

PELTIER-EFFECT-INDUCED CORRECTION TO OHMIC RESISTANCE

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The standard ohmic measurements by means of two extra leads contain an additional thermal correction to resistance. The current results in heating (cooling) at the first (second) sample contact because of the Peltier effect. The contact temperatures are different. The measured voltage is the sum of the ohmic voltage swing and the Peltier-effect-induced thermoelectromotive force that is linear in the current. As a result, the thermal correction to the resistance measured exists as $I \rightarrow 0$. The correction could be comparable with the ohmic resistance. Above some critical frequency depending on thermal inertial effects, the thermal correction disappears.

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It is well known that ohmic measurements (see Fig. 1) are carried out at low current density in order to prevent heating. Usually, only the Joule heat is considered to be important. In contrast to the Joule heat, the Peltier and Thomson effects are linear in the current. The crucial point of the present paper is that the Peltier effect which is linear in current influences the ohmic measurements and results in a correction to the resistance measured. Under current carrying conditions, one of the sample contacts is heated and the other is cooled because of Peltier effect. The temperature gradient established is proportional to the current. The Thomson heat is then proportional to square of the current and can therefore be neglected. Finally, the voltage swing across the circuit includes the thermoelectromotive force induced by the Peltier effect, which is linear in current. Accordingly, there exists a thermal correction to the ohmic resistance of the sample.

First we consider an isotropic (or of cubic symmetry) conductor that can be in the thermodynamic nonequilibrium with respect to conducting electrons. In general, the current density \mathbf{j} and the energy flux density \mathbf{q} of the inhomogeneous conductor are given by [1]

$$\mathbf{j} = \sigma(\mathbf{E} - \alpha \nabla T), \quad (1)$$

$$\mathbf{q} = (\phi + \alpha T)\mathbf{j} - \varkappa \nabla T, \quad (2)$$

where σ is the conductivity, α is the thermopower, and \varkappa is the thermal conductivity. For an inhomogeneous conductor, the potential $\phi = \varphi + \mu/e$ is the sum of the electric potential φ and the chemical potential μ of conducting electrons. For a homogeneous conductor, the above definition of the potential differs from φ by a constant, and the average microscopic electric field $-\nabla\varphi$ therefore coincides with $\mathbf{E} = -\nabla\phi$. The first term in Eq. (1) corresponds to the conventional Ohm's law and the second term describes thermoelectric phenomena. For the steady state,

$$\operatorname{div} \mathbf{j} = 0, \quad (3)$$

$$Q = -\operatorname{div} \mathbf{q} = \operatorname{div}(\varkappa \nabla T) + j^2/\sigma - jT\nabla\alpha = 0, \quad (4)$$

where Q is the total amount of heat evolved per unit time and unit volume of the conductor. The current flow is accompanied by both the Joule and Thomson heats that are proportional to the second and first power of the current, respectively. Using Eqs. (1)–(4), one can find the potential $\phi(\mathbf{r})$ and the temperature $T(\mathbf{r})$ for the conductor under given boundary conditions.

Now we consider the thermal effects in connection with ohmic measurements of the conductor resistance (Fig. 1). The conductor is connected by means of two

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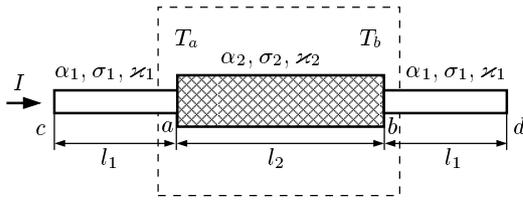


Fig. 1. The circuit for standart ohmic measurements. The dashed square represents the sample chamber

identical extra leads to the current source (not shown). Both contacts assumed to be ohmic; $\alpha, \sigma, \varkappa$, the length l , and the conductor cross-section S are different for the leads and the sample. The voltage is measured between the open ends c and d that are kept at the temperature T_0 of the external thermal reservoir. In general, the contacts a and b could be at different respective temperatures T_a and T_b .

It is well known that Peltier heat is generated by the current crossing the contact of two different conductors. At the contact (for example, a in Fig. 1), the temperature T_a , the electrochemical potential ϕ , the normal components of the current $I = jS$, and the total energy flux qS are continuous. There exists the difference of thermopowers $\Delta\alpha = \alpha_1 - \alpha_2$. For $\Delta\alpha > 0$, the charge intersecting contact a gains the energy $e\Delta\alpha T_a$. Consequently, $Q_a = I\Delta\alpha T_a$ is the amount of the Peltier heat evolved per unit time in contact a . We emphasize that Q_a can be calculated directly through the Thomson term in Eq. (4):

$$Q_a \equiv \int -IT\nabla\alpha dx,$$

where the integration is taken over the contact length. In fact, the Peltier effect is equivalent to the Thomson effect established at the contact.

For $\Delta\alpha > 0$ and the current direction shown in Fig. 1, contact a is heated and contact b is cooled. Thus, the contacts are at different temperatures and $T_a - T_b = \Delta T > 0$. Now we show that the standard ohmic measurements always result in a thermal correction to the resistance measured. Using Eq. (1), we find the voltage swing U between ends c and d as

$$U = \int_c^d \left(\frac{j}{\sigma} + \alpha\nabla T \right) dx = RI + \varepsilon_T, \quad (5)$$

where

$$R = 2R_1 + R_2 = \frac{2l_1}{S_1\sigma_1} + \frac{l_2}{S_2\sigma_2}$$

is the total resistance of the circuit. The first term in Eq. (5) corresponds to the Ohm's law¹⁾. The second term,

$$\varepsilon_T = \int_c^d \alpha dT,$$

coincides with the expression for the conventional thermoelectromotive force under zero current conditions [1]. We notice that ε_T is a universal value because it only depends on the contact temperatures for arbitrary cooling conditions. There exists a correlation between the thermoelectromotive force and the Peltier and Thomson heats. The total power evolved in the circuit, UI , is the sum of the Joule heat RI^2 and the power $\varepsilon_T I$ related to the thermal effects. The product $\varepsilon_T I$ is then exactly the sum of the Peltier heat

$$Q_P = Q_a - Q_b = I\Delta\alpha\Delta T$$

evolved at both contacts and the Thomson heat

$$Q_T = - \int_c^d IT\nabla\alpha dx$$

in the conductor bulk:

$$\varepsilon_T I = Q_P + Q_T. \quad (6)$$

It follows from Eq. (6) that for an arbitrary circuit under the same contact temperatures (T_a, T_b , and T_0), the zero-current measurements of the thermoelectromotive force allow one to find the total amount of both the Peltier and Thomson heats at $I \neq 0$.

We recall that the sample contacts are always extra heated (or cooled) because of the Peltier effect. The difference of the contact temperatures ΔT is linear in current, and therefore, there exists a thermal correction to the ohmic resistance:

$$\Delta R = \varepsilon_T / I = U / I - R.$$

For simplicity, we assume that the conductivity σ , the thermopower α , and the thermal conductivity \varkappa are temperature independent. The thermoelectromotive force is then given by $\varepsilon_T = \Delta\alpha\Delta T$.

Using Eqs. (4) and (5), one can easily find the voltage swing U and, thus, the thermal correction ΔR for an arbitrary circuit. We emphasize that the real cooling conditions strongly influence ΔR . Now we specify the

¹⁾ Ohm's law: «The amount of current flowing in a circuit made up of pure resistances is directly proportional to the electromotive forces (voltages) impressed on the circuit and inversely proportional to the total resistance of the circuit» (1827).

cooling conditions of the circuit shown in Fig. 1. We consider the adiabatic conditions with the sample being thermally isolated from the environment. For example, the sample can be placed into the vacuum chamber (see Fig. 1) surrounded by the thermal reservoir kept at T_0 . We neglect the heat transfer within the leads considering a self-isolated sample. We emphasize that under the above conditions, the sample is not heated. In fact, at small current we have $T_a \approx T_b \approx T_0$, and hence, the amount of Peltier heat evolved at contact a is equal to the one absorbed at contact b . The energy flux qS is continuous at each contact, and therefore,

$$Q_a = -Q_b = I\Delta\alpha T_0 = -\kappa_2 \frac{dT}{dx} S_2. \quad (7)$$

Using Eq. (7), we find the thermal correction to resistivity as

$$\Delta R = \frac{T_0(\Delta\alpha)^2 l_2}{S_2 \kappa_2}. \quad (8)$$

According to Eqs. (5) and (8), ΔR depends on the reservoir temperature and on the geometry and heat conductivity of the sample. We emphasize that the thermal correction is always positive, because the total amount of the Peltier heat $Q_P = \Delta R I^2 > 0$.

Now we estimate the magnitude of the thermal correction ΔR to resistivity in the case where both the conductor and leads are metals. At room temperature, the electron heat conductivity and thermopower of the electron gas are given by

$$\kappa = L\sigma T, \quad \alpha = \frac{\pi^2 k}{2e} \xi,$$

where $L = \pi^2 k^2 / 3e^2$ is the Lorentz number and $\xi = kT/E_F \ll 1$ is the degeneracy parameter. The difference $\Delta\alpha$ is of the order $k\xi/e$. It follows from the above assumptions that $\Delta R/R \sim \xi^2 \ll 1$. Thus, the thermal correction is small compared with the ohmic resistance because the electron gas is degenerate. For semimetals (bismuth, $E_F \approx 35$ meV), the thermal correction can be greater. In contrast to strongly degenerate electron gas, the thermal correction must be greater for a non-degenerate semiconductor because we then have

$$\Delta\alpha \approx \alpha_2 = \frac{k}{e} \left(\frac{5}{2} + r - \xi^{-1} \right) \sim \frac{k}{e},$$

where r is the parameter related to the scattering mechanism of the electrons. For example, we consider the non-degenerate n -InSb at $T = 0.5$ K. The Fermi energy lies between the conduction band and the shallow

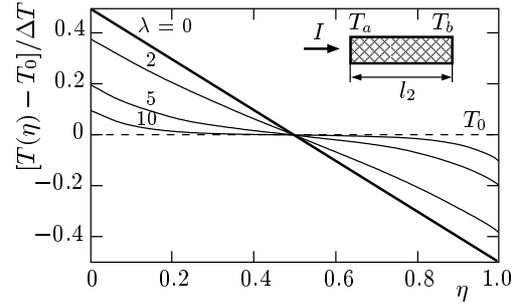


Fig. 2. The dimensionless $T(\eta)$ relation given by Eq. (9) for fixed current, $\lambda = 0, 2, 5, 10$, and contact temperature difference ΔT found at $\lambda = 0$

donor impurity level $\Delta E_d \approx 7$ K. For phonon scattering ($r = 3/2$) we obtain $\Delta\alpha = 11k/e$. At low temperatures, the electron heat conductivity is then less than the phonon-related Debye one $\kappa_{ph} = 0.05T^3$ W/cm·K. For n -InSb with the electron concentration $n = 10^{13}$ cm $^{-3}$ and mobility $\mu = 5 \cdot 10^6$ cm 2 /V·s, we obtain the thermal correction to resistivity $\Delta R/R \sim 0.01$.

In reality, the cooling conditions can be different from those assumed above. Now we consider a more realistic case where the local cooling of the sample is important, for example, with the sample chamber containing the gas. One can take the cooling effects into account using Eq. (4) with the linear term $-\beta(T - T_0)$ included, where β denotes the strength of the sample-to-gas thermal exchange. Under small current the spatial temperature distribution (Fig. 2) is given by

$$T(\eta) = \frac{(T_a - T_0) \text{sh}[\lambda(1-\eta)] + (T_b - T_0) \text{sh}[\lambda\eta]}{\text{sh} \lambda} + T_0, \quad (9)$$

where $\eta = x/l_2$ is the dimensionless coordinate. The sample local cooling is therefore governed by the dimensionless parameter $\lambda = \sqrt{\beta/\kappa_2} l_2$. Actually, λ is the ratio of the outgoing and internal (within the sample) heat fluxes. When $\lambda \ll 1$, the local cooling can be neglected, and hence, $T(\eta)$ is linear (Fig. 2). In the opposite intensive cooling case where $\lambda \gg 1$, the $T(\eta)$ dependence is sharp near the contacts.

The above results allow us to calculate the thermal correction to the sample resistance. Using Eqs. (5), (7), and (9) and omitting cumbersome algebraic calculations, we calculate the thermal correction to the resistance as

$$\Delta R = \frac{T_0(\Delta\alpha)^2 l_2}{S_2 \kappa_2} \frac{\text{th}(\lambda/2)}{\lambda/2}. \quad (10)$$

For small cooling as $\lambda \rightarrow 0$, Eqs. (8) and (10) coincide. In the opposite strong-cooling case, where $\lambda \rightarrow \infty$,

the difference ΔT and, thus, the thermal correction decrease (see Fig. 2).

Now we estimate ΔR given by Eq. (10) for natural air convection cooling. For a sample with the typical dimension $d \sim \sqrt{S}$, the outgoing thermal flux is given by $\kappa_{gas}(T - T_0)Nu/S$, where $Nu \sim 10$ is the Nusselt number. For the n -InSb sample ($0.5 \times 0.5 \times 0.5$ cm), the heat conductivity is $\kappa_{ph} = 0.15$ W/K·cm ($T = 293$ K). Assuming the air heat conductivity $\kappa_{gas} = 2.6$ W/cm·K, we find $\lambda = 14$. Thus, the thermal correction to resistivity is approximately seven times less than the one in the absence of convection.

We emphasize that both dc and ac ohmic measurements lead to a thermal correction. However, ΔR diminishes at high frequencies because of the thermal inertial effects. In fact, Eq. (8) is valid below some critical frequency $f_{cr} = \chi/d^2$, where χ is the temperature diffusive coefficient of the sample. For example, at room temperature for a metal conductor $\chi = \kappa/C \approx 10^2$ cm²/s, where C is the specific heat

of the electron gas. For the typical metal conductor with $d = 1$ mm, we obtain the critical frequency $f_{cr} = 10^4$ Hz. We suggest that the spectral dependence of the thermal correction can be used to estimate the magnitude of the thermal correction.

In conclusion, the ohmic measurements of a conductor resistance contain the thermal correction caused by the Peltier effect. The thermal correction always exists, while its magnitude depends on the actual cooling conditions of the circuit. Above some critical frequency depending on thermal inertial effects, the thermal correction disappears.

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