ЖЭТФ, 2001, том 119, вып. 6, стр. 1077–1086 © 2001

TAGGED-PHOTON EVENTS IN POLARIZED DIS PROCESSES

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Submitted 31 October 2000

Deep inelastic scattering events of the longitudinally polarized electron by the polarized proton with a tagged collinear photon radiated from the initial-state electron are considered. The corresponding cross-section is derived in the Born approximation. The model-independent radiative corrections to the Born cross-section are also calculated. The obtained result is applied to the elastic scattering.

PACS: 12.20.-m, 13.40.-f, 13.60.Hb, 13.88.+e

1. INTRODUCTION

The idea to use radiative events (events with the emission of an additional tagged photon) in lepton–hadron interaction to expand the experimental possibilities for studying different topics in high-energy physics has become quite attractive recently.

Photon radiation from the initial e^+e^-–state, in the events with missing energy, has been successfully used at LEP for measuring the number of light neutrinos and for searching for new physics signals (for a recent publication, see, e.g., [1]). The possibility to undertake the bottonium spectroscopy studies at B-factories by using the hard photon emission from the electron or the positron was considered in Ref. [2]. An important physical problem of the total hadronic cross-section scanning in the electron–positron annihilation process at low and intermediate energies by means of the initial-state radiative events was extensively discussed in Ref. [3].

The initial-state collinear radiation is very important in certain regions of the deep inelastic scattering (DIS) at the HERA kinematic domain. It leads to a reduction of the projectile electron energy, and therefore, to a shift of the effective Bjorken variables in the hard scattering process compared to those determined from the actual measurement of the scattered electron alone. That is why the radiative events in the DIS process

$$e^-(k_1) + p(p_1) \to e^-(k_2) + \gamma(k) + X(p_x) \qquad (1)$$

must be carefully taken into account [4].

In addition, the measurement of the energy of the photon emitted very close to the incident electron beam direction [5, 6] allows studying the overlap of the kinematical photoproduction region ($Q^2 = -(k_1 - k_2)^2 \approx 0$) and the DIS region with small transferred momenta (Q^2 about several GeV^2) within the high-energy HERA experiments. These radiative events can also be used for independently determining the proton structure functions F_1 and F_2 in a single run without lowering the beam energy [5, 7]. The high-precision calculation of the corresponding cross-section (taking the radiative corrections (RC) into account) was performed in Ref. [8].

In this paper, we investigate the events for deep-inelastic radiative process (1) with a longitudinally polarized electron beam and a polarized proton as a target. As in Ref. [8], we suggest that the hard photon is emitted very close to the direction of the incoming electron beam ($\theta_\gamma = \widehat{\mathbf{p}_1\mathbf{k}_1} \le \theta_0$ with $\theta_0 \ll 1$) and the photon detector (PD) measures the energy of all photons inside the narrow cone with the opening angle $2\theta_0$ around the electron beam. The scattered electron 3-momentum is fixed simultaneously.

We consider the longitudinal (along the electron beam direction) and perpendicular (in the plane $(\mathbf{k}_1, \mathbf{k}_2)$) polarizations of the proton. In Sec. 2, we derive the corresponding cross-sections in the Born approximation and in Sec. 3, we calculate the different RC contributions to the Born cross-section. The total radiative correction for different (exclusive and calorimeter) experimental conditions for the scattered electron

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measurement is given in Sec. 4. Our results can be applied to the cross-section of process (1). We consider the target proton at rest and also the colliding electron–proton beams. In Sec. 5, we apply the results obtained in Sec. 4 to describe the quasi-elastic scattering using the relation between the spin-dependent proton structure functions and the proton electromagnetic form factors in this limiting case.

2. BORN APPROXIMATION

The spin-independent part of the DIS cross-section for the experimental setup considered here was recently investigated in detail [8]. We now consider the spin-dependent part of the corresponding cross-section that is described by the proton structure functions g_1 and g_2. Because the opening angle of the forward PD is very small and we consider only the cross-section where the tagged photon is integrated over the solid angle covered by the PD, we can apply the quasi-real electron method [9] and parameterize these radiative events using the standard Bjorken variables

$$x = \frac{Q^2}{2p_1(k_1-k_2)}, \quad y = \frac{2p_1(k_1-k_2)}{V}, \quad V = 2p_1k_1, \tag{2}$$

and the energy fraction of the electron after the initial-state radiation of a collinear photon

$$z = \frac{2p_1(k_1-k)}{V} = \frac{\varepsilon-\omega}{\varepsilon}, \tag{3}$$

where ε is the initial-electron energy and ω is the energy deposited in the PD.

An alternative set of kinematic variables that is specially adapted to the collinear photon radiation is given by the shifted Bjorken variables [5, 10]

$$\widehat{Q}^2 = -(k_1-k_2-k)^2, \quad \hat{x} = \frac{\widehat{Q}^2}{2p_1(k_1-k_2-k)}, \quad \hat{y} = \frac{2p_1(k_1-k_2-k)}{2p_1(k_1-k)}. \tag{4}$$

The shifted and the standard Bjorken variables are related by

$$\widehat{Q}^2 = zQ^2, \quad \hat{x} = \frac{xyz}{z+y-1}, \quad \hat{y} = \frac{z+y-1}{z}. \tag{5}$$

At fixed values of x and y, the lower limit of z can be derived from the constraint on the shifted variable $\hat{x}$,

$$\hat{x} < 1 \quad \to \quad z > \frac{1-y}{1-xy}.$$

In the Born approximation, we determine the DIS cross-section in radiative process (1) in terms of the contraction of the leptonic and hadronic tensors as[1)]

$$\frac{d\sigma}{\hat{y}d\hat{x}\,d\hat{y}} = \frac{4\pi\alpha^2(\widehat{Q}^2)}{\widehat{Q}^4} zL^B_{\mu\nu}H_{\mu\nu}, \tag{6}$$

where $\alpha(\widehat{Q}^2)$ is the running electromagnetic coupling constant that takes the vacuum polarization effects into account and the Born leptonic current tensor is given by [11]

$$L^B_{\mu\nu} = \frac{\alpha}{4\pi^2}\int\limits_{\Omega} 2i\varepsilon_{\mu\nu\lambda\rho}q_\lambda\big(k_{1\rho}R_t + k_{2\rho}R_s\big)\frac{d^3k}{\omega}, \quad q = k_1-k_2-k, \tag{7}$$

where Ω covers the solid angle of the PD.

For the initial-state collinear radiation considered in this paper, the quantities R_t and R_s can be written as

$$R_t = -\frac{1}{(1-z)t} - \frac{2m^2}{t^2}, \quad R_s = -\frac{z}{(1-z)t} + \frac{2m^2(1-z)}{t^2}, \quad t = -2kk_1, \quad q = zk_1 - k_2. \tag{8}$$

In accordance with the quasi-real electron approximation [9], the trivial angular integration of the Born leptonic tensor gives

$$L^B_{\mu\nu} = \frac{\alpha}{2\pi}P(z,L_0)dz i\varepsilon_{\mu\nu\lambda\rho}q_\lambda k_{1\rho}, \quad L_0 = \ln\frac{\varepsilon^2\theta_0^2}{m^2}, \quad P(z,L_0) = \frac{1+z^2}{1-z}L_0 - \frac{2(1-z+z^2)}{1-z}, \tag{9}$$

where m is the electron mass.

We write the spin-dependent part of the hadronic tensor in the right-hand side of Eq. (6) as

$$H_{\mu\nu} = -iM\frac{\varepsilon_{\mu\nu\lambda\rho}q_\lambda}{2p_1q}\left[(g_1+g_2)S_\rho - g_2\frac{Sq}{p_1q}p_{1\rho}\right], \tag{10}$$

where M is the proton mass and S is the proton polarization 4-vector. In writing expressions (7) and (10), we assume that the polarization degree of both the electron and the proton is equal to 1.

Our normalization is such that the proton structure functions g_1 and g_2 are dimensionless and in the limiting case of the elastic scattering ($\hat{x} \to 1$) they are

[1)] In what follows, we are only interested in the spin-dependent part of the cross-section.

expressed in terms of the proton electric (G_E) and magnetic (G_M) form factors as

$$g_1(\hat{x},\widehat{Q}^2) \to \delta(1-\hat{x}) \times \left[G_M G_E + \frac{\lambda}{1+\lambda}(G_M - G_E)G_M\right], \quad \lambda = \frac{\widehat{Q}^2}{4M^2}, \tag{11}$$

$$g_2(\hat{x},\widehat{Q}^2) \to -\delta(1-\hat{x})\frac{\lambda}{1+\lambda}(G_M - G_E)G_M, \quad G_{M,E} = G_{M,E}(\widehat{Q}^2).$$

It is convenient to parameterize the proton polarization 4-vector in terms of the 4-momenta of the reaction under study [12],

$$S_\mu^{\parallel} = \frac{2M^2 k_{1\mu} - V p_{1\mu}}{MV}, \quad S_\mu^{\perp} = \frac{u p_{1\mu} + V k_{2\mu} - [2u\tau + V(1-y)]k_{1\mu}}{\sqrt{-uV^2(1-y) - u^2M^2}}, \tag{12}$$

where $u = -Q^2$, $\tau = M^2/V$, and we neglect the electron mass. The 4-vector of the longitudinal proton polarization has the respective components

$$S_\mu^{\parallel} = (0, \mathbf{n}_1), \quad S_\mu^{\parallel} = \left(-\frac{|\mathbf{p}_1|}{M}, \frac{\mathbf{n}_1 E_1}{M}\right) \tag{13}$$

for the target at rest and the colliding beams. Here, $E_1(\mathbf{p}_1)$ is the proton energy (3-momentum) and $\mathbf{n}_1$ is the unit vector along the initial electron 3-momentum direction. The 4-vector of the perpendicular proton polarization $S_\mu^{\perp}$ is the same in both these cases,

$$S_\mu^{\perp} = \left(0, \ \frac{\mathbf{n}_2 - \mathbf{n}_1(\mathbf{n}_1\cdot\mathbf{n}_2)}{\sqrt{1-(\mathbf{n}_1\cdot\mathbf{n}_2)^2}}\right), \tag{14}$$

where $\mathbf{n_2}$ is the unit vector along the scattered electron 3-momentum direction. It is easy to verify that $S^{\parallel}S^{\perp} = 0$.

Using the definitions of the DIS cross-section in Eq. (6), leptonic and hadronic tensors (9) and (10), and the parameterization of the proton polarization in Eq. (12), after simple calculations, we derive the spin-dependent part of the cross-section of process (1) with a tagged collinear photon radiated from the initial state,

$$\frac{d\sigma^B_{\parallel,\perp}}{\hat{y}\,d\hat{x}\,d\hat{y}\,dz} = \frac{\alpha}{2\pi}P(z,L_0)\Sigma_{\parallel,\perp}(\hat{x},\hat{y},\widehat{Q}^2), \tag{15}$$

$$\Sigma_{\parallel} = \frac{4\pi\alpha^2(\widehat{Q}^2)}{\widehat{V}\hat{y}}\left(\hat{\tau} - \frac{2-\hat{y}}{2\hat{x}\hat{y}}\right) \times g_1(\hat{x},\widehat{Q}^2)\left[1 + e_1\hat{R}(\hat{x},\widehat{Q}^2)\right], \tag{16}$$

$$\Sigma_{\perp} = -\frac{4\pi\alpha^2(\widehat{Q}^2)}{\widehat{V}\hat{y}}\sqrt{\frac{M^2}{\widehat{Q}^2}(1-\hat{y}-\hat{x}\hat{y}\hat{\tau})} \times g_1(\hat{x},\widehat{Q}^2)\left[1 + e_2\hat{R}(\hat{x},\widehat{Q}^2)\right], \tag{17}$$

where

$$e_1 = \frac{4\hat{\tau}\hat{x}}{2\hat{x}\hat{y}\hat{\tau} + \hat{y} - 2}, \quad e_2 = \frac{2}{\hat{y}},$$

$$\widehat{R} = \frac{g_2(\hat{x},\widehat{Q}^2)}{g_1(\hat{x},\widehat{Q}^2)}, \quad \hat{\tau} = \frac{M^2}{\widehat{V}}, \quad \widehat{V} = zV.$$

It is useful to recall that the unpolarized DIS cross-section is proportional to $\sigma_T(1+eR)$, where $R = \sigma_L/\sigma_T$ and for the events with the tagged collinear photon [5], we have

$$e = \frac{2(1-\hat{y})}{1+(1-\hat{y})^2}.$$

Because the quantities e_1 and e_2 strongly depend on z, the determination of the proton structure functions g_1 and g_2 is possible by measuring the z-dependence of cross-section (15) in a single run without lowering the electron beam energy. The quantity e_1 is proportional to τ and is therefore very small at the HERA conditions. Thus, the separation of g_1 and g_2 in the DIS process with the longitudinally polarized proton is possible in experiments with the target at rest and low values of V (up to 20 GeV^2). At HERA, the cross-section of this process can be used for measuring the structure function g_1 only. This can be seen in Fig. 1. On the other hand, Fig. 2 shows that the experiments with the tagged photon and the perpendicular proton polarization can be used to measure both g_1 and g_2 in a wide range of energies (provided that Q^2 is not large).

3. RADIATIVE CORRECTIONS

We restrict ourselves to the model-independent QED radiative corrections related to the radiation of the real and virtual photons by leptons. The remaining sources of RC in the same order of the perturbation theory, such as the virtual corrections with a double photon exchange mechanism and the bremsstrahlung of the proton and partons, are more involved and model-dependent. They are not considered here. Our approach to the calculation of the RC is based on the account of all the essential Feyman diagrams that describe the observed cross-section in the chosen approximation. To avoid cumbersome expressions, we retain the terms accompanied by at least one power of large

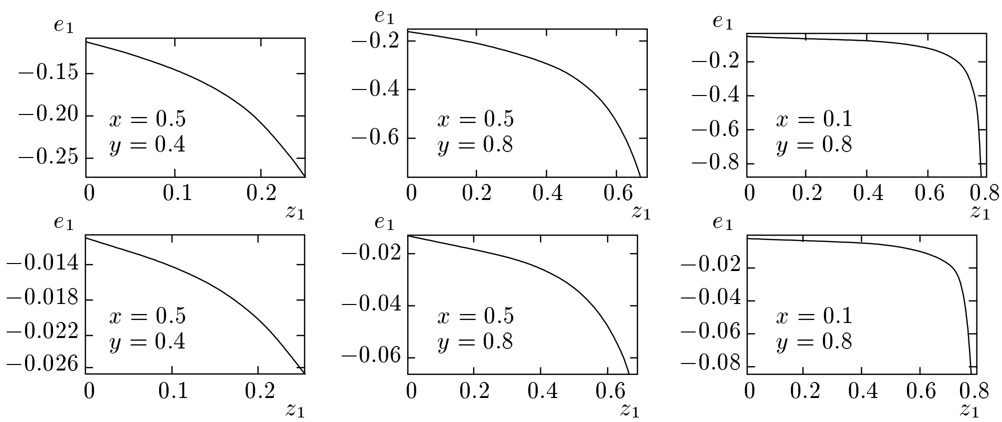

Fig. 1. The dependence of e_1 on the energy fraction of the tagged photon $z_1 = 1 - z$ for different values of x, y, and V. The upper set corresponds to $V = 10$ GeV^2 and the lower one to $V = 100$ GeV^2. The maximum value of z_1 is $y(1-x)/(1-xy)$

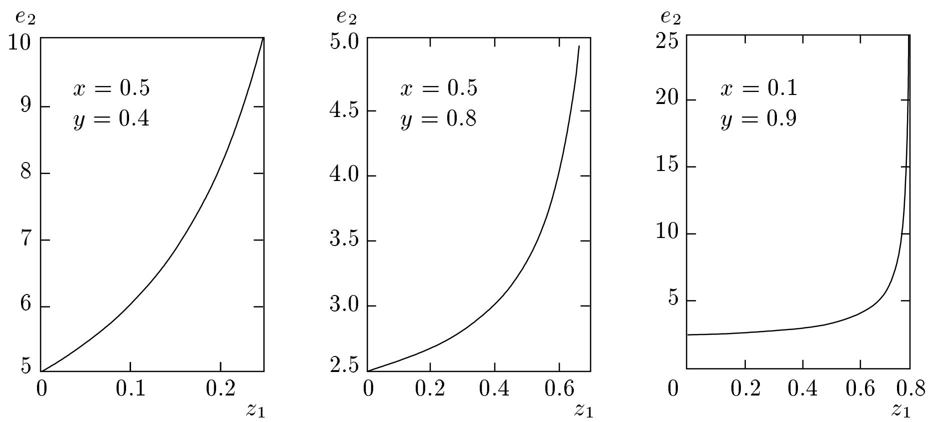

Fig. 2. The quantity e_2 at different values of x and y as a function of $z_1 = 1 - z$

logarithms in RC. In the considered case, three different types of such logarithms appear,

$$L_0, \quad L_Q = \ln \frac{Q^2}{m^2}, \quad L_\theta = \ln \frac{\theta_0^2}{4}. \tag{18}$$

In the chosen approximation, we in addition neglect the terms of the order θ_0^2, $m^2/\varepsilon^2\theta_0^2$, and m^2/Q^2 in the cross-section.

The total RC to cross-section (15) includes the contributions of the virtual and soft photon emission and also the hard photon radiation.

As one can see, we use the standard gauge-invariant expression for the hadronic tensor. The leptonic tensor was calculated in accordance with the QED rules. The complete set of Feynman diagrams for the calculation of the radiative correction caused by the real photon emission is taken into account. Taking the loop correction into account involves the gauge invariant method for solving both the infrared and the ultraviolet divergence problems. The results obtained in our paper are therefore gauge invariant. We begin with calculating the virtual and soft corrections.

3.1. Virtual and soft corrections

To calculate the virtual- and soft-photon emission corrections, we start from the expression for the one-loop corrected Compton tensor with a heavy photon for the longitudinally polarized electron [13]. For the hard collinear initial-state radiation considered here, this Compton tensor can be written as

$$L_{\mu\nu}^V = \frac{\alpha}{2\pi}\rho L_{\mu\nu}^B + \frac{\alpha^2}{4\pi^3}\times$$

$$\times \int\limits_{\Omega} i\varepsilon_{\mu\nu\lambda\rho} q_\lambda k_{1\rho} \frac{d^3k}{\omega} \times$$
$$\times \left[\frac{T}{-t} + \frac{4m^2(1-z+z^2)}{t^2} L_Q \ln z \right], \quad (19)$$

$$T = \frac{1+z^2}{1-z} \{ 2\ln z \,[l_t - \ln(1-z) - L_Q] - 2F(z) \} + + \frac{1+2z-z^2}{2(1-z)},$$

$$F(z) = \int\limits_1^{1/z} \frac{dx}{x} \ln|1-x|,$$
$$l_t = \ln\frac{-t}{m^2},$$
$$\rho = 4(L_Q-1)\ln\frac{\delta}{m} - L_Q^2 + 3L_Q + 3\ln z + \frac{\pi^2}{3} - \frac{9}{2},$$

where δ is the fictitious photon mass and the tensor $L^B_{\mu\nu}$ is defined in Eq. (9).

To eliminate the photon mass, we must add the contribution of the additional soft photon emission with the energy less than $\Delta\varepsilon$, $\Delta \ll 1$. This contribution was found in Ref. [14] and the corresponding procedure of the photon mass elimination was described in Ref. [15]. The result is

$$L^{V+S}_{\mu\nu} = L^V_{\mu\nu}(\rho \to \tilde\rho), \quad (20)$$

$$\tilde\rho = 2(L_Q-1)\ln\frac{\Delta^2}{Y} + 3L_Q + 3\ln z - \ln^2 Y - - \frac{\pi^2}{3} - \frac{9}{2} + 2\mathrm{Li}_2\left(\cos^2\frac{\theta}{2}\right),$$
$$Y = \frac{\varepsilon_2}{\varepsilon},$$

where ε_2 is the scattered electron energy and θ is the electron scattering angle ($\theta = \widehat{\mathbf{k}_1\mathbf{k}_2}$).

The angular integration with respect to the hard tagged photon over the solid angle of the PD gives (within the chosen accuracy)

$$\text{Ł}^{V+S}_{\mu\nu} = \left(\frac{\alpha}{2\pi}\right)^2 [\tilde\rho P(z,L_0) + G]\, dz\, i\varepsilon_{\mu\nu\lambda\rho} q_\lambda k_{1\rho}, \quad (21)$$

$$G = \left\{ \frac{1+z^2}{1-z} [\ln z(L_0 - 2L_Q) - 2F(z)] + + \frac{1+2z-z^2}{2(1-z)} \right\} L_0 + \frac{4(1-z+z^2)}{1-z} L_Q \ln z.$$

Using the right-hand side of Eq. (21) instead of $L^B_{\mu\nu}$ in the right side of Eq. (6), we derive the contribution of the virtual and soft corrections to Born cross-section (15) as

$$\frac{d\sigma^{V+S}_{\|,\perp}}{\hat y d\hat y\, d\hat x\, dz} = = \left(\frac{\alpha}{2\pi}\right)^2 [\tilde\rho P(z,L_0) + G]\, \Sigma_{\|,\perp}(\hat x, \hat y, \widehat{Q}^2), \quad (22)$$

where $\Sigma_{\|,\perp}(\hat x, \hat y, \widehat{Q}^2)$ are defined in Eqs. (16) and (17).

3.2. Double hard bremsstrahlung

We now consider the emission of an additional hard photon with the 4-momentum $\tilde k$ and the energy higher than $\Delta\varepsilon$. To calculate the contribution from the real hard bremsstrahlung, which in our case corresponds to the double hard photon emission with at least one photon seen in the forward PD, we specify three kinematical domains:

i) both hard photons hit the forward PD, i.e., both are emitted within a narrow cone around the electron beam ($\widehat{\mathbf{k}\mathbf{k}_1}, \widehat{\tilde{\mathbf{k}}\mathbf{k}_1} \le \theta_0$);

ii) one hard photon is tagged by the PD and the other is collinear to the outgoing electron momentum ($\widehat{\tilde{\mathbf{k}}\mathbf{k}_2} \le \theta_0'$, $\theta_0' \ll 1$);

iii) an additional photon is emitted at large angles (i.e., outside both narrow cones defined above) with respect to both incoming and outgoing electron momenta.

The contributions of regions *i*) and *ii*) contain quadratic terms in the large logarithms L_0 and L_Q, whereas region *iii*) contains terms of the order $L_0 L_\theta$, which can give an even larger numerical contribution if $2\theta_0 > \varepsilon\theta_0/m$.

We refer to the third kinematical region as the semi-collinear one. Beyond the leading logarithmic accuracy, the calculation can be performed using the results in [16] for the leptonic current tensor with the longitudinally polarized electron for the collinear as well as semi-collinear regions.

The contribution of kinematical region *i*), where both hard photons hit the PD and each has the energy higher than $\Delta\varepsilon$, can be written as

$$\frac{d\sigma^{i)}_{\|,\perp}}{\hat y d\hat y\, d\hat x\, dz} = \left(\frac{\alpha}{2\pi}\right)^2 L_0 \times$$
$$\times \left\{ \left[\frac{1}{2} P^{(2)}_\theta(z) + \frac{1+z^2}{1-z}\left(\ln z - \frac{3}{2} - 2\ln\Delta\right)\right] L_0 + + 7(1-z) - 2(1-z)\ln z + \frac{3+z^2}{2(1-z)}\ln^2 z - - 2\frac{3-2z+3z^2}{1-z}\ln\frac{1-z}{\Delta} \right\} \Sigma_{\|,\perp}(\hat x, \hat y, \widehat{Q}^2). \quad (23)$$

The double-logarithm terms in the right side of Eq. (23) are the same for the polarized and unpolarized cases, whereas the one-logarithm terms are different. In Eq. (23), we use the notation $P_\theta^{(2)}(z)$ for the Θ part of the second-order electron structure function $D(z, L)$ [17],

$$D(z,L) = \delta(1-z) + \frac{\alpha}{2\pi}P^{(1)}(z)L + \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2 P^{(2)}(z)L^2 + \ldots,$$

$$P^{(i)}(z) = P_\theta^{(i)}(z)\Theta(1-z-\Delta) + \delta(1-z)P_\delta^{(i)}, \quad \Delta \to 0,$$

$$P_\theta^{(1)}(z) = \frac{1+z^2}{1-z}, \quad P_\delta^{(1)} = \frac{3}{2} + 2\ln\Delta,$$

$$P_\theta^{(2)}(z) = 2\left[\frac{1+z^2}{1-z}\left(2\ln(1-z) - \ln z + \frac{3}{2}\right) + \frac{1}{2}(1+z)\ln z - 1 + z\right]. \quad (24)$$

To calculate the contribution of kinematical region ii), we can use the quasi-real electron method to describe the radiation of both collinear photons. This contribution to the observed cross-section depends on the event selection, in other words, on the method of measuring the scattered electron.

For the exclusive event selection, where only the scattered electron is detected, but the photon emitted almost collinearly (i.e., within the opening angle $2\theta_0'$ around the scattered electron momentum) goes unnoticed or is not taken into account in calculating the kinematical variables, we have, in accordance with Ref. [9],

$$\frac{d\sigma_{\parallel,\perp}^{ii),excl}}{\hat{y}d\hat{y}\,d\hat{x}\,dz} = \frac{\alpha^2}{4\pi^2}P(z,L_0) \times \int\limits_{\Delta/Y}^{y_{1max}} \frac{dy_1}{1+y_1}\left[\frac{1+(1+y_1)^2}{y_1}(\widetilde{L}-1) + y_1\right] \times \Sigma_{\parallel,\perp}(x_s, y_s, Q_s^2), \quad (25)$$

where y_1 is the energy fraction of the photon radiated along the 3-momentum $\mathbf{k}_2$ relative to the scattered electron energy ($y_1 = \tilde{\omega}/\varepsilon_2$) and

$$\widetilde{L} = \ln\frac{\varepsilon^2\theta_0'^2}{m^2} + 2\ln Y,$$

$$x_s = \frac{xyz(1+y_1)}{z-(1-y)(1+y_1)},$$

$$y_s = \frac{z-(1-y)(1+y_1)}{z}, \quad Q_s^2 = Q^2 z(1+y_1).$$

The upper integration limit in Eq. (25) can be found from the condition of the inelastic process occurrence $p_x^2 = (M+\mu)^2$, where μ is the pion mass. Taking into account that $q = zk_1 - (1+y_1)k_2$ for kinematics ii), we obtain

$$y_{1max} = \frac{2z\varepsilon[M - \varepsilon_2(1-c)] - 2M\varepsilon_2 - \mu^2 - 2M\mu}{2\varepsilon_2[M + z\varepsilon(1-c)]}$$

for the proton target at rest and

$$y_{1max} = \frac{2z - Y(1+c)}{Y(1+c)}$$

for the HERA collider, where $c = \cos\theta$. In writing this limit for HERA, we neglect the electron energy and the proton mass compared to the proton beam energy. We note that for the exclusive event selection, the parameter θ_0' is purely auxiliary and does not enter the final result when the contribution of region iii) is added.

From the experimental point of view, a more realistic measurement method is the calorimeter event selection, where the photon and the electron cannot be distinguished inside a narrow cone with the opening angle $2\theta_0'$ along the outgoing electron momentum direction. Therefore, only the sum of the photon and the electron energies can be measured if the photon belongs to this cone. In this case, we obtain

$$\frac{d\sigma_{\parallel,\perp}^{ii),cal}}{\hat{y}d\hat{y}\,d\hat{x}\,dz} = \frac{\alpha^2}{4\pi^2}P(z,L_0) \times \int\limits_{\Delta/Y}^{\infty} \frac{dy_1}{(1+y_1)^3}\left[\frac{1+(1+y_1)^2}{y_1}(\widetilde{L}-1) + y_1\right] \times \Sigma_{\parallel,\perp}(\hat{x},\hat{y},\widehat{Q}^2) = \frac{\alpha^2}{4\pi^2}P(z,L_0)\left[(\widetilde{L}-1)\left(2\ln\frac{Y}{\Delta} - \frac{3}{2}\right) + \frac{1}{2}\right] \times \Sigma_{\parallel,\perp}(\hat{x},\hat{y},\widehat{Q}^2). \quad (26)$$

For the calorimeter event selection, the parameter θ_0' is physical and the final result depends on it (see below).

To calculate the contribution of region iii), we can use the quasi-real electron method [9] and write the leptonic tensor in this region (that describes the collinear photon radiation with the energy fraction $1-z$ and the noncollinear photon radiation with the 4-momentum $\tilde{k}$) as

$$L_{\mu\nu}(k_1, k_2, (1-z)k_1, \tilde{k}) = \frac{\alpha}{2\pi}P(z,L_0)\frac{dz}{z}L_{\mu\nu}(zk_1, k_2, \tilde{k}), \quad (27)$$

$$L_{\mu\nu}(zk_1, k_2, \tilde{k}) = \frac{\alpha}{4\pi^2}\frac{d^3\tilde{k}}{\tilde{\omega}}L_{\mu\nu}^{\gamma}(zk_1, k_2, \tilde{k}),$$

$$L^{\gamma}_{\mu\nu}(zk_1,k_2,\tilde{k}) = 2i\varepsilon_{\mu\nu\lambda\rho}\tilde{q}_\lambda \left[\frac{(\tilde{u}+\tilde{t})z}{\tilde{s}\tilde{t}}k_{1\rho} + \frac{\tilde{s}+\tilde{u}}{\tilde{s}\tilde{t}}k_{2\rho}\right],$$

$$\tilde{q} = zk_1 - k_2 - \tilde{k}, \quad \tilde{u} = -2zk_2k_1,$$

$$\tilde{s} = 2\tilde{k}k_2, \quad \tilde{t} = -2z\tilde{k}k_1.$$

In the general case of the noncollinear photon radiation with the 4-momentum k, the contraction of the leptonic tensor $L^{\gamma}_{\mu\nu}(k_1,k_2,k)$ and the hadronic one is given by

$$L^{\gamma}_{\mu\nu}(k_1,k_2,k)H^{\parallel}_{\mu\nu} = -\frac{1}{st}\times \\ \times\left\{(2\tau A_t+q^2B)g_1+2\tau[A_t-x'(u+t)B]g_2\right\}\frac{x'}{q^2}, \quad (28)$$

$$L^{\gamma}_{\mu\nu}(k_1,k_2,k)H^{\perp}_{\mu\nu} = -\frac{1}{st}\left\{\left[A_s - \frac{uq^2}{V}B - \right.\right. \\ \left.-A_t\left(1-y+\frac{2u\tau}{V}\right)\right]g_1 + \left[A_s + x'(s+u)B + \right. \\ \left.\left.+\left(1-y+\frac{2u\tau}{V}\right)[x'(u+t)B - A_t]\right]g_2\right\}\times \\ \times\frac{x'}{q^2}\sqrt{\frac{M^2}{Q^2}\left(1-y+\frac{u\tau}{V}\right)^{-1}}, \quad (29)$$

$$A_t = (u+t)^3 + (uq^2 - st)(u+s),$$

$$B = (u+t)\left(2V + \frac{u+t}{x'}\right) + \\ + (u+s)\left(2V(1-y) - \frac{u+s}{x'}\right),$$

$$A_s = (u+s)^3 + (uq^2-st)(u+t), \quad q = k_1 - k_2 - k,$$

$$x' = \frac{-q^2}{2p_1q}, \quad g_{1,2} = g_{1,2}(x',q^2).$$

The contraction of the shifted leptonic tensor $L^{\gamma}_{\mu\nu}(zk_1,k_2,\tilde{k})$ entering the definition of the leptonic tensor in region *iii*) and the hadronic tensor can be obtained from Eqs. (28) and (29) by the substitution

$$(k_1,k) \to (zk_1,\tilde{k}), \\ (s,t,u,q,x') \to (\tilde{s},\tilde{t},\tilde{u},\tilde{q},\tilde{x}), \quad \tilde{x} = \frac{-\tilde{q}^2}{2p_1\tilde{q}}. \quad (30)$$

We use the approach developed in Ref. [8] to extract the leading contributions (those proportional to $\ln\theta_0$ and $\ln\theta'_0$) to the respective cross-section and to separate the infrared singularities. We write the cross-section as

$$\frac{d\sigma^{iii)}_{\parallel,\perp}}{\hat{y}d\hat{x}\,d\hat{y}\,dz} = \frac{\alpha^2}{4\pi^2}\times \\ \times\left\{P(z,L_0)\left[\int\limits_{\Delta}^{x_{1max}}\frac{dx_1[z^2+(z-x_1)^2]}{x_1z(z-x_1)}\times\right.\right. \\ \times\ln\frac{2(1-c)}{\theta_0^2}\Sigma_{\parallel,\perp}(x_t,y_t,Q_t^2) + \\ + \int\limits_{\Delta/Y}^{y_{1max}}\frac{dy_1[1+(1+y_1)^2]}{y_1(1+y_1)}\times \\ \left.\left.\times\ \ln\frac{2(1-c)}{\theta_0'^2}\Sigma_{\parallel,\perp}(x_s,y_s,Q_s^2)\right] + \frac{1+z^2}{1-z}L_0Z_{\parallel,\perp}\right\}, \quad (31)$$

where

$$x_t = \frac{xy(z-x_1)}{z-x_1+y-1}, \quad y_t = \frac{z-x_1+y-1}{z-x_1},$$

$$Q_t^2 = Q^2(z-x_1).$$

For the proton target at rest, we have

$$x_{1max} = \frac{2z\varepsilon[M-\varepsilon_2(1-c)] - 2M\varepsilon_2 - \mu^2 - 2\mu M}{2\varepsilon[M-\varepsilon_2(1-c)]}$$

and for the HERA collider conditions,

$$x_{1max} = z - \frac{Y(1+c)}{2}.$$

The dependence on the infrared auxiliary parameter Δ and on the angles θ_0 and θ'_0 is contained in the first two terms on the right-hand side of Eq. (31), whereas the quantities $Z_{\parallel,\perp}$ do not contain the infrared and collinear singularities. They can be written as

$$Z_{\parallel,\perp} = -\frac{2(1-c)}{zQ^2}\int\limits_0^\infty\frac{du}{1+u^2}\left\{\int\limits_0^1\frac{dt_1}{t_1|t_1-a|}\times\right. \\ \times\left[\int\limits_0^{x_m}\frac{dx_1}{x_1}\Phi_{\parallel,\perp}(t_1,t_2(t_1,u)) - \int\limits_0^{y_{1m}}\frac{dx_1}{x_1}\Phi_{\parallel,\perp}(a,0)\right] + \\ + \int\limits_0^a\frac{dt_1}{t_1a}\left[\int\limits_0^{y_{1m}}\frac{dx_1}{x_1}\Phi_{\parallel,\perp}(a,0) - \right. \\ \left.\left.-\int\limits_0^{x_{1m}}\frac{dx_1}{x_1}\Phi_{\parallel,\perp}(0,a)\right]\right\}, \quad (32)$$

where we use the same notation as in Ref. [8], namely

$$t_{2,1} = \frac{1-c_{1,2}}{2}, \quad a = \frac{1-c}{2},$$

$$t_2(t_1,u) = \frac{(a-t_1)^2(1+u^2)}{x_+ + u^2 x_-}, \quad c_{1,2} = \cos\theta_{1,2},$$

$$\theta_{1,2} = \widehat{\tilde{\mathbf{k}}\mathbf{k}}_{1,2},$$

$$x_\pm = t_1(1-2a) + a \pm 2\sqrt{a(1-a)t_1(1-t_1)}.$$

The quantity $\Phi_{\|,\perp}(t_1,t_2)$ is given by

$$\Phi_{\|,\perp}(t_1,t_2) = \frac{\alpha^2(\tilde{q}^2)\tilde{x}}{\widetilde{Q}^6} G_{\|,\perp}, \qquad (33)$$

$$G_\| = g_1(2\hat{\tau}\widetilde{A}_t + \tilde{q}^2\widetilde{B}) + 2g_2\hat{\tau}\left[\widetilde{A}_t - \tilde{x}(\tilde{u}+\tilde{t})\widetilde{B}\right],$$

$$g_{1,2} = g_{1,2}(\tilde{x},\tilde{q}^2),$$

$$G_\perp = \sqrt{\frac{M^2}{\widehat{Q}^2}(1-\hat{y}-\hat{x}\hat{y}\hat{\tau})^{-1}} \times \\ \times \left\{ g_1\left[\widetilde{A}_s - \frac{u\tilde{q}^2}{V}\widetilde{B} - \widetilde{A}_t\left(1-\hat{y}+\frac{2u\hat{\tau}}{V}\right)\right] + \right. \\ + g_2\left[\widetilde{A}_s + \tilde{x}(\tilde{s}+\tilde{u})\widetilde{B} + \left(1-\hat{y}+\frac{2u\hat{\tau}}{V}\right) \times \right. \\ \left.\left. \times \left[\tilde{x}(\tilde{u}+\tilde{t})\widetilde{B} - \widetilde{A}_t\right]\right]\right\}.$$

For the proton target at rest and the HERA collider, the respective upper integration limits in the right-hand side of Eq. (32) are

$$x_m = \frac{2Mz\varepsilon - 2M\varepsilon_2 - 2z\varepsilon\varepsilon_2(1-c) - \mu^2 - 2M\mu}{2\varepsilon[M + z\varepsilon(1-c_1) - \varepsilon_2(1-c_2)]},$$

$$x_m = \frac{2z - Y(1+c)}{1+c_1}.$$

4. THE TOTAL RADIATIVE CORRECTION

The total RC to Born cross-section (15) is given by the sum of the virtual and soft corrections and the hard-photon emission contribution. The last one is different for the exclusive and calorimeter event selection. In the considered approximation, it is convenient to write this RC as

$$\frac{d\sigma^{RC}_{\|,\perp}}{\hat{y}d\hat{x}\,d\hat{y}\,dz} = \frac{\alpha^2}{4\pi^2}\left(\Sigma_{i\|,\perp} + \Sigma_{f\|,\perp}\right). \qquad (34)$$

The first term Σ_i is independent of the experimental selection rules for the scattered electron and is given by

$$\Sigma_{i\|,\perp} = L_0\left\{\frac{1}{2}L_0 P^{(2)}_\theta(z) + \frac{1+z^2}{1-z} \times \right. \\ \times \left[5\ln z - 2F(z) + \ln^2 Y - 2\ln z\ln Y - \frac{\pi^2}{3} + \right. \\ \left. + 2\mathrm{Li}_2\left(\frac{1+c}{2}\right)\right] + \\ + \frac{3+z^2}{2(1-z)}\ln^2 z - \frac{2(3-2z+3z^2)}{1-z}\ln(1-z) + \\ \left. + \frac{3-20z+z^2}{2(1-z)}\right\}\Sigma_{\|,\perp}(\hat{x},\hat{y},\widehat{Q}^2) + P(z,L_0)\ln\frac{2(1-c)}{\theta_0^2} \times \\ \times \int\limits_0^{u_0}\frac{du}{1-u}P^{(1)}(1-u)\Sigma_{\|,\perp}(x_t,y_t,Q_t^2) + \\ + \frac{1+z^2}{1-z}L_0 Z_{\|,\perp}, \quad u_0 = \frac{x_{1max}}{z}, \qquad (35)$$

where the quantity $P^{(1)}(x)$ is defined by relations (24) and the quantities x_t, y_t, and Q_t^2 depend on $u = x_1/z$.

On the other hand, the second term in the right-hand side of Eq. (34), denoted by Σ_f, explicitly depends on the event selection rule. It includes the main effect of the scattered-electron radiation. For the exclusive event selection, where only the scattered bare electron is measured and any photon that is collinear to its momentum direction is ignored, this contribution is

$$\Sigma^{excl}_{f\|,\perp} = P(z,L_0) \times \\ \times \int\limits_0^{y_{1max}} dy_1\left[(L_Q + \ln Y - 1)P^{(1)}\left(\frac{1}{1+y_1}\right) + \frac{y_1}{1+y_1}\right] \times \\ \times \Sigma_{\|,\perp}(x_s,y_s,Q_s^2). \qquad (36)$$

In this case, as mentioned above, the parameter θ_0' that separates kinematical regions *ii*) and *iii*) is not physical, and we see that the final result does not contain it. But the mass singularity that is related to the scattered electron radiation exhibits itself through L_Q in the right-hand side of Eq. (36).

The situation is quite different for the calorimeter event selection, where the detector cannot distinguish between the events involving a bare electron and events where the scattered electron is accompanied by a hard photon emitted within a narrow cone with the opening angle $2\theta_0'$ around the scattered electron momentum direction. For this experimental setup, we derive

$$\Sigma^{cal}_{f\|,\perp} = P(z, L_0) \times \times \left[\ln \frac{2(1-c)}{\theta_0'^2} \int\limits_0^{y_{1max}} dy_1 P^{(1)} \left(\frac{1}{1+y_1} \right) \times \times \Sigma_{\|,\perp}(x_s, y_s, Q_s^2) + \frac{1}{2} \Sigma_{\|,\perp}(\hat{x}, \hat{y}, \widehat{Q}^2) \right]. \quad (37)$$

For the calorimeter setup, the parameter θ_0' defines the event selection rule and is therefore physically meaningful. The final result depends on it. However, the mass singularity due to the photon emission by the final electron is cancelled in accordance with the Kinoshita–Lee–Nauenberg theorem [18]. The absence of the mass singularity clearly indicates that the term containing $\ln \theta_0'$ in the right-hand side of Eq. (37) arises due to the contribution of kinematical region *iii*), where the scattered electron and the photon radiated from the final state are well separated. That is why no question arises as to determining the quantity ε_2 that enters the expression for y_{1max}.

Comparing our analytical results for the RC due to the real and virtual photon emission with similar calculations for the unpolarized case [8], we see that within the leading-log accuracy (double-logarithm terms in our case), these RC are the same for the spin-dependent and spin-independent parts of the cross-section of radiative DIS process (1). The difference appears at the level of the next-to-leading-log accuracy (single logarithmic terms in our case). That is true for the photonic corrections in an arbitrary order of the perturbation theory.

We note that the correction to the usually measured asymmetry, which is the ratio of the spin-dependent part of the cross-section to the spin-independent one, is not large because the main factorized contribution due to the virtual and soft photon emission trends to cancellation in this case. If the experimental information about the spin observables is extracted directly from the spin-dependent part of the cross-section (see Ref. [19] for the corresponding experimental method), this cancellation does not occur and the factorized correction gives the basic contribution.

5. THE CASE OF QUASI-ELASTIC SCATTERING

In the previous sections, we considered the tagged-photon events in the DIS process. These events can be used to measure the spin-dependent proton structure functions g_1 and g_2 in a single run without lowering the electron beam energy. In the quasi-elastic case, where the target proton is scattered elastically,

$$e^-(k_1) + p(p_1) \to e^-(k_2) + \gamma(k) + p(p_2), \quad (38)$$

the tagged-photon events can also be used to measure the proton electromagnetic form factors G_E and G_M. Our final results obtained in Sec. 4 can then be applied using relation (11) between the spin-dependent proton structure functions g_1 and g_2 and the proton electromagnetic form factors in this limit. In this case, we can therefore use all the formulas in Sec. 4 with $\Sigma_{\|,\perp}$ and $G_{\|,\perp}$ entering the definition of $Z_{\|,\perp}$ replaced by $\Sigma^{el}_{\|,\perp}$ and $G^{el}_{\|,\perp}$, respectively,

$$\Sigma^{el}_{\|}(x, y, Q^2) = \frac{4\pi\alpha^2(Q^2)}{y(4M^2+Q^2)} \times \times \left[4\tau \left(\tau + 1 - \frac{1}{y} \right) G_M G_E - \left(1 - \frac{y}{2} \right) (1+2\tau) G_M^2 \right] \times \times \delta(1-x), \quad (39)$$

$$\Sigma^{el}_{\perp}(x, y, Q^2) = \frac{8\pi\alpha^2(Q^2)}{y(4M^2+Q^2)} \sqrt{\frac{M^2}{Q^2}[1-y(1+\tau)]} \times \times \left[\left(1 - \frac{y}{2} \right) G_M^2 - (1+2\tau) G_M G_E \right] \delta(1-x), \quad (40)$$

$$G^{el}_{\|,\perp} = \frac{\widetilde{Q}^2}{4M^2 + \widetilde{Q}^2} \times \times \left(D_{\|,\perp} G_M^2 + E_{\|,\perp} G_M G_E \right) \delta(1-\tilde{x}), \quad (41)$$

$$D_{\|} = \bar{B} \left[\tilde{q}^2 + 2\hat{\tau}(\tilde{u} + \tilde{t}) \right],$$

$$E_{\|} = 2\hat{\tau} \left[\left(1 + \frac{4M^2}{\widetilde{Q}^2} \right) \widetilde{A}_t - \bar{B}(2\hat{V} + \tilde{u} + \tilde{t}) \right],$$

$$D_{\perp} = -K\bar{B} \left[\frac{u\tilde{q}^2}{V} + \tilde{s} + \tilde{u} + (\tilde{u} + \tilde{t}) \left(1 - \hat{y} + \frac{2u\hat{\tau}}{V} \right) \right],$$

$$E_{\perp} = K \left\{ \left(1 + \frac{4M^2}{\widetilde{Q}^2} \right) \left[\widetilde{A}_s - \left(1 - \hat{y} + \frac{2u\hat{\tau}}{V} \right) \widetilde{A}_t \right] + + \bar{B} \left[\tilde{s} + \tilde{u}(1+4\hat{\tau}) + (\tilde{u} + \tilde{t}) \left(1 - \hat{y} + \frac{2u\hat{\tau}}{V} \right) \right] \right\},$$

where

$$\bar{B} = (\tilde{u} + \tilde{t})(2\hat{V} + \tilde{u} + \tilde{t}) + (\tilde{u} + \tilde{s}) \left[2\hat{V}(1-\hat{y}) - \tilde{u} - \tilde{s} \right],$$

$$K = \sqrt{\frac{M^2}{\widehat{Q}^2}(1 - \hat{y} - \hat{x}\hat{y}\hat{\tau})^{-1}}$$

and the form factors in the right-hand side of Eq. (41) depend on $\tilde{q}^2$.

The description of the form factors is a very important test for any theoretical model of strong interactions [20]. The proton magnetic form factor G_M is known with a high accuracy in a wide range of the momentum transfer, while the data about the electric form factor G_E are very poor. The recent experiment in the Jefferson Lab on the measurement of the ratio of the recoil proton polarizations performed by the Hall A Collaboration [21] improves the situation in the region up to $Q^2 \approx 3.5$ GeV^2, but the higher momentum transfer region remains unexplored. The use of radiative events (38), with both the polarized and unpolarized proton target, on accelerators with a high-intensity electron beam (for example, CEBAF) can open new possibilities in the measurement of G_E as compared to both the Rosenbluth method [22] and the method based on measuring the recoil proton polarization ratio [23].

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