$$

where $\mathscr{F}_1$ and $\mathscr{F}_2$ are the Landau amplitudes for the first and second droplets, and $\mathscr{G}$ is the Landau amplitude describing the interaction of the droplets. The dispersion equation is

$$(1-A\mathscr{F}_1)(1-B\mathscr{F}_2)-AB\mathscr{G}^2=0. \quad (11)$$

Its solution is most easily found when $\mathscr{G}$ is small compared to $\mathscr{F}_1$ and $\mathscr{F}_2$. Then in the zeroth approximation the dispersion equation assumes the form $(1-A\mathscr{F}_1)\times(1-B\mathscr{F}_2)=0$. Let as assume that $\mathscr{F}_1>0$. Then, according to Ref. 5, zero-point sound can propagate in the droplet at rest:

$$1-AF_1=0, \quad \omega=kv_F\eta, \quad (12)$$

where the dimensionless speed of sound

$$\eta=\frac{\omega}{kv_F}=\frac{s}{v_F}, \quad s=\frac{\omega}{k},$$

is determined from the equation

$$\frac{1}{2}\eta\ln\frac{\eta+1}{\eta-1}-1=\frac{1}{\mathscr{F}_1},$$

with $\eta>1$. Now let us allow for the interaction between the liquids and set $\omega=kv_F\eta+\delta$ in Eq. (12). This leads to the following expression for δ:

$$\delta=-\left.\frac{AB\mathscr{G}^2}{(1-B\mathscr{F}_2)(\partial A/\partial\omega)\mathscr{F}_1}\right|_{\omega=kv_F\eta}.$$

Generally δ is a complex-valued quantity. Its imaginary part determines the damping (or growth) rate of the oscillations

$$\operatorname{Im}\delta=-\left(\frac{\mathscr{G}}{\mathscr{F}_1}\right)^2\frac{1}{\partial A/\partial\omega}\left.\frac{\operatorname{Im}B}{|1-B\mathscr{F}_2|^2}\right|_{\omega=kv_F\eta}.$$

Clearly [see Eq. (5)],

$$\frac{\partial A}{\partial\omega}=\frac{1}{kv_F}\left(\ln\frac{\eta+1}{\eta-1}-\frac{2\eta}{\eta^2-1}\right)<0.$$

According to Eq. (6),

$$\operatorname{Im}B|_{\omega=sk}=\frac{\pi y\theta}{2}(1-y^2), \quad \operatorname{Re}B|_{\omega=sk}=\frac{y}{2}\ln\frac{1+y}{1-y}-1,$$

where

$$y=\frac{v_0}{v_F'}\cos\vartheta_0-\frac{s}{v_F'}=\frac{v_0}{v_F'}\cos\vartheta_0-\eta\zeta, \quad \zeta=\frac{v_F}{v_F'},$$

with v_F and v_F' the Fermi velocities in the first (at rest) and second (incident) droplets.

Damping corresponds to the condition $\operatorname{Im}\delta<0$, or $s>v_0\cos\vartheta_0$. Growth corresponds to $\operatorname{Im}\delta>0$, or $s<v_0\cos\vartheta_0$, where ϑ_0 is the angle between $\mathbf{k}$ and $\mathbf{v}_0$. The interval $0<y<1$ corresponds to growth.

The maximum value of the oscillation growth rate occurs for angles ϑ_0 at which the function $\operatorname{Im}B/|1-B\mathscr{F}_2|^2$

reaches its maximum. Since this is a positive function that vanishes at $y=0$ and $y=1$, the conditions for its maximum is

$$\frac{\partial}{\partial y}\frac{\mathrm{Im}\,B}{|1-B\mathscr{F}_2|^2}=0.$$

According to (6), the last equation can be written as

$$1+\frac{2\mathscr{F}_2}{1-y^2}-\mathscr{F}_2^2\left[\left(\frac{y}{2}\ln\frac{1+y}{1-y}-1\right)\left(\frac{y}{2}\ln\frac{1+y}{1-y}+\frac{1+y^2}{1-y^2}\right)+\frac{\pi^2y^2}{4}\right]=0,\quad 0<y<1. \tag{13}$$

This equation yields y as a function of $\mathscr{F}_2$.

Let us first assume that $\mathscr{F}_2$ is small. For $\mathscr{F}_2<0$ (natural zero-point sound vibrations do not propagate in the second droplet), the solution of Eq. (13) is

$$y=-\zeta\eta+\frac{v_0}{v_F'}\cos\vartheta_0=1+\mathscr{F}_2. \tag{14}$$

But for $\mathscr{F}_2>0$ (natural zero-point sound vibrations propagate in the second droplet), the solution of Eq. (13) becomes

$$y=-\zeta\eta+\frac{v_0}{v_F'}\cos\vartheta_0=1-2\exp\left(-\frac{2}{\mathscr{F}_2}-2\right). \tag{15}$$

We see that the maximum growth rate is realized in the direction (in relation to the velocity of the incident Fermi liquid) defined by the following condition:

$$\cos\vartheta_0=\begin{cases}\dfrac{v_F'}{v_0}(1+\mathscr{F}_2+\zeta\eta), & \mathscr{F}_2<0,\\[2ex] \dfrac{v_F'}{v_0}\left[1-2\exp\left(-\dfrac{2}{\mathscr{F}_2}-2\right)+\zeta\eta\right], & \mathscr{F}_2>0.\end{cases} \tag{16}$$

In the region where $\mathscr{F}_2$ is large, Eq. (13) for finding the directions in which the growth rate is largest assumes the form

$$f(y)\equiv\left(\frac{y}{2}\ln\frac{1+y}{1-y}-1\right)\left[(1-y^2)\frac{y}{2}\ln\frac{1+y}{1-y}+1\right]+\pi^2y^2(1-y^2)=0, \tag{17}$$

with $y=-\zeta\eta+(v_0/v_F')\cos\vartheta_0$, and $0<y<1$. Since we have $f(0)>0$ and $f(1)<0$, Eq. (17) always has a solution $y=y_0$ in the interval $0<y<1$ (y_0 is a numerical constant). Thus,

$$y_0=-\zeta\eta+\frac{v_0}{v_F'}\cos\vartheta_0,$$

which implies that

$$\cos\vartheta_0=\frac{v_F'}{v_0}(\zeta\eta+y_0). \tag{18}$$

Estimating the growth rate requires estimating the Landau amplitudes. To this end we assume that the Fourier transform of the quark–quark interaction potential has the form

$$U_0\sim\frac{g^2\hbar^2}{|\mathbf{p}-\mathbf{p}'|^2},$$

where $\mathbf{p}-\mathbf{p}'$ is the momentum transfer, and g is a constant defined below. Assuming $|\mathbf{p}-\mathbf{p}'|\sim p_F$, we can estimate the dimensionless Landau amplitude $\mathscr{F}$ at

$$\mathscr{F}\sim\alpha_s\frac{\lambda_q}{a}\sim\alpha_s\frac{c}{v_F},$$

where $\alpha_s=g^2/\hbar c$ is the strong interaction constant, λ_q is the de Broglie wavelength of the quark, and a is the average separation of quarks in a quark droplet ($\alpha_s\sim0.1$ and $v_F\sim0.1c$). Using this estimate of the Landau amplitude and assuming that qualitatively the formula for the growth rate is applicable to the case where all amplitudes are quantities of the same order, we arrive at the following estimate for the growth rate:

$$\frac{\mathrm{Im}\,\delta}{skv_F}\sim1.$$

But if the amplitudes are small,

$$\frac{\mathrm{Im}\,\delta}{skv_F}\sim\pi\exp\left(-\frac{1}{\mathscr{F}}\right).$$

Up to this point we have examined the case of small Landau amplitudes $\mathscr{G}$. But what happens when $\mathscr{G}$ is large? Equation (11) shows that in this case the wave frequencies are large, too. Hence, expanding the functions $A(\eta)$ and $B(\eta)$ in power series in $1/\eta=kv_F/\omega$, we can write the dispersion equation as

$$3\eta y=\pm\mathscr{G},\quad y=-\zeta\eta+\frac{v_0}{v_F'}\cos\vartheta_0. \tag{19}$$

The solutions of this equation have the form

$$\eta=\frac{v_0}{2\zeta v_F'}\cos\vartheta_0\pm\frac{1}{\zeta}\sqrt{\frac{1}{4}\left(\frac{v_0}{v_F'}\cos\vartheta_0\right)^2\mp\frac{\zeta}{3}\mathscr{G}},\quad |\mathscr{G}|\gg1. \tag{20}$$

We see that instability develops in directions specified by the condition

$$\frac{1}{4}\left(\frac{v_0}{v_F'}\cos\vartheta_0\right)^2<\frac{\zeta}{3}|\mathscr{G}|.$$

We conclude this section by studying the behavior in momentum space of the nonequilibrium distribution functions of two colliding quark droplets of different species. This clarifies the conditions for correlations to emerge between the direction of propagation of the outgoing quarks and the direction of the relative velocity $\mathbf{v}_0$ of droplet motion.

Equations (1) and (2) suggest that for constant Landau amplitudes $\mathscr{F}_1$, $\mathscr{F}_2$, and $\mathscr{G}$ the deviations $\bar{g}_1(\omega,\mathbf{k},\mathbf{p})$ and $\bar{g}_2(\omega,\mathbf{k},\mathbf{p})$ of the distribution functions from their equilibrium values are determined by the following equations:

$$\bar{g}_1(\omega,\mathbf{k},\mathbf{p})=-\frac{\mathbf{k}\partial f_{01}(\mathbf{p})/\partial\mathbf{p}}{\omega-\mathbf{k}\partial\varepsilon_1(\mathbf{p})/\partial\mathbf{p}}\left[\mathscr{F}_1\int d\tau'\bar{g}_1(\omega,\mathbf{k},\mathbf{p}')+\mathscr{G}\int d\tau'\bar{g}_2(\omega,\mathbf{k},\mathbf{p}')\right], \tag{21}$$

$$\bar{g}_2(\omega,\mathbf{k},\mathbf{p})=-\frac{\mathbf{k}\partial f_{02}(\mathbf{p}-m_2\mathbf{v}_0)/\partial\mathbf{p}}{\omega-\mathbf{k}\partial\varepsilon_2(\mathbf{p})/\partial\mathbf{p}}\left[\mathscr{F}_2\int d\tau'\bar{g}_2(\omega,\mathbf{k},\mathbf{p}')+\mathscr{G}\int d\tau'\bar{g}_1(\omega,\mathbf{k},\mathbf{p}')\right], \tag{22}$$

which imply that the nonequilibrium distribution functions $f_{(1,2)}=f_{0(1,2)}+g_{(1,2)}$ can have a maximum if

$$\omega-\mathbf{k}\frac{\partial\varepsilon_1(\mathbf{p})}{\partial\mathbf{p}}=0 \quad \text{or} \quad \omega-\mathbf{k}\frac{\partial\varepsilon_2(\mathbf{p})}{\partial\mathbf{p}}=0,$$

which can be written as

$$\cos\vartheta=\eta, \quad \cos\vartheta=\zeta\eta, \tag{23}$$

where ϑ is the angle between $\mathbf{k}$ and $\mathbf{v}$.

The first condition cannot be satisfied because we have $\eta>1$, which corresponds to the propagation of zero-point sound vibrations in the droplet that is at rest. The condition in (23) cannot be met if $\zeta\eta>1$ holds, because in this case there can be no correlation between the direction of the quark momentum and that of $\mathbf{v}_0$, and the expected distribution of the outgoing quarks should be isotropic.

Features of the distribution function of the moving droplet in momentum space may emerge when $\zeta\eta<1$. For $\zeta\eta\approx1$, the quark momentum $\mathbf{p}$ is directed practically along the wave vector $\mathbf{k}$ (here we have $\vartheta\ll1$ and $v_F/v_F'<1$). In this case the direction of the quark momentum $\mathbf{p}$ in relation to the beam velocity $\mathbf{v}_0$ is determined by the angle ϑ_0. Therefore, along the direction specified by condition (16), where the growth rates of zero-point sound waves are at their maximum, we should expect quark jets to emerge as a result of heavy-ion collisions.

For $\zeta\eta\leqslant1$, in spite the presence of such structure, we should expect a uniform distribution of the outgoing quarks, since the angles ϑ and ϑ_0 are of the same order.

4. THE DISPERSION EQUATION IN THE CASE OF IDENTICAL DROPLETS

Up till now we have assumed that the droplets consist of quarks of different species. But, as noted earlier, if the droplets consist of particles of one species, we must use a single distribution function $f(\mathbf{x},\mathbf{p})$ and one kinetic equation instead of two. Let us denote the equilibrium distribution function of the two droplets by $f_0(\mathbf{p})$. Then the Fourier transform $g_{\omega,\mathbf{k}}(\mathbf{p})$ of the deviation of the distribution function $f(\mathbf{x},\mathbf{p})$ from the equilibrium value, $g=f-f_0$, obeys the linearized kinetic equation

$$\left(\omega-\mathbf{k}\frac{\partial\varepsilon(\mathbf{p})}{\partial\mathbf{p}}\right)g_{\omega,\mathbf{k}}(\mathbf{p})+\mathbf{k}\frac{\partial f_0}{\partial\mathbf{p}}\int d\tau' F(\mathbf{p},\mathbf{p}')g_{\omega,\mathbf{k}}(\mathbf{p}')=0.$$

When the Landau amplitude is constant, this equation yields a dispersion equation for determining the oscillations spectrum of a quark liquid:

$$1-\mathscr{F}\underline{A}(\omega,\mathbf{k})=0, \tag{24}$$

$$\underline{A}(\omega,\mathbf{k})=-\frac{1}{\nu}\int d\tau\frac{\mathbf{k}\partial f_0/\partial\mathbf{p}}{\omega-\mathbf{kv}}. \tag{25}$$

Now let us discuss the problem of determining the equilibrium distribution function of two identical quark droplets. We assume that this function has the form

$$f_0(\mathbf{p})=p_1f_0^{(1)}(\mathbf{p})+p_2f_0^{(2)}(\mathbf{p}-m\mathbf{v}_0), \tag{26}$$

where $f_0^{(1)}(\mathbf{p})$ is the distribution function of the quarks in the first droplet, and $f_0^{(2)}(\mathbf{p})$ is the distribution function of the quarks in the second (moving) droplet, with $p_1+p_2=1$ and $0\leqslant p_i\leqslant1$ $(i=1,2)$. These conditions ensure $0\leqslant f_0(\mathbf{p})\leqslant1$ (the Pauli principle) if the functions $f_0^{(1)}$ and $f_0^{(2)}$ lie between zero and unity. Here the particle number density

$$n=\frac{1}{V}\sum_{\mathbf{p}}f_0(\mathbf{p})$$

can be written as $n=n_1p_1+n_2p_2$, where n_1 and n_2 are determined by the chemical potentials of the drops, μ_1 and μ_2, from the conditions

$$n_i=\frac{1}{V}\sum_{\mathbf{p}}f_0^{(i)}(\mathbf{p}).$$

Plugging (26) into the expression (25) for $\underline{A}(\omega,\mathbf{k})$, we obtain

$$\underline{A}(\omega,\mathbf{k})=p_1A(\omega,\mathbf{k})+p_2B(\omega,\mathbf{k}), \tag{27}$$

where the quantities $A(\omega,\mathbf{k})$ and $B(\omega,\mathbf{k})$ are given by formulas similar to (5) and (6):

$$A(\omega,\mathbf{k})=-\frac{1}{\nu}\int d\tau\frac{\mathbf{k}\partial f_0^{(1)}(\mathbf{p})/\partial\mathbf{p}}{\omega-\mathbf{kv}},$$

$$B(\omega,\mathbf{k})=-\frac{1}{\nu}\int d\tau\frac{\mathbf{k}\partial f_0^{(2)}(\mathbf{p})/\partial\mathbf{p}}{\omega-\mathbf{kv}(\mathbf{p}+m\mathbf{v}_0)}.$$

Thus, according to (24) and (27), the dispersion equation for the case of two colliding droplets consisting of quarks of one species assumes the form

$$1-p_1\mathscr{F}A(\omega,\mathbf{k})-p_2\mathscr{F}B(\omega,\mathbf{k})=0. \tag{28}$$

This equation can be analyzed for $p_2\ll p_1$. Then, taking Eq. (28) in covert order,

$$1-p_1\mathscr{F}A(\omega,\mathbf{k})=0,$$

we can find the spectrum of zero-point sound waves $\omega=kv_F\eta$, where the dimensionless speed of sound η can be determined from the equation

$$\frac{\eta}{2}\ln\frac{\eta+1}{\eta-1}-1=\frac{1}{p_1\mathscr{F}}.$$

Assuming that $\omega=kv_F\eta+\delta$ and performing the necessary calculations similar to those done in connection with Eq. (11), we find that

$$\delta=-\frac{p_2}{p_1}\frac{B}{\partial A/\partial\omega}\bigg|_{\omega=kv_F\eta}, \quad \mathrm{Im}\ \delta=-\frac{p_2}{p_1}\frac{\mathrm{Im}\ B}{\partial A/\partial\omega}\bigg|_{\omega=kv_F\eta},$$

$$\mathrm{Im}\ B=\frac{\pi}{2}\left(\frac{v_0}{v_F}\cos\vartheta_0-\zeta\eta\right).$$

The condition $\mathrm{Im}\ \delta>0$, or $s<v_0\cos\vartheta_0$, corresponds to growth.

In conclusion, we note that the present method of studying instabilities that develop in the collision of two degenerate liquids can be applied not only to a quark–gluon plasma but also to other degenerate Fermi systems.

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[1)] A description of collisions of heavy ions that uses the Vlasov equation for a colorless plasma can be found in Ref. 2.

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