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RADIATIVE CORRECTIONS TO POLARIZED INELASTIC SCATTERING IN THE COINCIDENCE SETUP

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We completely analyze the model-independent leading radiative corrections to the cross-section and polarization observables in the semi-inclusive deep-inelastic electron–nucleus scattering with the detection of a proton and the scattered electron in coincidence. The calculations are based on representing the spin-independent and spin-dependent parts of the cross-section in terms of the electron structure functions similarly to the Drell–Yan representation. As the applications, we consider the polarization transfer effect from a longitudinally polarized electron beam to the detected proton and the scattering by a polarized target.

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1. INTRODUCTION

Current experiments at the new-generation electron accelerators reached a new level of precision. This precision requires a new approach to the data analysis and the inclusion of all possible systematic uncertainties. One of the important sources of systematic uncertainties are the electromagnetic radiative effects caused by physics processes in the next orders of perturbation theory.

The purpose of this paper is to develop a unified approach to the computation of radiative effects for the inelastic scattering of polarized electrons in the coincidence setup, namely, in the case where one produced hadron is detected in coincidence with the scattered electron. A broad range of measurements fall into the category of the coincidence electron scattering experiments. This includes deep-inelastic semi-inclusive leptoproduction of hadrons, $(e, e'h)$, and quasielastic nucleon knock-out processes, $(e, e'N)$. Experiments of the former class give access to the flavor structure of quark–parton distributions and fragmentation functions. They are in the focus of experimental programs at CERN, DESY, SLAC, and Jefferson Lab. Some experiments have already been completed and some are in preparation. The detailed modern review of the activities can be found in [1]. The quasielastic nucleon knock-out process allows studying single-nucleon properties in nuclear medium and probing the nuclear wave function [2, 3].

The different theoretical aspects of strong interactions in the semi-inclusive deep-inelastic scattering (DIS) were studied in a number of papers [4, 5]. The most direct experimental probe of the momentum distribution in nuclei that is presently available is provided by the reaction $A(e, e'N)B$ (see reviews [6]). Specific polarization effects in reactions of this type have been investigated in Ref. [7] at the level of the Born approximation with respect to the electromagnetic interaction.

There are several papers dealing with radiative effects for coincidence experiments. The lowest-order correction was treated in [8] using the approach of the covariant cancellation of infrared divergences. The leading logarithmic correction was studied in [9] for the charm production. Finally, the radiative correction in quasielastic scattering was recently studied in [10]. Different approaches were used in calculations and differ-

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ent approximations have been applied. These calculations adopted some specific models for the structure functions. Because the current experimental data do not cover sufficiently wide kinematical ranges, the extrapolation and interpolation procedures must be used in calculating radiative effects. Therefore, the model dependence of the results reduces their generality and hence their applicability. Furthermore, higher-order effects, which are important at the current level of the experimental accuracy, were not systematically considered.

The method of the electron structure functions [11] allows the same treatment to be applied to the observed cross-section in the lowest order and in higher orders. This results in clear and physically transparent formulas for radiative effects. In this paper, we restrict our consideration to the leading accuracy. This allows us to avoid choosing a preferred model for the hadron structure functions and thus to obtain some general formulas for a wide class of physical processes. Whenever needed, the next-to-leading order correction to some specific process can be obtained by the standard procedure. Good examples are the recent calculations of the leading order and the next-to-leading order corrections to polarization observables in DIS [12] and elastic [13] processes.

In this paper, we consider the model-independent radiative corrections to the cross-section and polarization observables in the semi-inclusive deep-inelastic scattering of the longitudinally polarized electron off nucleus targets in the case where the target and the detected hadron can be polarized. In Sec. 2, we use the electron structure function approach to calculate the radiative corrections and to derive the master formulas for the radiatively corrected spin-independent and spin-dependent parts of the corresponding cross-sections in the form of the Drell–Yan type representation in electrodynamics [14]. The result of this section is applicable to leptonic variables if the scattered electron is detected. In Sec. 3, we apply our master formulas to the case where the polarization of the final nucleon is measured. The radiative corrections to the semi-inclusive DIS on the nucleus target with a vector polarization are calculated in Sec. 4. In Sec. 5, we apply our approach to describe the effects of the polarization transfer from the target to the detected nucleon. These effects include the double spin (hadron–hadron) and triple spin (electron–hadron–hadron) correlations. In Sec. 6, we derive the modification of the master formulas for hadronic variables (when the total 4-momentum of all the hadrons is measured instead of the scattered electron) and consider some applications. In Conclusion, we briefly discuss the extension of our results for the radiatively corrected polarization observables beyond the leading-log accuracy.

2. THE MASTER FORMULA

In the recent experiment [15], the polarization transfer to the detected proton was measured in the process with the longitudinally polarized electron beam $^{16}\mathrm{O}(\vec{e}, e, \vec{p})^{15}\mathrm{N}$. This reaction is a particular case of the more general semi-inclusive deep-inelastic polarized process

$$\vec{e}^{\,-}(k_1) + A(p_1) \to e^-(k_2) + \vec{p}(p_2) + X. \qquad (1)$$

In this paper, we clarify the problem of calculating the electromagnetic radiative corrections to the cross-section and polarization observables in a process of this type within the framework of the electron structure function approach.

For process (1) with a definite spin orientation of the proton detected in the final state, we define the cross-section in terms of the leptonic and hadronic tensors as

$$d\sigma = \frac{\alpha^2}{(2S_A+1)V(2\pi)^3}\,\frac{L_{\mu\nu}H_{\mu\nu}}{2\hat{q}^4}\,\frac{d^3k_2}{\varepsilon_2}\,\frac{d^3p_2}{E_2}, \qquad (2)$$

where S_A is the target spin, $\varepsilon_2(E_2)$ is the energy of the scattered electron (the detected proton), and $\hat{q}$ is the 4-momentum of the virtual photon that probes the hadron block. The hadronic tensor can be expressed through the hadron electromagnetic current J_μ as

$$H_{\mu\nu} = \sum_X \langle p_1|J_\mu(\hat{q})|p_2, X\rangle \times \times \langle X, p_2|J_\nu(-\hat{q})|p_1\rangle \delta(P_x^2 - M_x^2),$$

$$P_x = \hat{q} + p_1 - p_2,$$

where P_x is the total 4-momentum of the undetected hadron system and M_x is its invariant mass.

The electron structure function approach leads to the summation of the leading-log contributions to the leptonic tensor in all orders of the perturbation theory. These contributions arise because of the radiation of the hard collinear as well as the soft and virtual photons and the electron–positron pairs by electrons in the initial and final states. In the leading approximation, the electron tensor in the right-hand side of Eq. (2) can be written as [16]

$$L_{\mu\nu}(k_1,k_2) = \iint \frac{dx_1 dx_2}{x_1 x_2^2} D(x_2,Q^2) \times \\ \times \left[D(x_1,Q^2)\hat{Q}^B_{\mu\nu}(\hat{k}_1,\hat{k}_2) + i\lambda D_\lambda(x_1,Q^2)\hat{E}^B_{\mu\nu}(\hat{k}_1,\hat{k}_2)\right], \tag{3}$$

$$Q^2 = -(k_1-k_2)^2, \quad \hat{k}_1 = x_1 k_1, \quad \hat{k}_2 = \frac{k_2}{x_2},$$

where the respective structure functions $D(x,Q^2)$ and $D_\lambda(x,Q^2)$ describe the radiation of the unpolarized and the longitudinally polarized electrons. At the level of the next-to-leading accuracy, these functions already differ in the first order of the perturbation theory, but in the framework of the leading approximation used here, they only differ in the second order. This difference is caused by the leading contribution of the e^+e^--pair production in the singlet channel to the D function (the effect of the final electron identity). For the unpolarized and longitudinally polarized electron, these contributions are different and are given by [16] (KMS), [17]

$$D^S = \left(\frac{\alpha L}{2\pi}\right)^2 \left[\frac{2(1-x^3)}{3x} + \frac{1-x}{2} + (1+x)\ln x\right],$$

$$L = \frac{Q^2}{m_e^2},$$

$$D^S_\lambda = \left(\frac{\alpha L}{2\pi}\right)^2 \left[\frac{5(1-x)}{2} + (1+x)\ln x\right],$$

where m_e is the electron mass.

Taking the singlet channel contribution into account usually leads to very small effects (of the order 10^{-4}) because, as one can see, the terms inside the brackets tend to compensate each other (see, e.g. [18]). In what follows, we do not distinguish between D and D_λ, which corresponds to taking only the nonsinglet channel contribution into account (for the corresponding D functions, see [17, 18]). This approximation allows us to write compact formulas for the radiatively corrected cross-sections. We also omit the quantity Q^2 from the arguments of the D functions.

The quantity λ entering the right-hand side of Eq. (3) is the degree of the longitudinal polarization of the electron beam. The integration limits are defined below. Representation (3) follows from the quasireal electron approximation [19]. The physical interpretation of the variables x_1 and x_2 is as follows: $1-x_1 = \omega/\varepsilon_1$ is the ratio of the energy of all the collinear photons and the e^+e^--pairs radiated by the initial electron to the energy of that electron and $(1-x_2)/x_2$ is a similar ratio for the scattered electron.

In the Born approximation, we have

$$Q^B_{\mu\nu}(k_1,k_2) = q^2 g_{\mu\nu} + 2(k_1k_2)_{\mu\nu}, \\ E^B_{\mu\nu}(k_1,k_2) = 2(\mu\nu k_1 k_2), \\ (\mu\nu k_1 k_2) = \epsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}, \tag{4}$$

$$(k_1k_2)_{\mu\nu} = k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}, \quad q = k_1 - k_2.$$

In the general case, the hadronic tensor in the right-hand side of Eq. (2) depends on the 4-momenta p_1, p_2, the virtual photon 4-momentum $\hat{q} = \hat{k}_1 - \hat{k}_2$, and the 4-vector of the hadron spin S that satisfies the conditions $S^2 = -1$ and $(Sp_2) = 0$. For example, in the case under consideration,

$$H_{\mu\nu} = H^{(u)}_{\mu\nu} + H^{(p)}_{\mu\nu},$$

$$H^{(u)}_{\mu\nu} = h_1\tilde{g}_{\mu\nu} + h_2\tilde{p}_{1\mu}\tilde{p}_{1\nu} + h_3\tilde{p}_{2\mu}\tilde{p}_{2\nu} + \\ + h_4(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ih_5[\tilde{p}_1\tilde{p}_2]_{\mu\nu}, \tag{5}$$

$$H^{(p)}_{\mu\nu} = (Sp_1)\left[h_6(\tilde{p}_1N)_{\mu\nu} + ih_7[\tilde{p}_1N]_{\mu\nu} + h_8(\tilde{p}_2N)_{\mu\nu} + \\ + ih_9[\tilde{p}_2N]_{\mu\nu}\right] + (S\hat{q})\left[h_{10}(\tilde{p}_1N)_{\mu\nu} + \\ + ih_{11}[\tilde{p}_1N]_{\mu\nu} + h_{12}(\tilde{p}_2N)_{\mu\nu} + ih_{13}[\tilde{p}_2N]_{\mu\nu}\right] + \\ + (SN)\left[h_{14}\tilde{g}_{\mu\nu} + h_{15}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + \\ + h_{16}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + h_{17}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + ih_{18}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}\right], \tag{6}$$

$$N_\mu = \epsilon_{\mu\nu\rho\sigma}p_{1\nu}p_{2\rho}\hat{q}_\sigma = (\mu p_1 p_2 \hat{q}), \quad [ab]_{\mu\nu} = a_\mu b_\nu - a_\nu b_\mu,$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\hat{q}_\mu\hat{q}_\nu}{\hat{q}^2}, \quad \tilde{p}_{i\mu} = p_{i\mu} - \frac{(\hat{q}p_i)\hat{q}_\mu}{\hat{q}^2}, \quad i = 1, 2,$$

where h_i ($i = 1$–18) are the hadron semi-inclusive structure functions that depend on four invariants in general. These invariants can be taken as $\hat{q}^2$, $(\hat{q}p_1)$, $(\hat{q}p_2)$, and (p_1p_2).

The j-component of the proton polarization P^j that could be measured experimentally is defined as the ratio of the spin-dependent part of cross-section (2) (which is caused by the contraction of the leptonic tensor with the spin-dependent part of the hadronic one $H^{(p)}_{\mu\nu}$, with the given j-component of the proton spin) to the spin-independent one (which is caused by the contraction of $L_{\mu\nu}$ with $H^{(u)}_{\mu\nu}$),

$$P^j = \frac{d\sigma^{(p)}(\lambda, S^j, k_1, k_2, p_1, p_2)}{d\sigma^{(u)}(\lambda, k_1, k_2, p_1, p_2)}. \tag{7}$$

We note that P^j is nonzero even if $\lambda = 0$ (the case of the unpolarized electron beam) because of nonzero single-spin correlations in semi-inclusive processes.

In principle, three independent components can be measured in process (1): P^l (longitudinal), P^t (transverse), and P^n (normal), which could be taken relative to the definite physical directions and planes created by 3-momenta of the particles participating in the process. If no additional particle (photons and e^+e^--pairs) radiated by electrons with the 4-momenta k_1 and k_2 are detected, there are three independent directions along $\mathbf{p}_2$, $\mathbf{k}_1$, and $\mathbf{k}_2$. In this case, any component of the proton polarization and the corresponding proton spin components S^j are defined for the Born kinematics and their directions are not affected by the radiation.

Combining formulas (2) for the cross-section, definitions (3) and (4) of the lepton and (5) and (6) of the hadron tensors and taking the above discussion into account, we can write the cross-section of process (1) as

$$\varepsilon_2 E_2 \frac{d\sigma(\lambda, S^j, k_1, k_2, p_1, p_2)}{d^3k_2 d^3p_2} = \iint \frac{dx_1 dx_2}{x_2^2} D(x_1)D(x_2)\hat{\varepsilon}_2 E_2 \times \frac{d\sigma^B(\lambda, S^j, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2}, \quad (8)$$

where $j = l,\ t,\ n$. The factor $1/x_1$ that enters the definition of $L_{\mu\nu}$ is absorbed into the flow in the reduced Born cross-section that is by definition given by (see Eq. (2))

$$\hat{\varepsilon}_2 E_2 \frac{d\sigma^B(\lambda, S^j, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2} = \frac{\alpha^2}{(2S_A+1)\hat{V}(2\pi)^3} \times \frac{L^B_{\mu\nu}(\hat{k}_1, \hat{k}_2, \lambda)H_{\mu\nu}(S^j, \hat{q}, p_1, p_2)}{2\hat{q}^4},$$

where $\hat{V} = x_1 V$. Within the chosen accuracy, representation (8) is valid for both the spin-dependent $(d\sigma^{(p)})$ and spin-independent $(d\sigma^{(u)})$ parts of the cross-section.

In theoretical calculations, it is often useful to parameterize the proton spin 4-vector, which enters the definition of the hadron tensor, in terms of the particle 4-momenta [20]. In our case, we have four 4-momenta to express any component of the proton spin S^j such that

$$S^j = S^j(k_1, k_2, p_1, p_2). \quad (9)$$

We temporarily imagine that the chosen parameterization in the right-hand side of Eq. (9) is stabilized by the relative substitution

$$k_1 \to \hat{k}_1, \quad k_2 \to \hat{k}_2,$$

$$S^{j_s}(k_1, k_2, p_1, p_2) = S^{j_s}(\hat{k}_1, \hat{k}_2, p_1, p_2).$$

(In what follows, we label such stabilized parameterizations by an index with a lower-case letter.) In this case, we can write the Born cross-section in the integrand in the right-hand side of Eq. (8) as

$$\hat{\varepsilon}_2 E_2 \frac{d\sigma^B(\lambda, S^j, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2} = \hat{\varepsilon}_2 E_2 \frac{d\sigma^B_j(\lambda, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2}. \quad (10)$$

If the proton spin S^J is unstable under the above substitution (in this case, we use a capital letter index), it can always be expressed in terms of the stabilized one by means of an orthogonal matrix,

$$S^J(k_1, k_2, p_1, p_2) = A_{Jj}(k_1, k_2, p_1, p_2)S^j(\hat{k}_1, \hat{k}_2, p_1, p_2), \quad A_{Jj} = -S^J S^j. \quad (11)$$

Using this formula and recalling that in the class of processes considered here, the hadron tensor depends on the proton spin linearly, we can write the master representation for the spin-dependent part $(d\sigma^{(p)})$ of the cross-section of process (1) for an arbitrary orientation of the proton spin as

$$\varepsilon_2 E_2 \frac{d\sigma(\lambda, S^J, k_1, k_2, p_1, p_2)}{d^3k_2 d^3p_2} = A_{Jj} \iint \frac{dx_1 dx_2}{x_2^2} D(x_1)D(x_2)\hat{\varepsilon}_2 E_2 \times \frac{d\sigma^B_j(\lambda, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2}, \quad (12)$$

where the summation over the index $j = l, t, n$ is implied.

This representation is the electrodynamical analogue of the Drell–Yan formula well known in QCD [14], which has previously been applied to calculate the electromagnetic radiative corrections to the total cross-section of the electron–positron annihilation into hadrons [17], to the small-angle Bhabha scattering cross-section at LEP1 [18], to unpolarized [21] and polarized deep-inelastic cross-sections [12], and to the polarized elastic electron–proton scattering [13]. In the next section, we show how this representation can be used to describe the leading radiative corrections in polarized semi-inclusive deep-inelastic events. Within the leading accuracy, we must find adequate parameterizations of the proton spin 4-vector, calculate the elements of the orthogonal matrix A_{Jj}, derive the spin-independent and spin-dependent parts of the Born cross-section for a given parameterization S^j, and determine the x_1 and x_2 integration limits in master formula (12).

3. THE ANALYSIS OF SEMI-INCLUSIVE DEEP-INELASTIC EVENTS WITH THE POLARIZATION TRANSFER

We begin with the parameterizations of the proton spin 4-vector in process (1). To describe this process, we use the set of invariant variables

$$\begin{aligned} z &= \frac{2p_1p_2}{V}, \quad z_{1,2} = \frac{2k_{1,2}p_2}{V}, \\ y &= \frac{2p_1(k_1-k_2)}{V}, \\ x &= \frac{-q^2}{2p_1q}, \quad V = 2p_1k_1, \quad q = k_1 - k_2. \end{aligned} \tag{13}$$

It is physically justified to determine the longitudinal component of the proton spin along the direction of $-\mathbf{p}_1$ as seen from the rest frame of the detected proton. This direction is not affected by the lepton collinear radiation and the corresponding parameterization is given by

$$S^l_\mu = \frac{zp_{2\mu} - 2\tau_2 p_{1\mu}}{m\sqrt{z^2-4\tau_1\tau_2}}, \quad \tau_1 = \frac{M^2}{V}, \quad \tau_2 = \frac{m^2}{V}, \tag{14}$$

where $M(m)$ is the mass of the target nucleus (detected proton). It is easy to verify that in the rest frame of the proton ($p_2 = (m, 0)$), this longitudinal component is equal to $(0, -\mathbf{n}_1)$, where $\mathbf{n}_1 = \mathbf{p}_1/|\mathbf{p}_1|$, and in the laboratory system ($p_1 = (M,\ 0)$), it is equal to $(|\mathbf{p}_2|,\ E_2\mathbf{n}_2)/m$, where $\mathbf{n}_2$ is the unit vector in the direction of the detected proton 3-momentum.

For the fixed longitudinal component, we have several possibilities to determine the transverse and normal components. We first take the transverse component in the plane $(\mathbf{k}_1,\ \mathbf{p}_2)$ and the normal component in the plane that is perpendicular to it. The orientations of these planes do not change under the substitution $\mathbf{k}_1 \to \hat{\mathbf{k}}_1$, and we therefore have

$$S^t_\mu = \frac{(z^2-4\tau_1\tau_2)k_{1\mu} + (2z_1\tau_1 - z)p_{2\mu} + (2\tau_2 - zz_1)p_{1\mu}}{\sqrt{V(z^2-4\tau_1\tau_2)[1]}}, \quad S^n_\mu = \frac{2(\mu k_1 p_1 p_2)}{\sqrt{V^3[1]}}, \tag{15}$$

$$[1] = zz_1 - \tau_2 - z_1^2\tau_1, \quad (S^jS^i) = -\delta_{ji}.$$

Totally similarly to the above procedure, we can determine another stabilized set of transverse and normal components relative to the plane $(\mathbf{k}_2,\ \mathbf{p}_2)$,

$$\begin{aligned} \tilde{S}^t_\mu &= \frac{(z^2-4\tau_1\tau_2)k_{2\mu} + (2z_2\tau_1 - z(1-y))p_{2\mu} + (2\tau_2(1-y) - zz_2)p_{1\mu}}{\sqrt{V(z^2-4\tau_1\tau_2)[2]}}, \\ \tilde{S}^n_\mu &= \frac{2(\mu k_2 p_1 p_2)}{\sqrt{V^3[2]}}, \quad [2] = zz_2(1-y) - \tau_2(1-y)^2 - z_2^2\tau_1. \end{aligned} \tag{16}$$

The sets in Eqs. (15) and (16) represent the complete list of stabilized parameterizations of the proton spin components under the condition that the longitudinal component is chosen in accordance with Eq. (14). There are many unstable parameterizations that can be taken relative to an arbitrary plane $(a\mathbf{k}_1 + b\mathbf{k}_2,\ \mathbf{p}_2)$ with arbitrary numbers a and b. In what follows, we consider only the physically favorable set with $a = -b = 1$. The corresponding transverse and normal components are given by

$$\begin{aligned} S^T_\mu &= \frac{(z^2-4\tau_1\tau_2)q_\mu + (2(z_1-z_2)\tau_1 - zy)p_{2\mu} + (2y\tau_2 - z(z_1-z_2))p_{1\mu}}{\sqrt{V(z^2-4\tau_1\tau_2)[q]}}, \\ S^N_\mu &= \frac{2(\mu q p_1 p_2)}{\sqrt{V^3[q]}}, \\ [q] &= zy(z_1-z_2) + xy(z^2-4\tau_1\tau_2) - (z_1-z_2)^2\tau_1 - y^2\tau_2. \end{aligned} \tag{17}$$

We now consider the relation between the stabilized set (for definiteness, we work with set (15)) and an unstable one. It is obvious that this relation can be written as

$$S^N = S^n\cos\theta + S^t\sin\theta, \quad S^T = -S^n\sin\theta + S^t\cos\theta, \tag{18}$$

where

$$\cos\theta = -(S^N S^n) = -(S^T S^t) = \frac{z(z_1(1+y)-z_2)+xy(z^2-4\tau_1\tau_2)-2z_1(z_1-z_2)\tau_1-2y\tau_2}{2\sqrt{[1][q]}},$$

$$\sin\theta = -(S^N S^t) = (S^T S^n) = \frac{\eta}{2}\sqrt{\frac{z^2-4\tau_1\tau_2}{[1][q]}},$$

$$\eta = \mathrm{sign}[(p_1p_2k_1k_2)]\sqrt{\frac{16}{V^4}(p_1p_2k_1k_2)^2},$$

$$(p_1p_2k_1k_2) = \epsilon_{\mu\nu\rho\sigma}p_{1\mu}p_{2\nu}k_{1\rho}k_{2\sigma},$$

$$\frac{16(p_1p_2k_1k_2)^2}{V^4} = x^2y^2(4\tau_1\tau_2 - z^2) + + 2xy[z(z_2+z_1(1-y)) - - 2z_1z_2\tau_1 - 2(1-y)\tau_2] - (z_2 - z_1(1-y))^2.$$

One can verify that the necessary condition $\cos^2\theta + + \sin^2\theta = 1$ is satisfied.

We can now write the spin-independent part (which is actually independent of the proton spin only) and the spin-dependent part of the cross-section of process (1) as

$$\varepsilon_2 E_2\frac{d\sigma_{(u),L}}{d^3k_2d^3p_2} = = \iint\frac{dx_1dx_2}{x_2^2}D(x_1)D(x_2)\hat\varepsilon_2E_2\frac{d\hat\sigma^B_{(u),l}}{d^3\hat k_2d^3p_2}, \quad (19)$$

$$\varepsilon_2 E_2\frac{d\sigma_N}{d^3k_2d^3p_2} = \int\int\frac{dx_1dx_2}{x_2^2}D(x_1)D(x_2)\hat\varepsilon_2E_2 \times \times\left[\cos\theta\frac{d\hat\sigma^B_n}{d^3\hat k_2d^3p_2} + \sin\theta\frac{d\hat\sigma^B_t}{d^3\hat k_2d^3p_2}\right], \quad (20)$$

$$\varepsilon_2 E_2\frac{d\sigma_T}{d^3k_2d^3p_2} = \iint\frac{dx_1dx_2}{x_2^2}D(x_1)D(x_2)\hat\varepsilon_2E_2 \times \times\left[-\sin\theta\frac{d\hat\sigma^B_n}{d^3\hat k_2d^3p_2} + \cos\theta\frac{d\hat\sigma^B_t}{d^3\hat k_2d^3p_2}\right], \quad (21)$$

where $d\hat\sigma^B$ with any lower-case index denotes the corresponding Born cross-section given at the shifted values of $k_{1,2}\to\hat k_{1,2}$. The corresponding shifted dimensionless variables introduced by relation (13) are given by

$$\hat x = \frac{x_1xy}{x_1x_2+y-1},\quad \hat y = \frac{x_1x_2+y-1}{x_1x_2},\quad \hat V = x_1V, \quad \hat z = \frac{z}{x_1},\quad \hat z_1 = z_1,\quad \hat z_2 = \frac{z_2}{x_1x_2}. \quad (22)$$

Equations (19)–(21) are the straightforward consequences of master representation (12). Obviously, in order to obtain $d\sigma_n$ and $d\sigma_t$ in the left-hand sides of Eqs. (20) and (21), we must set $\cos\theta = 1$, $\sin\theta = 0$.

Next, we must derive the Born cross-sections that enter the right-hand sides of Eqs. (19)–(21). The spin-independent part of the cross-section for the longitudinally polarized electron beam (with the degree λ) is expressed in terms of the hadron structure functions $h_1,\ldots,h_5$ as

$$\varepsilon_2E_2\frac{d\sigma^B_{(u)}}{d^3k_2d^3p_2} = \frac{\alpha^2V}{2(2S_A+1)(2\pi)^3q^4}H_1, \quad (23)$$

$$H_1 = -\frac{2xy}{V}h_1 + (1-y-xy\tau_1)h_2 + (z_1z_2 - xy\tau_2)h_3 + + (z_2 + z_1(1-y) - xyz)h_4 - \lambda\eta h_5.$$

We note that the phase space of the detected proton can also be expressed in terms of invariant variables (13) as

$$\frac{d^3p_2}{E_2} = \frac{V}{2|\eta|}dz_1dz_2dz. \quad (24)$$

If the proton spin is directed along S^l, the spin-dependent part of the Born cross-section is given by

$$\varepsilon_2E_2\frac{d\sigma^B_l}{d^3k_2d^3p_2} = -\frac{\alpha^2V^3\eta\sqrt{z^2-4\tau_1\tau_2}}{8(2S_A+1)m(2\pi)^3q^4}\times \times\left[H_2 + \frac{z(z_1-z_2)-2y\tau_2}{z^2-4\tau_1\tau_2}H_3\right], \quad (25)$$

$$H_2 = (2-y)h_6 + (z_1+z_2)h_8 + \frac{\lambda}{\eta}(\eta_1h_7 + \eta_2h_9),$$

$$H_3 = (2-y)h_{10} + (z_1+z_2)h_{12} + \frac{\lambda}{\eta}(\eta_1h_{11} + \eta_2h_{13}),$$

$$\eta_1 = y\left[z_2 - z_1(1-y) - xz(2-y) + 2x(z_1+z_2)\tau_1\right],$$

$$\eta_2 = (z_1-z_2)\left(z_2 - z_1(1-y)\right) + xyz(z_1+z_2) - 2xy(2-y)\tau_2.$$

For the transverse orientation of the spin (along S^t), we have

$$\varepsilon_2 E_2 \frac{d\sigma_t^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^2 \eta}{8(2S_A+1)(2\pi)^3 q^4} \times \times \sqrt{\frac{V}{z^2-4\tau_1\tau_2}} \left[\psi H_3 - \frac{z^2-4\tau_1\tau_2}{\sqrt{[1]}} H_4\right], \quad (26)$$

$$\psi = = \frac{xy(z^2-4\tau_1\tau_2)+(z-2z_1\tau_1)(z_1-z_2)+(zz_1-2\tau_2)y}{\sqrt{[1]}} = = 2\sqrt{[q]}\cos\theta,$$

where H_4 can be obtained from H_1 by the simple replacement $h_i \to h_{i+13}$.

Finally, for the normal orientation of the proton spin (along S^n) the spin-dependent part of the cross-section of process (1) is given by

$$\varepsilon_2 E_2 \frac{d\sigma_n^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^2 \sqrt{V}}{8(2S_A+1)(2\pi)^3 q^4} \times \times \left[-\frac{\eta^2}{\sqrt{[1]}} H_3 - \psi H_4\right]. \quad (27)$$

We must also determine the integration limits for x_1 and x_2 in master representation (12). They can be obtained from the condition that the semi-inclusive deep-inelastic process occurs. For the electron–proton scattering, this is possible under the condition that the hadron state involves at least a proton and a pion. This leads to the inequality

$$x_1x_2+y-1-x_1xy \geq x_2\delta, \quad \delta = \frac{(m+m_\pi)^2-m^2}{V}, \quad (28)$$

where m_π is the pion mass. For the integration limits, we then have

$$1 > x_2 > \frac{1-y+xyx_1}{x_1-\delta}, \quad 1 > x_1 > \frac{1+\delta-y}{1-xy}. \quad (29)$$

For the electron–nucleus scattering process (1) considered here, we must change the pion mass entering the definition of δ by the bound energy of the ejected proton in a given nucleus.

It is interesting to note that in the case where the final proton polarizations are measured relative to stabilized orientations, the corresponding Born values and the leading radiative corrections to them are expressed in terms of the same hadron structure functions. The situation changes radically if the polarizations are measured relative to unstable orientations. In this case, the contributions to the polarizations caused by the radiative corrections due to the hard collinear radiation are expressed in terms of different sets of hadron structure functions compared to those used in the Born polarizations. To make this more transparent, we write the spin-dependent part of the Born cross-section for the orientations of the proton spin along S^N and S^T,

$$\varepsilon_2 E_2 \frac{d\sigma_T^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^2 \eta}{4(2S_A+1)(2\pi)^3 q^4} \times \times \sqrt{\frac{V[q]}{z^2-4\tau_1\tau_2}} H_3, \quad (30)$$

$$\varepsilon_2 E_2 \frac{d\sigma_N^B}{d^3k_2 d^3p_2} = -\frac{\alpha^2 V^2 \sqrt{V[q]}}{4(2S_A+1)(2\pi)^3 q^4} H_4. \quad (31)$$

These formulas can be derived from Eqs. (20) and (21) if the $D(x_i)$ functions are taken to be the δ-function, which corresponds to the radiationless process (or to the Born approximation).

4. SEMI-INCLUSIVE DEEP-INELASTIC SCATTERING ON A POLARIZED TARGET

In this section, we apply the master representation to the analysis of polarized phenomena in the semi-inclusive deep-inelastic scattering of the polarized nucleus,

$$\vec{e}^{\,-}(k_1) + \vec{A}(p_1) \to e^-(k_2) + H(p_2) + X, \quad (32)$$

where H is an arbitrary hadron and the nucleus A has a definite vector polarization P. In this case, the leptonic tensor is the same as above (see Eqs. (3) and (4)) and the hadronic tensor has the same structure as defined by Eqs. (5) and (6), where the nucleus polarization P must be used instead of the proton spin S and (Sp_1) must be replaced with (Pp_2). We also use the notation $g_1, \ldots, g_{18}$ for the corresponding hadron structure functions.

To find the various asymmetries measured in studying the polarization phenomena, it is necessary to know the polarization-independent and polarization-dependent parts of the cross-section at different orientations of the target polarization. The corresponding analysis can therefore be performed in the same way as in Sec. 2.

We first define parameterizations of the nucleus polarization 4-vector in terms of the 4-momenta. As a stabilized set, we can choose the longitudinal and transverse components given in Ref. [12],

$$P_\mu^l = \frac{2\tau_1 k_{1\mu} - p_{1\mu}}{M},$$
$$P_\mu^t = \frac{k_{2\mu} - (1-y-2xy\tau_1)k_{1\mu} - xyp_{1\mu}}{\sqrt{Vxy(1-y-xy\tau_1)}}, \tag{33}$$

and for the normal component, we use

$$P_\mu^n = \frac{2(\mu k_1 k_2 p_1)}{\sqrt{V^3 xy(1-y-xy\tau_1)}}. \tag{34}$$

It is easy to verify that parameterizations (33) and (34) are not changed after the substitution $k_{1,2} \to \hat{k}_{1,2}$. In the laboratory system, this set corresponds to the longitudinal polarization directed along $\mathbf{k}_1$, the transverse polarization in the plane $(\mathbf{k}_1, \mathbf{k}_2)$, and the normal one in the plane that is perpendicular to the $(\mathbf{k}_1,\ \mathbf{k}_2)$ plane.

Another set of polarizations can be chosen such that the longitudinal component is along the $\mathbf{q}$ direction in the laboratory system and the transverse one is in the plane $(\mathbf{q},\ \mathbf{k}_1)$. In this case, the normal component coincides with (34) and

$$P_\mu^L = \frac{2\tau_1(k_{1\mu} - k_{2\mu}) - yp_{1\mu}}{M\sqrt{y^2 + 4xy\tau_1}},$$
$$P_\mu^T = \tag{35}$$
$$= \frac{(1+2x\tau_1)k_{2\mu} - (1-y-2x\tau_1)k_{1\mu} - x(2-y)p_{1\mu}}{\sqrt{Vx(1-y-xy\tau_1)(y+4x\tau_1)}}.$$

The transformation between sets (35) and (33) is implemented by the orthogonal matrix

$$P^L = \cos\theta_1 P^l + \sin\theta_1 P^t,$$
$$P^T = -\sin\theta_1 P^l + \cos\theta_1 P^t, \tag{36}$$

$$\cos\theta_1 = \frac{y(1+2x\tau_1)}{\sqrt{y(y+4x\tau_1)}},$$

$$\sin\theta_1 = -2\sqrt{\frac{x\tau_1(1-y-xy\tau_1)}{y+4x\tau_1}}.$$

Master equation (12) can be applied to the polarization-independent part of cross-section (32) and to the polarization-dependent part. Therefore, we must derive the Born cross-section for the stabilized set. A simple calculation gives

$$\varepsilon_2 E_2 \frac{d\sigma_{(u)}^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V}{(2S_A+1)(2\pi)^3 q^4} G_1. \tag{37}$$

We note that the numerical coefficient in front of G_1 is twice the coefficient in front of H_1 in the right-hand side of Eq. (23). The reason is that we do not fix the spin state of the final hadron H in this case.

The polarization-dependent part of the cross-section for the longitudinal stabilized polarization is given by

$$\varepsilon_2 E_2 \frac{d\sigma_l^B}{d^3k_2 d^3p_2} = -\frac{\alpha^2 V^3 \eta}{4(2S_A+1)M(2\pi)^3 q^4} \times$$
$$\times \left[(2\tau_1 z_1 - z)G_2 - y(1+2x\tau_1)G_3 + 2\tau_1 G_4\right], \tag{38}$$

where the functions G_i, $i = 1, \ldots, 4$, can be derived from H_i by replacing the hadron structure functions h_j with g_j.

For the transverse polarization, the corresponding part of the cross-section can be written as

$$\varepsilon_2 E_2 \frac{d\sigma_t^B}{d^3k_2 d^3p_2} = -\frac{\alpha^2 V^2 \eta\sqrt{Vxy(1-y-xy\tau_1)}}{4(2S_A+1)(2\pi)^3 q^4} \times$$
$$\times \left[\frac{z_2 - xyz - z_1(1-y-2xy\tau_1)}{xy(1-y-xy\tau_1)} G_2 + \right.$$
$$\left. + 2G_3 + \frac{1+2x\tau_1}{x(1-y-xy\tau_1)} G_4\right]. \tag{39}$$

For the normal polarization, the spin-dependent part of the cross-section is

$$\varepsilon_2 E_2 \frac{d\sigma_n^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^2}{4(2S_A+1)(2\pi)^3 q^4} \times$$
$$\times \sqrt{\frac{V}{xy(1-y-xy\tau_1)}} \times$$
$$\times \left[\eta^2 G_2 - y\big(z_2(1+2x\tau_1) - \right.$$
$$\left. - z_1(1-y-2x\tau_1) - xz(2-y)\big)G_4\right]. \tag{40}$$

Using master representation (12) leads to the radiatively corrected contributions (within the leading accuracy) to the cross-section of process (32),

$$\varepsilon_2 E_2 \frac{d\sigma_{(u),N}}{d^3k_2 d^3p_2} =$$
$$= \iint \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \frac{d\hat{\sigma}_{(u),n}^B}{d^3\hat{k}_2 d^3p_2}, \tag{41}$$

$$\varepsilon_2 E_2 \frac{d\sigma_L}{d^3k_2 d^3p_2} = \iint \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \times$$
$$\times \left[\cos\theta_1 \frac{d\hat{\sigma}_l^B}{d^3\hat{k}_2 d^3p_2} + \sin\theta_1 \frac{d\hat{\sigma}_t^B}{d^3\hat{k}_2 d^3p_2}\right], \tag{42}$$

$$\varepsilon_2 E_2 \frac{d\sigma_T}{d^3k_2 d^3p_2} = \iint \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \times$$
$$\times \left[-\sin\theta_1 \frac{d\hat{\sigma}_l^B}{d^3\hat{k}_2 d^3p_2} + \cos\theta_1 \frac{d\hat{\sigma}_t^B}{d^3\hat{k}_2 d^3p_2}\right]. \tag{43}$$

We also write the cross-sections in the left-hand sides of Eqs. (42) and (43) in the Born approximation,

$$\varepsilon_2 E_2 \frac{d\sigma_L^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^3 \eta}{4(2S_A+1)(2\pi)^3 M q^4} \times \left[\frac{yz - 2(z_1 - z_2)\tau_1}{\sqrt{y(y+4x\tau_1)}} G_2 + \sqrt{y(y+4x\tau_1)} G_3 \right], \quad (44)$$

$$\varepsilon_2 E_2 \frac{d\sigma_T^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^2 \eta \sqrt{V}}{4(2S_A+1)(2\pi)^3 q^4} \times \left[-\sqrt{\frac{y+4x\tau_1}{x(1-y-xy\tau_1)}} G_4 + \frac{xz(2-y) - z_2 + z_1(1-y) - 2x\tau_1(z_1+z_2)}{\sqrt{x(y+4x\tau_1)(1-y-xy\tau_1)}} G_2 \right]. \quad (45)$$

Thus, the polarization-dependent parts of the Born cross-section involve fewer hadron structure functions than the radiatively corrected cross-sections.

We can also use the 4-vector p_2 to parameterize the nucleus polarization 4-vector. With the longitudinal polarization chosen along $\mathbf{p}_2$ in the laboratory system, the stabilized set can be defined with respect to the plane $(\mathbf{k}_1, \mathbf{p}_2)$ and the unstable set with respect to the plane $(\mathbf{q}, \mathbf{p}_2)$ as in Sec. 2; the corresponding calculations are very similar to those in Sec. 2. But the parameterizations used in this section look more physical and can also be used to describe the polarization phenomena in inclusive deep-inelastic events.

5. POLARIZATION TRANSFER FROM THE TARGET TO THE DETECTED PROTON

We now consider the effects of the polarization transfer from the vector polarized target to the detected proton in the process

$$\vec{e}^{\,-}(k_1) + \vec{A}(p_1) \to e^-(k_2) + \vec{p}(p_2) + X \quad (46)$$

for the longitudinally polarized electron beam and the vector polarization of the target. In this case, the general form of the hadronic tensor is given by

$$H_{\mu\nu} = H_{\mu\nu}^{(u)} + H_{\mu\nu}^{(S)} + H_{\mu\nu}^{(W)} + H_{\mu\nu}^{(SW)}, \quad (47)$$

where $S(W)$ labels the vector polarization of the target (the spin of the detected proton). All the effects caused by the first three terms in the right-hand side of Eq. (47) were considered in previous sections and we now investigate the radiative corrections to the hadron double-spin correlations that precisely arise due to the last term,

$$\begin{aligned} H_{\mu\nu}^{(SW)} = {} & (Sp_2)(Wp_1)\big[f_1\tilde{g}_{\mu\nu} + f_2\tilde{p}_{1\mu}\tilde{p}_{1\nu} + f_3\tilde{p}_{2\mu}\tilde{p}_{2\nu} + \\ & + f_4(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + if_5[\tilde{p}_1\tilde{p}_2]_{\mu\nu}\big] + \\ & + (Sp_2)(Wq)\big[f_6\tilde{g}_{\mu\nu} + f_7\tilde{p}_{1\mu}\tilde{p}_{1\nu} + f_8\tilde{p}_{2\mu}\tilde{p}_{2\nu} + \\ & + f_9(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + if_{10}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}\big] + \\ & + (Sp_2)(WN)\big[f_{11}(\tilde{p}_1N)_{\mu\nu} + if_{12}[\tilde{p}_1N]_{\mu\nu} + \\ & + f_{13}(\tilde{p}_2N)_{\mu\nu} + if_{14}[\tilde{p}_2N]_{\mu\nu}\big] + \\ & + (Sq)(Wp_1)\big[f_{15}\tilde{g}_{\mu\nu} + f_{16}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + f_{17}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + \\ & + f_{18}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + if_{19}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}\big] + \\ & + (Sq)(Wq)\big[f_{20}\tilde{g}_{\mu\nu} + f_{21}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + f_{22}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + \\ & + f_{23}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + if_{24}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}\big] + \\ & + (Sq)(WN)\big[f_{25}(\tilde{p}_1N)_{\mu\nu} + if_{26}[\tilde{p}_1N]_{\mu\nu} + \\ & + f_{27}(\tilde{p}_2N)_{\mu\nu} + if_{28}[\tilde{p}_2N]_{\mu\nu}\big] + \\ & + (SN)(Wp_1)\big[f_{29}(\tilde{p}_1N)_{\mu\nu} + if_{30}[\tilde{p}_1N]_{\mu\nu} + \\ & + f_{31}(\tilde{p}_2N)_{\mu\nu} + if_{32}[\tilde{p}_2N]_{\mu\nu}\big] + \\ & + (SN)(Wq)\big[f_{33}(\tilde{p}_1N)_{\mu\nu} + if_{34}[\tilde{p}_1N]_{\mu\nu} + \\ & + f_{35}(\tilde{p}_2N)_{\mu\nu} + if_{36}[\tilde{p}_2N]_{\mu\nu}\big] + \\ & + (SN)(WN)\big[f_{37}\tilde{g}_{\mu\nu} + f_{38}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + f_{39}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + \\ & + f_{40}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + if_{41}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}\big]. \quad (48) \end{aligned}$$

Thus, the coefficients of the polarization transfer from the target to the detected proton are described, in general, by 41 structure functions. If the electron beam is unpolarized, only the symmetric part of the hadronic tensor contributes, which corresponds to double-spin (hadron–hadron) correlations in the cross-section of process (46). The antisymmetric part of the hadron tensor contributes in the case of the longitudinally polarized electron beam due to triple-spin (electron–hadron–hadron) correlations.

The corresponding radiatively corrected parts of the cross-section for the unstable orientations of the target nucleus polarization S^J (given by Eq. (35)) and the detected proton spin W^I (given by Eq. (17)) can be written as

$$\varepsilon_2 E_2 \frac{d\sigma_{JI}}{d^3k_2 d^3p_2} = \sum_{j,i} A_{Jj} B_{Ii} \times \iint \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \frac{d\hat{\sigma}_{ji}^B}{d^3\hat{k}_2 d^3p_2}, \quad (49)$$

where the Born cross-section in the integrand is defined for the stable orientations of S^j (given by Eqs. (33) and (34)) and W^i (given by Eqs. (14) and (15)) and depends on the shifted variables

$$\hat{\varepsilon}_2 E_2 \frac{d\hat{\sigma}^B_{ji}}{d^3\hat{k}_2 d^3 p_2} = \hat{\varepsilon}_2 E_2 \frac{d\sigma^B(\lambda, S^j, W^i, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3 p_2}.$$

In accordance with the calculations in Secs. 3 and 4, the matrices A_{Jj} and B_{Ii} are given by

$$A_{Jj} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \quad B_{Ii} = \begin{pmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{50}$$

$$I, J = L, T, N; \quad i, j = l, t, n.$$

If we write the hadron–hadron spin correlations entering the Born cross-section as

$$\varepsilon_2 E_2 \frac{d\sigma^B_{ji}}{d^2 k_2 d^3 p_2} = \frac{\alpha^2 V^4 X_{ji}}{16(2\pi)^3 2(2S_A+1)q^4}, \tag{51}$$

the quantities X_{ji} can be written as

$$X_{ll} = 2\sqrt{\frac{f\tau_1}{\tau_2}}\left\{\eta^2(R_{29}+\xi R_{33}) + \frac{2}{V^2\tau_1}\left[b(F_1+\xi F_6) - d(F_{15}+\xi F_{20})\right]\right\}, \tag{52}$$

$$X_{lt} = \eta^2\sqrt{\frac{f}{\tau_1[1]}}\left[bR_{11} - dR_{25} + 2\tau_1 F_{37} - \frac{2\psi}{\eta^2 V^2 f}\sqrt{[1]}(2bF_6 - 2dF_{20} + \eta^2 V^2\tau_1 R_{33})\right], \tag{53}$$

$$X_{ln} = \frac{\eta}{\sqrt{\tau_1}}\left[\psi(bR_{11} - dR_{25} + 2\tau_1 F_{37}) + \frac{2}{V^2\sqrt{[1]}}(2bF_6 - 2dF_{20} + \eta^2 V^2 \tau_1 R_{33})\right], \tag{54}$$

$$X_{tl} = \sqrt{\frac{f}{r\tau_2}}\left\{\eta^2 d(R_{29}+\xi R_{33}) + \frac{4}{V^2}\left[2r(F_{15}+\xi F_{20}) + \zeta(F_1+\xi F_6)\right]\right\}, \tag{55}$$

$$X_{tt} = \eta^2\sqrt{\frac{f}{r[1]}}\left[\zeta R_{11} + 2rR_{25} + dF_{37} - \frac{\psi\sqrt{[1]}}{\eta^2 V^2 f}(\eta^2 V^2 dR_{33} + 4\zeta F_6 + 8rF_{20})\right], \tag{56}$$

$$X_{tn} = \frac{\eta}{\sqrt{r}}\left[\psi(\zeta R_{11} + 2rR_{25} + dF_{37}) + \frac{1}{V^2\sqrt{[1]}}(\eta^2 V^2 dR_{33} + 4\zeta F_6 + 8rF_{20})\right], \tag{57}$$

$$X_{nl} = \eta\sqrt{\frac{f}{r\tau_2}}\left[\eta_1(R_{29}+\xi R_{33}) - \frac{4}{V^2}(F_1 + \xi F_6)\right], \tag{58}$$

$$X_{nt} = \frac{\eta}{\sqrt{fr}}\left[\psi\left(\frac{4}{V^2}F_6 - \eta_1 R_{33}\right) + \frac{f}{\sqrt{[1]}}(\eta_1 F_{37} - \eta^2 R_{11})\right], \tag{59}$$

$$X_{nn} = -\frac{\eta^2}{\sqrt{r}}\left[\frac{1}{\sqrt{[1]}}\left(\frac{4}{V^2}F_6 - \eta_1 R_{33}\right) - \psi\left(\frac{\eta_1}{\eta^2}F_{37} - R_{11}\right)\right], \tag{60}$$

where we used the notation

$$b = 2z_1\tau_1 - z, \quad d = y(1+2x\tau_1), \quad f = z^2 - 4\tau_1\tau_2, \quad r = xy(1 - y - xy\tau_1),$$

$$\zeta = z_2 - z_1(1 - y - 2xy\tau_1) - xyz, \quad \xi = \frac{z(z_1 - z_2) - 2y\tau_2}{z^2 - 4\tau_1\tau_2}.$$

The functions R_l and F_l entering the expressions for X_{ji} are defined by the hadron structure functions f_n in Eq. (48) as

$$R_l = (2 - y) f_l + (z_1 + z_2) f_{l+2} + \frac{\lambda}{\eta}(\eta_1 f_{l+1} + \eta_2 f_{l+3}), \tag{61}$$

$$F_l = -\frac{2xy}{V} f_l + (1 - y - xy\tau_1) f_{l+1} + (z_1 z_2 - xy\tau_2) f_{l+2} + (z_2 + z_1(1 - y) - xyz) f_{l+3} - \lambda\eta f_{l+4}. \tag{62}$$

6. HADRONIC VARIABLES

There exists the exprerimental possibility of measuring the total 4-momentum of the hadron system X instead of recording the scattered electron in semi-inclusive reactions. In such experiments, the momentum q_h of the heavy intermediate photon that probes the hadron structure can be determined explicitly. The corresponding set of dynamical variables is usually referred to as the hadronic one.

For the hadronic variables, we must eliminate the phase space of the scattered electron and introduce the heavy photon phase space using the identities

$$\begin{gathered} \frac{d^3k_2}{\varepsilon_2} = 2x_2^2 x_h \frac{d^4 q_h}{Q_h^2}\delta(x_1 - x_h), \quad \frac{d^4 q_h}{Q_h^2} = \frac{dQ_h^2 dx_h dy_h dz_h}{4x_h^2|\eta_h|}, \\ x_h = -\frac{Q_h^2}{2k_1 q_h}, \quad y_h = \frac{2p_1 q_h}{V}, \quad z_h = \frac{2p_2 q_h}{V}, \quad Q_h^2 = -q_h^2, \end{gathered} \tag{63}$$

$$\eta_h^2 = \frac{Q_h^2}{V}\left[(4\tau_1\tau_2 - z^2)\frac{Q_h^2}{x_h^2 V} + 2\left(1 - \frac{y_h}{x_h}\right)(zz_1 - 2\tau_2) + 2\left(z_1 - \frac{z_h}{x_h}\right)(z - 2z_1\tau_1)\right] - (z_h - z_1 y_h)^2.$$

Therefore, combining this with representation (3) for the leptonic tensor and also bearing in mind that the hadronic tensor is independent of x_2, we can express the quantity $L_{\mu\nu} d^3k_2/\varepsilon_2$ through the hadronic variables as

$$\frac{d^3k_2}{\varepsilon_2} L_{\mu\nu} = \frac{D(x_h, Q_h^2)}{x_h^2} L^B_{\mu\nu}(\hat{k}_1, \hat{k}_1 - q_h, \lambda)\frac{dx_h dy_h dz_h dQ_h^2}{2|\eta_h|}. \tag{64}$$

We note that for the events with the undetected scattered electron, the lower integration limit with respect to x_2 in Eq. (3) is equal to 0. In accordance with the Kinoshita–Lee–Nauenberg theorem [22], the mass singularities caused by the final-state radiation must disappear in this case. In the language of the electron structure functions, this fact exhibits itself due to the relation

$$\int_0^1 D(x, Q^2)dx = 1,$$

which was used in writing Eq. (64).

In the Born approximation, the lepton tensor can be rewritten as

$$L^B_{\mu\nu}(k_1, k_1 - q_h) = 2(k_1 q_h)\tilde{g}_{\mu\nu} + 4\tilde{k}_{1\mu}\tilde{k}_{1\nu} - 2i\lambda(\mu\nu k_1 q_h), \tag{65}$$

and the physically founded parameterizations for S^j in process (1) and for P^j in process (32) remain stable with respect to the scaling transformation $k_1 \to x_h k_1$. For example, one set can be chosen as in Eqs. (14) and (15) and the other as

$$\begin{gathered} S^L_{h\mu} = S^l_\mu, \quad S^T_{h\mu} = \frac{(z^2 - 4\tau_1\tau_2)q_{h\mu} + (2z_h\tau_1 - zy_h)p_{2\mu} + (2y_h\tau_2 - zz_h)p_{1\mu}}{\sqrt{V(z^2 - 4\tau_1\tau_2)[q_h]}}, \quad S^N_{h\mu} = \frac{2(\mu q_h p_1 p_2)}{\sqrt{V^3[q_h]}}, \\ [q_h] = zz_h y_h + \frac{Q_h^2}{V}(z^2 - 4\tau_1\tau_2) - z_h^2\tau_1 - y_h^2\tau_2, \end{gathered} \tag{66}$$

with the transverse component belonging to the plane $(\mathbf{q}_h, \mathbf{p}_2)$ in the laboratory system.

Two physical sets of the target polarizations, each with the normal component perpendicular to the plane $(\mathbf{k}_1, \mathbf{q}_h)$, can be chosen as

$$P^l_{h\mu} = \frac{2\tau_1 k_{1\mu} - p_{1\mu}}{M}, \quad P^t_{h\mu} = \left[-q_{h\mu} + \left(y_h + \frac{2Q_h^2\tau_1}{x_h V}\right)k_{1\mu} - \frac{Q_h^2}{x_h V}p_{1\mu}\right]K^{-1}, \quad P^n_{h\mu} = \frac{-2(\mu k_1 q_h p_1)}{VK}, \tag{67}$$

with the longitudinal component along $\mathbf{k}_1$ in the laboratory system and

$$\begin{gathered} P^L_{h\mu} = \frac{2\tau_1 q_{h\mu} - y_h p_{1\mu}}{MG}, \\ P^T_{h\mu} = \left[\left(y_h^2 + 4\tau_1\frac{Q_h^2}{V}\right)k_{1\mu} - \left(y_h + \frac{2Q_h^2\tau_1}{x_h V}\right)q_{h\mu} - \frac{Q_h^2}{V}\left(2 - \frac{y_h}{x_h}\right)p_{1\mu}\right](KG)^{-1}, \\ P^N_{h\mu} = P^n_{h\mu}, \; K = \sqrt{Q_h^2\left(1 - \frac{y_h}{x_h} - \frac{Q_h^2\tau_1}{x_h^2 V}\right)}, \quad G = \sqrt{y_h^2 + 4\frac{Q_h^2\tau_1}{V}}, \end{gathered} \tag{68}$$

with the longitudinal component along $\mathbf{q}_h$. The different components of P_h^J in the laboratory system are

$$P_h^L = (0, \; \mathbf{n}_q), \quad P_h^T = \left(0, \frac{\mathbf{n}_1 - (\mathbf{n}_1\cdot\mathbf{n}_q)\mathbf{n}_q}{\sqrt{1 - (\mathbf{n}_1\cdot\mathbf{n}_q)^2}}\right),$$

$$P_h^N = \left(0, \frac{\mathbf{n}_q \times \mathbf{n}_1}{\sqrt{1 - (\mathbf{n}_1\cdot\mathbf{n}_q)^2}}\right),$$

$$\mathbf{n}_q = \frac{\mathbf{q}_h}{|\mathbf{q}_h|}, \quad \mathbf{n}_1 = \frac{\mathbf{k}_1}{|\mathbf{k}_1|}.$$

All these sets of the proton spin and target polarizations given by Eqs. (66), (67), and (68) are stable with respect to the initial-state collinear radiation. This can be verified by replacing k_1 with $x_h k_1$, which implies

$$\begin{gathered} k_1 \to x_h k_1, \quad x_h \to 1, \quad y_h \to \frac{y_h}{x_h}, \\ z_h \to \frac{z_h}{x_h}, \quad z \to \frac{z}{x_h}, \quad V \to x_h V, \quad \tau_{1,2} \to \frac{\tau_{1,2}}{x_h}. \end{gathered} \tag{69}$$

To make the invariance of P^j $(j = l, t, n)$ and $P^J (J = L, T, N)$ under replacement (69) more transparent, we express x_h in terms of Q_h^2 and $(k_1 q_h)$. Then, e.g.,

$$K = \sqrt{Q_h^2 + y_h 2(k_1 q_h) - \frac{4(k_1 q_h)^2\tau_1}{V}},$$

and it is easy to see that this quantity is not changed under substitution (69). We also note that the quantity η_h can be derived using the rule

$$\eta_h = x_h \eta^*,$$

where η^* is determined from η with xy replaced by Q_h^2/V, z_2 replaced with $z_1 - z_h$, and with the subsequent replacement (69).

For hadronic variables, the cross-section for both the spin-independent and spin-dependent parts can therefore be written as

$$E_2\frac{d\sigma^j}{d^3p_2 dQ_h^2 dx_h dy_h dz_h} = \\ = \frac{D(x_h, Q_h^2)}{x_h^2}E_2\frac{d\hat{\sigma}_j^B}{d^3p_2 dQ_h^2 d\hat{y}_h d\hat{z}_h}, \tag{70}$$

where

$$E_2\frac{d\hat{\sigma}_j^B}{d^3p_2 dQ_h^2 d\hat{y}_h d\hat{z}_h} = \frac{\alpha^2 C}{(2\pi)^3(2S_A+1)\hat{V}Q_h^4 2|\eta^*|} \times \\ \times L_{\mu\nu}(\hat{k}_1, \hat{k}_1 - q_h, \lambda)H_{\mu\nu}(q_h, p_1, p_2; S^j(P^j))$$

and C is equal to $1/2$ or 1 for the respective process (1) or (32).

Representation (70) shows that using the hadron variables allows us to tag the initial-state radiated photon. Indeed, for a fixed 4-momentum P_x, we can reconstruct the 4-momentum q_h, and consequently, the variable x_h that is the energy fraction of the photon radiated by the initial electron (see Eq. (63)).

The Born cross-section in the right-hand side of Eq. (70) has the form that is very similar to the corresponding cross-section for the leptonic variables. We can formulate the following rules to write it:

i) change the phase space differentials in the left-hand sides of the expressions valid for the leptonic variables as

$$\frac{\varepsilon_2}{d^3k_2} \to \frac{2|\eta_{1h}|}{dQ_h^2 dy_h dz_h}, \quad \eta_{1h} = \eta_h(x_h = 1),$$

ii) apply the substitution

$$xy \to \frac{Q_h^2}{V}, \quad y \to y_h, \quad z_2 \to z_1 - z_h$$

to the right-hand sides.

These rules lead, e.g., to the formula for the spin-dependent part of the cross-section of process (1) in the case of the longitudinal polarization (which follows from Eq. (25))

$$E_2 \frac{d\sigma_L^B}{d^3p_2 dQ_h^2 dy_h dz_h} = = -\frac{\alpha^2 V^3 \eta_{1h}\sqrt{z^2-4\tau_1\tau_2}}{8m(2S_A+1)(2\pi)^3 Q_h^4 2|\eta_{1h}|} \times \times \left[H_2^{(h)} + \frac{zz_h - 2y_h\tau_2}{z^2-4\tau_1\tau_2} H_3^{(h)}\right], \qquad (71)$$

$$H_2^{(h)} = (2-y_h)h_6 + (2z_1-z_h)h_8 + \frac{\lambda}{\eta_{1h}}(\eta_1^{(h)}h_7 + \eta_2^{(h)}h_9),$$

$$\eta_1^{(h)} = \frac{Q_h^2}{V}[2(2z_1-z_h)\tau_1 - z(2-y_h)] + z_1 y_h^2 - z_h y_h,$$

$$\eta_2^{(h)} = \frac{Q_h^2}{V}[z(2z_1-z_h) - 2(2-y_h)\tau_2] - z_h^2 + z_1 z_h y_h,$$

where $H_3^{(h)}$ is derived from $H_2^{(h)}$ by the replacement $h_i \to h_{i+4}$.

For the normal target polarization that follows from Eq. (40), the spin-dependent part of the cross-section of process (32) is given by

$$E_2 \frac{d\sigma_N^B}{d^3p_2 dQ_h^2 dy_h dz_h} = = -\frac{\alpha^2 V^3}{4(2S_A+1)(2\pi)^3 Q_h^4 K(x_h=1) 2|\eta_{1h}|} \times \times \left\{\eta_{1h}^2 G_2^{(h)} - \left[y_h(z_1 y_h - z_h) + + \frac{Q_h^2}{V}\left(2\tau_1(2z_1-z_h) - z(2-y_h)\right)\right] G_4^{(h)}\right\}. \qquad (72)$$

The remaining spin-dependent and spin-independent parts of the cross-sections for processes (1) and (32) can be obtained totally similarly using the above rules and the results in Secs. 3 and 4.

The variable x_h characterizes the inelasticity of the initial-state electron and is equal to 1 in the absence of radiation. The electron structure function $D(x_h, Q_h^2)$ has a singularity at $x_h = 1$ and representation (70) shows that this singularity is such that

$$\lim_{x_h \to 1} D(x_h, Q_h^2) dx_h = 1, \qquad (73)$$

because in this limiting case, the left-hand side of Eq. (70) multiplied by dx_h must coincide with the Born cross-section.

7. CONCLUSION

In this paper, we consider radiative corrections to the polarization observables in a wide class of semi-inclusive deep-inelastic processes. We restrict ourselves to the leading-log accuracy and neglect the contribution of the pair production in the singlet channel. This allows us to write a compact formulas for the radiatively corrected spin-independent and spin-dependent parts of the corresponding cross-sections in the form of the Drell–Yan representation in electrodynamics by means of the electron structure functions. The parameterization of the hadron spin 4-vectors in terms of the particle 4-momenta is very important in the calculations. If the momentum of the intermediate photon that probes the hadron structure is determined in terms of the hadronic variables, the traces of the final-state radiation disappear in the final result within the adopted approximation.

In practice, the corrections can be computed adopting some specific model for the structure functions. The correction then acquires some model dependence that can contribute to the systematical error in experimental measurements. Another possibility is related to some iteration procedure, where the fit of the processed experimental data is used for the chosen model. We note that the obtained leading-log formulas have a partly factorized form, which is very convenient for this procedure. The examples for the DIS case can be found in [20, 23].

Apart from the classes of experiments discussed above, the results can also be adopted to exclusive electroproduction processes, where the unobservable hadron state is one particle. In this case, the structure functions involve an additional δ-function, and therefore, some analytical manipulations could be necessary.

The accuracy that is higher than the leading one sometimes becomes necessary. To go beyond the leading accuracy, one must modify the master representations. This modification affects both the electron structure function and the cross-section (the hard part) that depends on the shifted variables. To improve the hard part, it sufficies to take the radiation of a single additional noncollinear photon into account and to add the non-leading part of the one-loop correction. The corresponding procedure is described in Ref. [21] for the unpolarized deep-inelastic scattering and in the second of

Refs. [20] for the quasi-elastic polarized electron–proton scattering. To be complete, one must also improve the structure functions by adding the second-order next-to-leading contributions of the double collinear photon emission and the pair production. The non-leading contributions to the D function caused by the one-loop corrected collinear single-photon emission and the two-loop correction must also be added properly. These contributions are different for symmetric and asymmetric parts of the leptonic tensor and can be extracted from the results in Ref. [16] (for the two-loop correction, see [24]). In this case, we must therefore distinguish between D and D_λ at the level of the nonsinglet channel contribution. The specific calculations will be done elsewhere.

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