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(QUASI)ELASTIC LARGE-ANGLE ELECTRON–MUON SCATTERING IN THE TWO-LOOP APPROXIMATION: CONTRIBUTIONS OF THE EIKONAL TYPE

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A part of the eikonal-type contributions to the $e\mu$ large-angle high-energy scattering cross-section is considered in a quasi-elastic experimental set-up. In addition to virtual corrections, we examine inelastic processes with emission of one and two soft real photons and soft lepton and pion pairs. Virtual photon contributions are given within a logarithmic accuracy. Box-type Feynman amplitudes with leptonic and a hadronic vacuum polarization insertion and double-box ones are considered explicitly. Wherever appropriate, the analytic expressions obtained are compared with those predicted by the structure function approach.

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1. INTRODUCTION

The need for evalution of the radiative corrections at the two-loop order is dictated by the experimental data on observables for a collider calibration process of electron–positron scattering that has reached an impressive level of accuracy. Inspired by this, we consider the determination of the second-order radiative corrections to the cross-section of Bhabha scattering to be our ultimate goal. At the same time, because the task of two-loop calculus is rather involved, it appears to be easier to consider the electron–muon scattering first, despite different masses of interacting particles. The latter process is also important in itself because it forms a background to the rare processes, in particular those violating lepton number (for more details, see [1] and references therein). Improving theoretical predictions on its observables could therefore impose more stringent bounds on the physics beyond the Standard Model.

The aim of this investigation is to calculate the next-to-leading order contributions to the large-angle electron–muon high-energy cross-section

$$e^-(p_1) + \mu^-(p_2) \to e^-(p_1') + \mu^-(p_2'), \qquad (1)$$

in a quasielastic experimental set-up,

$$\frac{2\varepsilon - \varepsilon_1' - \varepsilon_2'}{2\varepsilon} = \frac{\Delta\varepsilon}{\varepsilon} \equiv \Delta \ll 1, \quad \Delta\varepsilon \gg m_\mu(m_\pi)\,, \qquad (2)$$

where ε, ε_1', and ε_2' are the energies of the initial and scattered leptons in the center-of-mass reference frame and the Mandelstam variables are much larger than the mass squared of any particle involved in the process. The quantity $\Delta\varepsilon$ indicates the energy resolution of detectors that are supposed to track final particles. In the leading logarithmic approximation, the cross-section is that of the Drell–Yan process [2],

$$d\sigma(s,t) = \int \prod_{i=1}^{4} dx_i \mathcal{D}(x_i,\rho_t)\, d\sigma_0(sx_1x_2, tx_1x_3) \times \times \left(1 + \frac{\alpha}{\pi}K\right)\,, \qquad (3)$$

where

$$\rho_t = \ln\frac{-t}{m_e m_\mu}, \quad t = (p_1 - p_1')^2, \quad s = (p_1+p_2)^2, \quad u = (p_1 - p_2')^2\,. \qquad (4)$$

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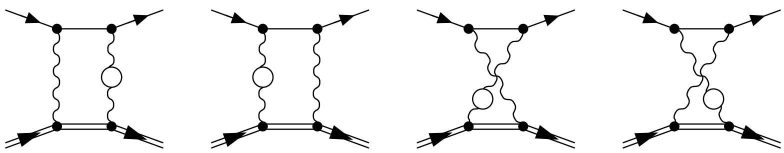

Fig. 1. Box-type graphs with a vacuum polarization insertion

In the above expression, the quantities $\mathcal{D}(x_i,\rho_t)$ are the nonsinglet structure functions that satisfy the renormalization group (RG) evolution equations. Their expansion in the leading logarithmic approximation

$$(\alpha/\pi) \ll 1, \quad (\alpha/\pi)\rho_t \sim 1$$

can be written as

$$\mathcal{D}(x,\rho_t) = \delta(1-x) + \sum_{n=1}^{\infty} \frac{1}{n!}\left(\frac{\alpha\rho_t}{2\pi}\right)^n \mathcal{P}^{(n)}(x). \quad (5)$$

In a quasielastic set-up, it is appropriate to use only the δ-part of the splitting function $\mathcal{P}^{(n)}(x)$ denoted by $\mathcal{P}_\Delta^{(n)}(x)$,

$$\begin{aligned} &\mathcal{P}^{(n)}(x) = \int_x^1 \frac{dy}{y}\mathcal{P}^{(1)}(y)\mathcal{P}^{(n-1)}\left(\frac{x}{y}\right), \quad n \geq 2, \\ &\mathcal{P}^{(1)}(x) = \left(\frac{1+x^2}{1-x}\right)_+ = \lim_{\Delta\to 0}\left[\mathcal{P}_\Delta^{(1)}(x)+\mathcal{P}_\theta^{(1)}(x)\right], \\ &\mathcal{P}_\Delta^{(1)}(x) = \mathcal{P}_\Delta^{(1)}\delta(1-x), \quad \mathcal{P}_\Delta^{(1)} = 2\ln\Delta + \frac{3}{2}, \\ &P_\theta^{(1)}(x) = \frac{1+x^2}{1-x}\Theta(1-x-\Delta). \end{aligned} \quad (6)$$

The structure function then becomes

$$\mathcal{D}(x,\rho_t) = \delta(1-x)\left[1 + \sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{\alpha\rho_t}{2\pi}\right)^n \mathcal{P}_\Delta^{(n)}\right]. \quad (7)$$

Because the structure function approach outlined is capable of providing only the leading logarithmic corrections, we need to explicitly calculate the so-called K-factor entering Eq. (3) in the one- and two-loop approximations.

Broadly speaking, the radiative corrections to the differential cross-section in the adopted mass regularization scheme are of two types. The first ones are those arising from the virtual photon emission up to the second order of perturbation theory, which requires calculating, among others, the real two-loop Feynman amplitudes. They suffer from infrared divergences, which are regularized by assigning the photon a negligibly small mass λ that is set to zero at the end of the calculations. Contributions of the second type come from the emission of soft real photons and charged particle pairs.

The general structure of the correction to the cross-section can be represented as a sum of three types: vertex, eikonal, and decorated box type. Each of them contains virtual and real soft photon contributions, is free of infrared divergences, and preserves the structure of the leading log correction predicted on the basis of RG ideas through the contributions of individual diagrams containing up to the fourth power of the large logarithm ρ_t at the two-loop order. In this regard, we recall that in our previous paper [1], it was shown that the vertex contributions already provide a result consistent with the RC approach. Because the first-order radiative corrections coming from box-type diagrams are given in our previous work devoted to the evaluation of vertex-type contributions [1], we here concentrate on the investigation of some eikonal box-type diagrams at the second order of perturbation theory. In the case of elastic processes, they correspond to graphs with one, two (box diagram), and three (double box diagram) virtual photons mediated between interacting leptons. Box-type graphs with a vacuum polarization insertion of either of the virtual exchange photons into the Green's function must also be taken into account (see Fig. 1). A single soft photon approximation must be applied to the one-loop corrected Feynman amplitudes in order to obtain another set of contributions. Finally, the emission of two soft photons (pairs of charged particles) must also be taken into account at this order.

We briefly describe the contents of the paper. In Sec. 2, we consider the vacuum polarization effects in box-type Feynman amplitudes with lepton ($\mu\bar{\mu}, e\bar{e}$) and pion ($\pi^-\pi^+$) pairs running a loop. Also in this section we consider the corresponding contribution coming from a soft lepton pair and a soft charged pion pair production with one soft photon emission (see Fig. 2) associated with the one-loop self-energy amplitudes of the virtual exchange photon. In Sec. 3, the results of evaluation of the corrections corresponding to a single and double soft photon emission (see Fig. 3) and to a

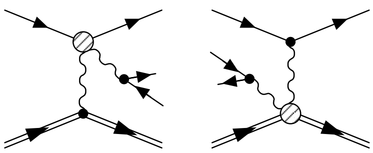

Fig. 2. Soft lepton and pion pair production

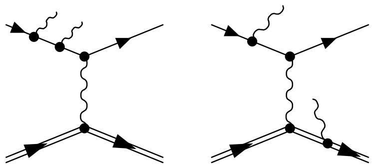

Fig. 3. Sample diagrams pertaining to the double soft photon emission

square of box-type diagrams are presented; they are followed by brief concluding remarks. In Appendix A, we present a set of scalar integrals for box-type diagrams with a vacuum polarization insertion. In Appendix B, we give some details of the derivation of radiative corrections coming from the squared box-type diagrams and all the integrals encountered during the calculation.

2. BOX-TYPE DIAGRAMS WITH A VACUUM POLARIZATION INSERTION

Vacuum polarization effects in the box-type Feynman amplitudes can be taken into account by replacing one of the photon propagators by the vacuum polarization insertion (see [3]). In the case where leptons with the mass μ run a loop, it is given by

$$\frac{1}{k^2} \to \frac{\alpha}{3\pi} \int\limits_0^1 \frac{\phi(v)dv}{(1-v^2)(k^2 - M^2(v))}, \quad M^2 = \frac{4\mu^2}{1-v^2}, \quad \phi(v) = 2 - (1-v^2)(2-v^2), \tag{8}$$

and for a pion–antipion pair in the loop, it is

$$\frac{1}{k^2} \to \frac{\alpha}{3\pi} \int\limits_{4m_\pi^2}^{\infty} \frac{dM^2}{M^2} \frac{\mathcal{R}(M^2)}{k^2 - M^2}, \quad \mathcal{R}(M^2) = \frac{\sigma^{e\bar{e}\to hadr}(M^2)}{\sigma^{e\bar{e}\to\mu\bar{\mu}}}. \tag{9}$$

Here, the quantity M is the invariant mass of the hadronic jet produced in a single-photon annihilation of a lepton pair and $\mathcal{R}(M^2)$ is the known experimental input ratio [4]. For the matrix element squared, we then obtain

$$\delta|\mathcal{M}|^2_{vp(lept)} = \frac{2^8\alpha^4}{3t} \times \int\limits_0^1 dv \frac{\phi(v)}{1-v^2}\left[S(s,t,M^2) - S(u,t,M^2)\right] \tag{10}$$

for the vacuum polarization induced by leptons, and

$$\delta|\mathcal{M}|^2_{vp(hadr)} = \frac{2^8\alpha^4}{3t} \times \int\limits_{4m_\pi^2}^{\infty} \frac{dM^2}{M^2}\mathcal{R}(M^2)\left[S(s,t,M^2) - S(u,t,M^2)\right] \tag{11}$$

for the hadronic vacuum polarization contribution.

The quantity $S(s,t,M^2)$ is universal irrespective of the virtual pair running a self-energy loop and is given by

$$S(s,t,M^2) = \int \frac{d^4k}{i\pi^2} \frac{\mathrm{Tr}(e)\,\mathrm{Tr}(\mu)}{(1)(2)(3)(4)}, \tag{12}$$

where

$$\begin{aligned} &(1) = k^2 - 2kp_1, \quad (2) = k^2 + 2kp_2, \\ &(3) = k^2 - 2kq + \tilde{t}, \quad (4) = k^2 - \lambda^2, \\ &\mathrm{Tr}(e) = \frac{1}{4}\,\mathrm{Sp}\{p_1\gamma_\mu p_1'\gamma_\nu(p_1 - k)\gamma_\lambda\}, \\ &\mathrm{Tr}(\mu) = \frac{1}{4}\,\mathrm{Sp}\{p_2\gamma_\mu p_2'\gamma_\nu(p_2 + k)\gamma_\lambda\}, \\ &p_1^2 = m_e^2, \quad p_2^2 = m_\mu^2, \\ &\tilde{t} = t - M^2, \quad q = p_1 - p_1'. \end{aligned} \tag{13}$$

Using a set of scalar, vector, and tensor box-type integrals given in Appendix A, we can express the quantity $S(s,t,M^2)$ through several basic integrals,

$$\begin{aligned} S(s,t,M^2) = u\left[\ln\frac{s}{-\tilde{t}} + \frac{M^2}{t}\ln\frac{-\tilde{t}}{M^2}\right] - \\ - \left(s(s-u) + \frac{t\tilde{t}}{2}\right)\left[I_{134} + I_{234}\right] + \\ + s(s^2+u^2)I + s\left(u + \frac{\tilde{t}}{2}\right)\left[-I_{123} - I_{124} + \tilde{t}I\right], \end{aligned} \tag{14}$$

where

$$I_{ijl} = \int \frac{d^4k}{i\pi^2}\frac{1}{(i)(j)(l)}, \quad I = \int \frac{d^4k}{i\pi^2}\frac{1}{(1)(2)(3)(4)}. \tag{15}$$

Performing loop-momentum integration and neglecting terms of the order of $m_\mu^2/(-t) \ll 1$, we find, in the limit of large invariant variables,

$$\left[S(s,t,M^2) - S(u,t,M^2)\right]\Big|_{|t|\gg M^2} = \\ = \frac{s^2+u^2}{t} L_{us}(\rho_m - 2\rho_t - \rho_\lambda) + \\ + (u-s)\left(\frac{1}{2}\pi^2 + \rho_m^2 - \frac{1}{2}L_{st}^2 - \frac{1}{2}L_{ut}^2 + \ln^2\frac{m_\mu}{m_e}\right) + \\ + uL_{st} - sL_{ut}, \quad (16)$$

$$\rho_m = \ln\frac{M^2}{m_e m_\mu}, \quad \rho_\lambda = \ln\frac{m_e m_\mu}{\lambda^2},$$

$$L_{st} = \ln\frac{s}{-t}, \quad L_{ut} = \ln\frac{u}{t}, \quad L_{us} = \ln\frac{-u}{s}.$$

In the opposite limit, the result is found to be

$$\left[S(s,t,M^2) - S(u,t,M^2)\right]\Big|_{M^2\gg|t|} = \\ = \frac{1}{M^2}\left[\frac{s^2+u^2}{2} L_{us}(\rho_s + \rho_u + 2\rho_\lambda) + \right. \\ + \frac{3}{2}(u^2\rho_s - s^2\rho_u) + t^2 L_{us} + t(u-s) \times \\ \left. \times \left(\frac{3}{2}\rho_m + \frac{7}{4}\right) + \frac{s^2+u^2}{2}\pi^2\right]. \quad (17)$$

For the leptonic vacuum polarization with the mass

$$M^2 = \frac{4\mu^2}{1-v^2}$$

(where both cases $\mu = m_e, m_\mu$ are taken into account), further integration leads to the following expression within the logarithmic accuracy:

$$\frac{d\sigma_{vp}^{box}}{d\sigma_0} = \frac{2\alpha^2}{3\pi^2}\rho_t\left\{2L_{su}\left(\frac{3}{2}\rho_t + \rho_\lambda - \frac{10}{3}\right) - \right. \\ - \frac{s^2-u^2}{s^2+u^2}(L_{st}^2 + L_{ut}^2 - 2\pi^2) + \\ \left. + \frac{2t}{s^2+u^2}[(t-s)L_{st} - (t-u)L_{ut}]\right\}. \quad (18)$$

To finalize this result, we must remove infrared divergences. For this, the interference between the soft photon emission tree-level amplitudes and those bearing a leptonic vacuum polarization insertion must be taken into account, with the result

$$\frac{d\sigma_{vp}^{\gamma}}{d\sigma_0} = -\frac{4\alpha^2}{3\pi^2}\left(\rho_t - \frac{5}{3}\right)\left[(2\ln\Delta + \rho_\lambda)L_{su} + \rho_t L_{su} - \right. \\ \left. - \frac{1}{2}(L_{ut}^2 - L_{st}^2) - \mathrm{Li}_2\left(\frac{1-c}{2}\right)\right], \quad (19)$$

where Δ is given in Eq. (2), $c = \cos\widehat{p_1, p_1'}$ is the cosine of the scattering angle in center-of-mass reference frame, and the dilogarithm function is defined by the standard formula

$$\mathrm{Li}_2(x) = -\int_0^x \frac{\ln(1-t)}{t}dt. \quad (20)$$

Next, we must consider the contribution coming from the soft lepton pair production with the total pair energy not exceeding $\Delta\varepsilon$ ($2\mu \ll \Delta\varepsilon \ll \varepsilon$). This can be read off, e.g., from Ref. [5],

$$\frac{d\sigma_{sp}}{d\sigma_0} = -\frac{2\alpha^2}{3\pi^2}\rho_t\left\{L_{su}\left[\rho_t + L_{st} + L_{ut} + \right.\right. \\ \left.\left. + 2\left(2\ln\Delta - \frac{5}{3}\right)\right] - 2\mathrm{Li}_2\left(\frac{1-c}{2}\right)\right\}. \quad (21)$$

The final logarithmically accurate result for the total correction given by the leptonic vacuum polarization and the soft $e\bar{e}, \mu\bar{\mu}$ pair production is then brought to the form (see Eqs. (18), (19), (21))

$$\frac{d\sigma_{vp+sp}}{d\sigma_0} = \frac{2\alpha^2}{3\pi^2}\rho_t\left[2(L_{ut}^2 - L_{st}^2) + 8L_{us}\ln\Delta - \right. \\ - \frac{s^2-u^2}{s^2+u^2}(L_{ut}^2 + L_{st}^2 - 2\pi^2) + \\ \left. + \frac{2t}{s^2+u^2}(tL_{su} - sL_{st} + uL_{ut}) + 4\mathrm{Li}_2\left(\frac{1-c}{2}\right)\right]. \quad (22)$$

This expression is seen to contain only a next-to-leading term (of the order of $\alpha^2\rho_t$) and to be free of infrared divergences.

We now consider the soft pion pair production with the total pair energy below $\Delta\varepsilon$ and the invariant mass squared M^2 bounded as

$$4m_\pi^2 \ll M^2 < (\Delta\varepsilon)^2 \ll \varepsilon^2 = s/4. \quad (23)$$

The corresponding contribution to the differential cross-section arises from the interference of the «up–down» pair production, which refers to pairs created by virtual photons emitted from the electron line and the muon line,

$$\frac{d\sigma}{dM^2 d\sigma_0}\Big|_{\pi^\pm} = 2\left(\frac{4\pi\alpha}{M^2}\right)^2 \frac{d^4 q}{M^2} \times \\ \times \int \frac{d^3\mathbf{q}_+ d^3\mathbf{q}_-}{2\varepsilon_+ 2\varepsilon_-}\delta^4(q_+ + q_- - q) \times \\ \times \left(\frac{Qp_1'}{qp_1'} - \frac{Qp_1}{qp_1}\right)\left(\frac{Qp_2'}{qp_2'} - \frac{Qp_2}{qp_2}\right), \\ q^2 = M^2, \quad Q = q_+ - q_-. \quad (24)$$

We first perform the invariant pion pair phase space integration,

$$\int \frac{d^3\mathbf{q}_+ d^3\mathbf{q}_-}{2\varepsilon_+ 2\varepsilon_-}\delta^4(q_+ + q_- - q)Q_\mu Q_\nu = \\ = \frac{1}{3}\frac{\pi\beta}{2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)Q^2, \quad \beta = \sqrt{1 - \frac{4m_\pi^2}{q^2}}. \qquad (25)$$

Upon rearranging the phase volume,

$$\int \frac{d^4 q}{dq^2} = \frac{1}{2}\int\limits_{\sqrt{q^2}}^{\Delta\varepsilon} dq_0 \sqrt{q_0^2 - q^2}\int d\Omega_q\,, \qquad (26)$$

the right-hand side of Eq. (24) can be recast to the form

$$\frac{\alpha^2}{3\pi^2}\int\limits_M^{\Delta\varepsilon} dq_0\sqrt{q_0^2 - M^2}\,\times \\ \times \int \frac{d\Omega_q}{4\pi}\left(\frac{p_1 p_2}{p_1 q\cdot p_2 q} - \frac{p_1 p_2'}{p_1 q\cdot p_2' q}\right) = \\ = \frac{\alpha^2}{3\pi^2}\left(L_{su}\ln\frac{2\Delta\varepsilon}{M} + \mathcal{O}(1)\right). \qquad (27)$$

The final result is then given by

$$\left.\frac{d\sigma}{dM^2 d\sigma_0}\right|_{\pi^\pm} = \frac{\alpha^2}{6\pi^2 M^2}\left[L_{su}(\rho_t - \rho_m) + \mathcal{O}(1)\right]. \qquad (28)$$

Obviously, the contribution coming from the box-type diagrams with the hadronic vacuum polarization cannot be obtained in analytic form because of the presence of the quantity $\mathcal{R}(M^2)$.

3. SQUARED BOX AND THE CORRESPONDING SOFT PHOTON CORRECTIONS

The «up–down» interference of the soft photon emission from the electron line and the muon line can be evaluated using the expression

$$I_{p_A p_B} = \frac{1}{4\pi}\int \frac{d^3\mathbf{k}}{\omega}\left.\frac{p_A p_B}{p_A k\cdot p_B k}\right|_{\omega<\Delta\varepsilon} = \\ = \left(\ln\Delta + \frac{1}{2}\rho_\lambda\right)L_{AB} + \frac{1}{4}\left(L_{AB}^2 - \ln^2\frac{m_\mu}{m_e}\right) - \\ - \frac{\pi^2}{6} + \frac{1}{2}\mathrm{Li}_2\left(\frac{1+c}{2}\right), \qquad (29)$$

where

$$L_{AB} = \ln\left(\frac{2p_A p_B}{m_e m_\mu}\right), \quad p_A p_B = \varepsilon^2(1-c),$$

$$p_A^2 = m_e^2, \quad p_B^2 = m_\mu^2, \quad \varepsilon_A = \varepsilon_B \equiv \varepsilon\,,$$

and the quantity ω is the soft photon energy. Using the known results for the interference of the Born and box-type elastic amplitudes (see Appendix B), we obtain that in the soft photon approximation, the single soft photon emission contribution is given by

$$\frac{d\sigma_{box}^\gamma}{d\sigma_0} = \left(\frac{\alpha}{\pi}\right)^2\left[2L_{su}(\rho_t + \rho_\lambda) + \frac{t^2}{s^2+u^2}\,\times\right. \\ \times\left(\frac{u}{t}L_{st} - \frac{s}{t}L_{ut} + \frac{s-u}{2t}(\pi^2 + L_{ut}^2 + L_{st}^2)\right)\Big]\,\times \\ \times\left[-L_{su}\rho_t + \frac{1}{2}(L_{ut}^2 - L_{st}^2) - 2L_{su}\left(\ln\Delta + \frac{1}{2}\rho_\lambda\right) + \right. \\ \left. + \mathrm{Li}_2\left(\frac{1-c}{2}\right)\right]. \qquad (30)$$

In the case of the emission of two soft photons with the total energy not exceeding $\Delta\varepsilon$, we have

$$\frac{d\sigma^{\gamma\gamma}}{d\sigma_0} = \left(\frac{2\alpha}{\pi}\right)^2\left\{\left[\frac{1}{2}\rho_t L_{su} + \frac{1}{4}(L_{st}^2 - L_{ut}^2) + \right.\right. \\ \left.\left. + L_{su}\left(\ln\Delta + \frac{1}{2}\rho_\lambda\right) - \frac{1}{2}\mathrm{Li}_2\left(\frac{1-c}{2}\right)\right]^2 - \frac{\pi^2}{6}L_{su}^2\right\}. \qquad (31)$$

Finally, from the evaluation of the squared box-type graphs in Appendix B, we infer the logarithmic contribution

$$\frac{d\sigma_{BB}}{d\sigma_0} = \frac{\alpha^2}{\pi^2}\frac{t^2}{s^2+u^2}\rho_t\left[A\rho_t + B\right], \qquad (32)$$

where the coefficients are given by

$$A = 2\frac{s^2+u^2}{t^2}(L_{us}^2 + \pi^2),$$

$$B = 4\frac{s^2+u^2}{t^2}(L_{us}^2+\pi^2)\rho_\lambda + 2L_{us}\left(\frac{s}{t}L_{ut} - \frac{u}{t}L_{st}\right) + \\ + \frac{s-u}{t}\left[\pi^2(2L_{st} - L_{us}) - L_{us}(L_{ut}^2 + L_{st}^2)\right] + \frac{8u}{t}\pi^2.$$

4. SUMMARY

This paper is devoted to the determination of a part of the second-order radiative corrections to the cross-section of the process of large-angle quasi-elastic $e\mu$ scattering, namely those corresponding to eikonal box-type diagrams. For box-type diagrams with a vacuum polarization insertion, we obtain the formulas in Eqs. (16), (17), and (28), which imply that the contributions coming from the interference between the tree-level diagram and those (bearing a vacuum polarization insertion) with the straight and crossed «legs» become in fact equal when we exchange $s \leftrightarrow u$ (with the accuracy up to terms of the order of π^2) and alternate the overall sign of the contribution. This is indeed a manifestation of the well-known symmetry relation between amplitudes corresponding to different channels of a given reaction.

The main results of this work are analytic formulas given in the logarithmic approximation, but intermediate formulas presented to a power accuracy allow at least a numeric evaluation of the impact of subleading terms on the overall value of the corrections. For example, in Sec. 2, we obtain two limiting cases of the leptonic vacuum polarization contribution, for a small (Eq. (16)) and large (Eq. (17)) lepton pair invariant mass M with constant accuracy.

As a consistency check of the calculation, the auxiliary infrared parameter λ is expected to completely cancel in the final results. Within the gauge invariant set of amplitudes considered in Sec. 2, we show that integrating over v and then adding the contribution given by the soft lepton pair production, we indeed obtain a result free of infrared divergences (Eq. (22)). The structure of this correction is in agreement with the RG predictions and does not contain large logarithms raised to the power higher than the second. But the same cannot be done for the contributions calculated in Sec. 3 because the analysis there is in fact incomplete. We also give the expression for the cross-section of a soft pion pair production (Eq. (28)). Here, we cannot explicitly show the cancellation of leading or next-to-leading logarithms to occur when the expression is combined with the corresponding virtual correction. This is because of a partially nonanalytic form of the expression for the radiative corrections caused by the hadronic vacuum polarization insertion.

In Sec. 3, we examined the contribution coming from squared box-type diagrams (see Eq. (32)) supplied by the corresponding one and two soft photon emission contributions with the explicit expressions given in Eqs. (30) and (31). To complete the picture, we must take the radiative corrections caused by genuine two-loop eikonal-type amplitudes into account. Keeping in mind the validity of the RG approach in the leading logarithmic approximation and the effect of cancellation of large logarithms in the expression for the lowest-order radiative corrections to eikonal-type diagrams (see Ref. [1]), we expect the interference between them and the Born-level amplitude to completely cancel when added to the contributions in Eqs. (30)–(32). Their explicit evaluation will be the subject of a forthcoming paper.

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APPENDIX A

In this appendix, we give a set of scalar integrals encountered in dealing with box-type diagrams with a vacuum polarization insertion in one of the exchange virtual photon propagators. Clearly, in this case, we need the integrals with a virtual exchange photon endowed with a mass M. In evaluating vector and tensor integrals, we therefore use the technique presented in Appendix B with the only change that all the scalar integrals with three (I_{ijk}) and four (I) denominators are replaced by the following ones:

1) in the case of a large mass M ($M^2 \gg s \sim -t$),

$$\begin{aligned}
I_{123} &= \frac{1}{M^2}\left\{-\ln\frac{M^2}{s} - 1 + \right. \\
&\left. + \frac{s}{M^2}\left[\frac{1}{2}\ln\frac{M^2}{s} + \frac{1}{4}\right]\right\}, \\
I_{134} &= -\frac{1}{M^2}\left\{\ln\frac{M^2}{m_e^2} + 1 + \right. \\
&\left. + \frac{t}{M^2}\left[\frac{1}{2}\ln\frac{M^2}{m_e^2} + \frac{1}{4}\right]\right\}, \\
I_{234} &= -\frac{1}{M^2}\left\{\ln\frac{M^2}{m_\mu^2} + 1 + \right. \\
&\left. + \frac{t}{M^2}\left[\frac{1}{2}\ln\frac{M^2}{m_\mu^2} + \frac{1}{4}\right]\right\}, \\
I &= -\frac{1}{2sM^2}\left\{2\rho_s\rho_\lambda + \rho_s^2 - \frac{4\pi^2}{3}\right\}, \\
I_3 &= \tilde{t}I - I_{124} = \frac{1}{M^2}\left\{-\frac{2t}{s}\rho_s + 1 - \ln\frac{s}{M^2} + \right. \\
&\left. + \frac{s}{M^2}\left[\frac{1}{2}\ln\frac{s}{M^2} - \frac{1}{4} + \left(\frac{t}{s}\right)^2\rho_s\right]\right\},
\end{aligned} \tag{A.1}$$

2) in the opposite limit $-t \gg M^2$, we must use the integrals

$$\begin{aligned}
I_{134} &= \frac{1}{t}\left[\ln\frac{M^2}{m_e^2}\ln\frac{-t}{M^2} + \frac{\pi^2}{6} + \frac{1}{2}\ln^2\frac{-t}{M^2}\right], \\
I_{234} &= \frac{1}{t}\left[\ln\frac{M^2}{m_\mu^2}\ln\frac{-t}{M^2} + \frac{\pi^2}{6} + \frac{1}{2}\ln^2\frac{-t}{M^2}\right],
\end{aligned}$$

$$I_{123} = -\frac{1}{2s}\left[2\rho_s\rho_m + \frac{4\pi^2}{3} - \rho_s^2 + \ln^2\frac{m_\mu}{m_e}\right],$$
$$I = \frac{1}{st}\rho_s\left[\rho_\lambda + 2\rho_t - \rho_m\right],$$
$$I_3 = \frac{1}{s}\left[2\rho_s\ln\frac{-t}{M^2} + \rho_s\rho_m - \frac{1}{2}\rho_s^2 + \right.$$
$$\left. + \frac{1}{2}\ln^2\frac{m_\mu}{m_e} + \frac{2\pi^2}{3}\right]. \qquad \text{(A.2)}$$

APPENDIX B

Here, we give the details of the box–box contribution calculation. First of all, we must distinguish three cases: two box squares with straight and crossed legs and one case with the interference of amplitudes with crossed and straight legs.

To calculate the contributions, we must evaluate tensor, vector, and scalar integrals with four and three denominators. We first consider the integral for the box with straight legs. The vector integral can be written as

$$\int\frac{d^4k k^\mu}{i\pi^2(1)(2)(3)(4)} = Ap_{1\mu} + Bp_{2\mu} + Cq_\mu, \qquad \text{(B.1)}$$

where quantities (1), (2), and (4) were defined in (13), and we use the notation $m = m_e$, $M = m_\mu$, and (3) is $k^2 - 2kq + t$ with

$$q = p_1 - p_1' = p_2' - p_2, \quad q^2 = t. \qquad \text{(B.2)}$$

The coefficients A, B, and C are determined as

$$A = \frac{1}{2stu}[-t^2a - t(2s+t)b - stc],$$
$$B = \frac{1}{2stu}[-t(2s+t)a - t^2b + stc],$$
$$C = \frac{1}{2stu}[-sta + stb - s^2c], \qquad \text{(B.3)}$$
$$a = I_{123} - I_{234}, \quad b = I_{134} - I_{123}, \quad c = tI.$$

The scalar integrals I and I_{ijk} are given by

$$I = \int\frac{d^4k}{i\pi^2(1)(2)(3)(4)} =$$
$$= \frac{2}{st}\left[\ln\frac{s}{mM} - i\pi\right]\ln\frac{-t}{\lambda^2},$$

$$I_{123} = \int\frac{d^4k}{i\pi^2(1)(2)(3)} =$$
$$= -\frac{1}{2s}\left[2\left[\ln\frac{s}{mM} - i\pi\right]\ln\frac{\lambda^2}{mM} + \right.$$
$$\left. + \frac{\pi^2}{3} - \left[\ln\frac{s}{mM} - i\pi\right]^2 + \ln^2\frac{M}{m}\right], \qquad \text{(B.4)}$$
$$I_{134} = \int\frac{d^4k}{i\pi(1)(3)(4)} = \frac{1}{t}\left[\frac{1}{2}\ln^2\frac{-t}{m^2} + \frac{2\pi^2}{3}\right],$$
$$I_{234} = \int\frac{d^4k}{i\pi(2)(3)(4)} = \frac{1}{t}\left[\frac{1}{2}\ln^2\frac{-t}{M^2} + \frac{2\pi^2}{3}\right].$$

To consider the tensor integral, we use the algebraical method,

$$\int\frac{d^4k k_\mu k_\nu}{i\pi^2(1)(2)(3)(4)} = a_g g_{\mu\nu} + a_{11}p_{1\mu}p_{1\nu} +$$
$$+ a_{22}p_{2\mu}p_{2\nu} + a_{12}(p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu}) +$$
$$+ a_{1q}(p_{1\mu}q_\nu + p_{1\nu}q_\mu) + a_{2q}(p_{2\mu}q_\nu + p_{2\nu}q_\mu) +$$
$$+ a_{qq}q_\mu q_\nu. \qquad \text{(B.5)}$$

Multiplying the above equation with four-vectors p_1, p_2, and q, we obtain a system of algebraic equations, whence the quantities a_{ij} are expressed through the scalar integrals,

$$a_{22} = \frac{1}{s}(A_2 - ta_{2q}),$$
$$a_{11} = \frac{1}{s}(A_4 + ta_{1q}),$$
$$a_{12} = \frac{1}{s}(A_1 - 2a_g - ta_{1q}),$$
$$a_g = \frac{1}{2}(A_9 - 2ta_{qq} - ta_{1q} + ta_{2q}), \qquad \text{(B.6)}$$
$$a_{1q} = \frac{1}{t}(A_1 - A_5 - ta_{2q}),$$
$$a_{2q} = \frac{1}{s+t}(A_3 + A_{10} - A_5 - A_9),$$
$$a_{qq} = \frac{1}{t(s+t)}(tA_3 + s(A_5 + A_9 - A_{10})).$$

The quantities A_j are given by

$$A_1 = \frac{1}{s}(I_{13} - I_{12}),$$
$$A_2 = I_{234} + \frac{1}{s}(I_{12} - I_{23}) +$$
$$+ \frac{1}{t}(2I_{34} - I_{23} - I_{24}),$$
$$A_3 = I_{123} + \frac{1}{s}(2I_{12} - I_{13} - I_{23}) +$$
$$= \frac{1}{t}(-I_{23} + I_{34}),$$

$$
\begin{aligned}
&A_4 = I_{134} + \frac{1}{s}(I_{12} - I_{13}) + \\
&+ \frac{1}{t}(2I_{34} - I_{13} - I_{14}), \\
&A_5 = \frac{1}{s}(I_{23} - I_{12}), \\
&A_9 = tC + I_{123} + \frac{1}{s}(2I_{12} - I_{13} - I_{23}), \\
&A_{10} = I_{123},
\end{aligned}
\tag{B.7}
$$

where I_{ij} denote scalar integrals with two denominators,

$$
\begin{aligned}
&I_{12} = \int \frac{d^4k}{i\pi^2(1)(2)} = \ln\frac{\Lambda^2}{M^2} - \ln\frac{s}{M^2} + i\pi + 1, \\
&I_{13} = \int \frac{d^4k}{i\pi^2(1)(3)} = I_{14} = \int \frac{d^4k}{i\pi^2(1)(4)} = \\
&= \ln\frac{\Lambda^2}{M^2} + \ln\frac{M^2}{m^2} + 1, \\
&I_{23} = \int \frac{d^4k}{i\pi^2(2)(3)} = I_{24} = \int \frac{d^4k}{i\pi^2(2)(4)} = \\
&= \ln\frac{\Lambda^2}{M^2} + 1, \\
&I_{34} = \int \frac{d^4k}{i\pi^2(3)(4)} = \ln\frac{\Lambda^2}{M^2} - \ln\frac{-t}{M^2} + 1,
\end{aligned}
\tag{B.8}
$$

and I_{ijk} and I are determined above. For crossed legs in a box-type diagram, we must evaluate the integrals

$$
\int \frac{d^4k\,k^\mu}{i\pi^2(1)(\tilde{2})(3)(4)} = \tilde{A}p_{1\mu} - \tilde{B}p'_{2\mu} + \tilde{C}q_\mu, \tag{B.9}
$$

where $(\tilde{2}) = k^2 + p'_2k$ and

$$
\begin{aligned}
&\tilde{A} = \frac{1}{2stu}[-t^2\tilde{a} - t(2u+t)\tilde{b} - u\,t\,\tilde{c}], \\
&\tilde{B} = \frac{1}{2stu}[-t(2u+t)\tilde{a} - t^2 b + u\,t\,\tilde{c}], \\
&\tilde{C} = \frac{1}{2stu}[-u\,t\,\tilde{a} + u\,t\,\tilde{b} - u^2\,\tilde{c}], \\
&\tilde{a} = I_{1\tilde{2}3} - I_{\tilde{2}34}, \quad \tilde{b} = I_{134} - I_{1\tilde{2}3}, \quad \tilde{c} = t\tilde{I}.
\end{aligned}
\tag{B.10}
$$

The integrals are given by

$$
\begin{aligned}
&\tilde{I} = \int \frac{d^4k}{i\pi^2(1)(\tilde{2})(3)(4)} = \frac{2}{ut}\ln\frac{-u}{mM}\ln\frac{-t}{\lambda^2}, \\
&I_{1\tilde{2}3} = \int \frac{d^4k}{i\pi^2(1)(\tilde{2})(3)} = -\frac{1}{2u} \times \\
&\times \left[2\ln\frac{-u}{mM}\ln\frac{\lambda^2}{mM} + \frac{\pi^2}{3} - \right. \\
&\left. - \ln^2\frac{-u}{mM} + \ln^2\frac{M}{m}\right], \\
&I_{\tilde{2}34} = \int \frac{d^4k}{i\pi(\tilde{2})(3)(4)} = \frac{1}{t}\left[\frac{1}{2}\ln^2\frac{-t}{M^2} + \frac{2\pi^2}{3}\right],
\end{aligned}
\tag{B.11}
$$

and I_{134} is given in (B.4).

For the tensor integral, we have

$$
\begin{aligned}
&\int \frac{d^4k\,k_\mu k_\nu}{i\pi^2(1)(\tilde{2})(3)(4)} = \tilde{a}_g g_{\mu\nu} + \tilde{a}_{11}p_{1\mu}p_{1\nu} + \\
&+ \tilde{a}_{22}p'_{2\mu}p'_{2\nu} - \tilde{a}_{12}(p_{1\mu}p'_{2\nu} + p_{1\nu}p'_{2\mu}) + \\
&+ \tilde{a}_{1q}(p_{1\mu}q_\nu + p_{1\nu}q_\mu) - \tilde{a}_{2q}(p'_{2\mu}q_\nu + p'_{2\nu}q_\mu) + \\
&+ \tilde{a}_{qq}q_\mu q_\nu,
\end{aligned}
\tag{B.12}
$$

where we use

$$
\begin{aligned}
&\tilde{a}_{22} = \frac{1}{u}(\tilde{A}_2 - t\tilde{a}_{2q}), \\
&\tilde{a}_{11} = \frac{1}{u}(\tilde{A}_4 + t\tilde{a}_{1q}), \\
&\tilde{a}_{12} = \frac{1}{u}(\tilde{A}_1 - 2\tilde{a}_g - t\tilde{a}_{1q}), \\
&\tilde{a}_g = \frac{1}{2}(\tilde{A}_9 - 2t\tilde{a}_{qq} - t\tilde{a}_{1q} + t\tilde{a}_{2q}), \\
&\tilde{a}_{1q} = \frac{1}{t}(\tilde{A}_1 - \tilde{A}_5 - t\tilde{a}_{2q}), \\
&\tilde{a}_{2q} = \frac{1}{u+t}(\tilde{A}_3 + \tilde{A}_{10} - \tilde{A}_5 - \tilde{A}_9), \\
&\tilde{a}_{qq} = \frac{1}{t(u+t)}(t\tilde{A}_3 + u(\tilde{A}_5 + \tilde{A}_9 - \tilde{A}_{10})).
\end{aligned}
\tag{B.13}
$$

The quantities $\tilde{A}_j$ are given by

$$
\begin{aligned}
&\tilde{A}_1 = \frac{1}{u}(I_{13} - I_{1\tilde{2}}), \\
&\tilde{A}_2 = I_{\tilde{2}34} + \frac{1}{u}(I_{1\tilde{2}} - I_{\tilde{2}3}) + \\
&+ \frac{1}{t}(2I_{34} - I_{\tilde{2}3} - I_{\tilde{2}4}), \\
&\tilde{A}_3 = I_{1\tilde{2}3} + \frac{1}{u}(2I_{1\tilde{2}} - I_{13} - I_{\tilde{2}3}) + \\
&+ \frac{1}{t}(-I_{\tilde{2}3} + I_{34}), \\
&\tilde{A}_4 = I_{134} + \frac{1}{u}(I_{1\tilde{2}} - I_{13}) + \\
&+ \frac{1}{t}(2I_{34} - I_{13} - I_{14}), \\
&\tilde{A}_5 = \frac{1}{u}(I_{\tilde{2}3} - I_{1\tilde{2}}), \\
&\tilde{A}_9 = t\tilde{C} + I_{1\tilde{2}3} + \frac{1}{u}(2I_{1\tilde{2}} - I_{13} - I_{\tilde{2}3}), \\
&\tilde{A}_{10} = I_{1\tilde{2}3},
\end{aligned}
\tag{B.14}
$$

where I_{ij} denote scalar integrals with two denominators,

$$
\begin{aligned}
I_{1\tilde{2}} &= \int \frac{d^4k}{i\pi^2(1)(\tilde{2})} = \ln\frac{\Lambda^2}{M^2} - \ln\frac{-u}{M^2} + 1,\\
I_{\tilde{2}3} &= \int \frac{d^4k}{i\pi^2(\tilde{2})(3)} = I_{\tilde{2}4} = \int \frac{d^4k}{i\pi^2(\tilde{2})(4)} = \\
&= \ln\frac{\Lambda^2}{M^2} + 1,
\end{aligned}
\tag{B.15}
$$

and I_{ijk} and I are determined above. The other integrals I_{13}, I_{14}, and I_{34} are given in (B.8).

With all these integrals, it is straightforward to obtain the final result for the squared box-type diagrams. With the intent to realize subsequent numeric calculations, we give it in the form where all terms not enhanced by large logarithms are retained,

$$
\sum |\mathcal{M}_{box}|^2 = 16\alpha^4 \mathcal{B}(s,t,u),
$$

$$
\begin{aligned}
\mathcal{B}(s,t,u) &= \frac{8(s^2+u^2)}{t^2}(L_{us}^2+\pi^2)\times\\
&\times \ln^2\left(\frac{-t}{\lambda^2}\right) - 4\ln\left(\frac{-t}{\lambda^2}\right)\times\\
&\times L_{us}\left[\frac{s-u}{t}(L_{ut}^2+L_{st}^2-L_{ut}-L_{st})+L_{us}\right]+\\
&+\frac{(s-u)^2}{2}\left[\frac{1}{s^2}L_{ut}^4+\frac{1}{u^2}L_{st}^4\right]+\\
&+2(s-u)\left[-\frac{1}{s}L_{ut}^3+\frac{1}{u}L_{st}^3\right]+2\left[L_{ut}^2+L_{st}^2\right]+\\
&+\pi^2\left\{4\ln\left(\frac{-t}{\lambda^2}\right)\left[\frac{s-u}{t}(2L_{st}-L_{us})+\frac{2u}{t}\right]+\right.\\
&+\left[L_{ut}\left(1-\frac{u}{s}\right)-1\right]^2+\\
&\left.+2\left[L_{st}\left(1-\frac{s}{u}\right)-1\right]^2-1\right\}+\\
&+\frac{\pi^4}{2}\left(1-\frac{u}{s}\right)^2.
\end{aligned}
\tag{B.16}
$$

For completeness, we here present a formula for the interference of a tree-level and a box-type diagram amplitudes,

$$
\begin{aligned}
2\sum \mathcal{M}_{Born}^{\star}\mathcal{M}_{box} &= \\
&= \sum |\mathcal{M}_{Born}|^2\frac{\alpha}{\pi}\left\{2L_{su}(\rho_t+\rho_\lambda)+\right.\\
&+\frac{t^2}{s^2+u^2}\left[\frac{u}{t}L_{st}-\frac{s}{t}L_{ut}+\right.\\
&\left.\left.+\frac{s-u}{2t}(\pi^2+L_{ut}^2+L_{st}^2)\right]\right\}.
\end{aligned}
\tag{B.17}
$$

Adding to this expression the contribution arising from the «up–down» interference of a soft photon emission by electron and muon lines, we arrive at the expression for the radiative corrections given in Eq. (16) in [1].

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