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RADIATIVE CORRECTIONS TO POLARIZATION OBSERVABLES IN THE ELASTIC ELECTRON–DEUTERON SCATTERING IN HADRONIC VARIABLES

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The model-independent QED radiative corrections to polarization observables in the elastic scattering of the unpolarized and longitudinally polarized electron beam by the deuteron target are calculated. Two experimental setups are considered: the deuteron target is arbitrarily polarized or the vector and/or tensor polarization of the recoil deuteron is measured. The calculations are based on taking all essential Feynman diagrams into account and on using the covariant parameterization of the deuteron polarization state. The radiative corrections are calculated for the hadronic variables using invariant integration of the leptonic tensor. Numerical estimates of the radiative corrections to the polarization observables are made for various values of the kinematical variables.

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1. INTRODUCTION

Recent progress in electron-scattering experiments has allowed measuring various polarization observables in the region of the momentum transfers where they can help to discriminate between different theoretical predictions. Much of this progress has been made possible by the modern high-energy electron accelerators with high duty cycle such as MAMI or JLAB and the development of polarized sources, targets, and polarimeters.

The electron scattering by few-body systems has shown that two-body terms of the nuclear electromagnetic operators give important contributions to the observables.

The deuteron, the only bound two-nucleon system, is one of the fundamental systems of nuclear physics. Accordingly, many studies, both experimental and theoretical, have been devoted to it. Of particular interest today is the degree to which the deuteron can be understood as a system of two nucleons interacting via the known nucleon–nucleon interaction.

When addressing the electromagnetic properties of the deuteron more specifically, the corresponding question concerns the ability to predict the three deuteron form factors starting from the calculated deuteron wave function and the nucleon form factors known from the electron–nucleon scattering. At low momentum transfers, predictions and data agree quite well when only one-body terms are taken into account; at higher momentum transfers, two-body contributions are known to be important. Whether quark degrees of freedom do need to be allowed for is still a matter of debate. An up-to-date status of the experimental and theoretical research of the deuteron can be found in reviews [1].

The deuteron electromagnetic form factors are most often studied in order to check our understanding of the two-nucleon system. In parallel, however, the deuteron form factors are also exploited to obtain a better handle on the neutron form factors. In the past, much of our knowledge on the neutron charge form factor $G_{En}(q^2)$ came from precision studies of the deuteron structure function $A(q^2)$ (see Eq. (16) for the definition). Only very recently, experiments involving both polarized electrons and polarized target/recoil nuclei have allowed accessing G_{En} via other observables. At large q^2, however, G_{En} is still largely unknown, which represents a serious handicap for the quantitative understanding of the deuteron charge form factors.

Elastic electron–deuteron scattering has been investigated in many experiments, and the cross section data today cover a large range of momentum transfers (see

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review [2]). Some of these data are obviously not very precise, other data, mainly more recent, have reached accuracies down to the 1 % level. During the last years, it has increasingly become possible to measure not only cross sections, but also spin observables. The knowledge of these spin observables is imperative if one wants to separate the contributions of the different form factors to the $A(q^2)$ structure function. On the side of experiment, good progress has been made. In particular, we now have a reasonably complete set of polarization data for electron–deuteron scattering that allows us to separate the deuteron charge and quadrupole form factors.

Two techniques are basically available to measure such spin observables.

1) At storage rings, one can use polarized, internal deuteron gas targets from an atomic beam source [3]. The high intensity of the circulating electron beam allows one to achieve acceptable luminosities despite the very low thickness of the gas target.

2) At facilities with external beams, one can use polarimeters to measure the polarization of the recoil deuterons. High beam intensities are a prerequisite because the polarization measurement, which requires a second reaction of the deuteron, involves a loss of a few orders of magnitude in count rate.

Current experiments at modern accelerators reached a new level of precision; this requires a new approach to data analysis and inclusion of all possible systematic uncertainties. An important source of such uncertainties is the electromagnetic radiative effects caused by physical processes that occur in higher orders of the perturbation theory with respect to the electromagnetic interaction. Previously, we calculated the radiative corrections to the polarization observables in deep inelastic scattering (due to the tensor-polarized deuteron target) [4] and in semi-inclusive deep inelastic scattering (due to the vector polarization of the target or/and outgoing hadron) [5].

In present paper, we calculate the model-independent $O(\alpha)$ QED corrections to the polarization observables in the scattering of the unpolarized or longitudinally polarized electron beam off the vector- or tensor-polarized deuteron target (or production of the arbitrarily polarized final deuteron),

$$e^-(k_1) + D(p_1) \to e^-(k_2) + D(p_2). \qquad (1)$$

The experimental setups also allow measuring the tensor polarization observables at scattering off the polarized deuteron target as well as by determination of the recoil deuteron polarizations. Different aspects of respective approaches [6] in JLAB were discussed recently in Ref. [7].

For the polarized target experiments, the scattered electron is usually detected, although the measurement of the recoil deuteron is also possible. In the first case, the leptonic variables, and in the second case, the hadronic ones are used to calculate the radiative corrections. In the leptonic variables, the virtuality of the heavy intermediate photon is not fixed due to the possibility to radiate a photon by the initial or scattered electron. As a result, the corresponding radiative correction involves some integrals with deuteron form factors over the intermediate photon mass, which cannot be computed in a model-independent way (without knowing the form factors). Contrarily, in hadron variables, the heavy photon mass is fixed, and the respective radiative correction caused by electromagnetic effects in the lepton part of the interaction can be calculated, in principle, in a model-independent way in any order of the perturbation theory.

The measurement of the recoil-deuteron polarization requires the analysis of the second scattering, which in turn suggests knowledge of the recoil-deuteron 3-momentum. Therefore, calculation of the radiative correction in this experimental setup requires using the hadronic variables, which we consider in this work. Our approach is based on the covariant parameterization of the polarization state of the deuteron target or recoil deuteron in terms of the 4-momenta of the particles in process (1), used first in [8–10] and recently in [4, 5]. In addition, we use invariant integration of the leptonic tensor to calculate the contribution to the radiative correction caused by the hard-photon radiation. Derived this way, the first-order QED correction is generalized by exponentiation of the most singular terms in the limiting case where the real photon energy is small. Our analytical final results are simple enough and have a physically transparent form.

2. BORN APPROXIMATION

Different polarization observables in the electron–deuteron elastic scattering have been studied in [11–16] and other papers, where the results were expressed in terms of the deuteron electromagnetic form factors. Here, we reproduce most of these results using the method of covariant parameterization of the deuteron polarization state in terms of the particle 4-momenta and demonstrate the advantage of this approach.

We first consider the scattering off the polarized deuteron target. In the one-photon exchange approximation, we define the cross section of process (1) in terms of the contraction of the leptonic $L_{\mu\nu}$ and hadronic $H_{\mu\nu}$ tensors (we neglect the electron mass wherever possible),

$$d\sigma = \frac{\alpha^2}{2Vq^4} L^B_{\mu\nu} H_{\mu\nu} \frac{d^3k_2}{\varepsilon_2}\frac{d^3p_2}{E_2}\delta(k_1+p_1-k_2-p_2), \quad (2)$$

where $V = 2k_1p_1$, ε_2 and E_2 are the respective energies of the scattered electron and the recoil deuteron, and $q = k_1 - k_2 = p_2 - p_1$ is the 4-momentum of the heavy virtual photon that probes the deuteron. For a longitudinally polarized electron beam, the leptonic tensor in the Born approximation is given by

$$L^B_{\mu\nu} = q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu}) + 2i\lambda(\mu\nu q k_1), \quad (\mu\nu ab) = \varepsilon_{\mu\nu\lambda\rho}a_\lambda b_\rho, \quad (3)$$

where λ is the degree of the beam polarization (in what follows, we assume that the electron beam is completely polarized, and consequently $\lambda = 1$).

The hadronic tensor can be expressed via the deuteron electromagnetic current J_μ describing the transition $\gamma^* d \to d$ as

$$H_{\mu\nu} = J_\mu J^*_\nu. \quad (4)$$

Because the deuteron is a spin-one nucleus, its electromagnetic current is completely described by three form factors. Assuming the P- and C-invariance of the hadron electromagnetic interaction, we can write this current as [17]

$$J_\mu = (p_1+p_2)_\mu \times \left[-G_1 U_1 U_2^* + \frac{G_3}{M^2}\left(U_1 q U_2^* q - \frac{q^2}{2}U_1U_2^*\right)\right] + G_2(U_{1\mu}U_2^* q - U^*_{2\mu}U_1 q), \quad (5)$$

where $U_{1\mu}(U_{2\mu})$ is the wave function of the initial (recoil) deuteron, M is the deuteron mass, and G_i ($i = 1, 2, 3$) are the deuteron electromagnetic form factors. Due to the current hermiticity, the form factors $G_i(q^2)$ are real functions in the region of space-like momentum transfer. They can be related to the standard deuteron form factors, G_C (the charge monopole), G_M (the magnetic dipole), and G_Q (the quadrupole), as

$$G_M = -G_2, \quad G_Q = G_1 + G_2 + 2G_3, \quad G_C = \frac{\rho}{6\tau}(G_2 - G_3) + \left(1 + \frac{\rho}{6\tau}\right)G_1, \quad \rho = -\frac{q^2}{V}, \quad \tau = \frac{M^2}{V}. \quad (6)$$

The standard form factors have the normalizations

$$G_C(0) = 1, \quad G_M(0) = \frac{M}{m_n}\mu_d, \quad G_Q(0) = M^2 Q_d, \quad (7)$$

where m_n is the nucleon mass, $\mu_d(Q_d)$ is deuteron magnetic (quadrupole) moment, and their values are

$$\mu_d = 0.857, \quad Q_d = 0.2859 fm^2.$$

In calculating the expression for the hadron tensor $H_{\mu\nu}$ in terms of the deuteron electromagnetic form factors, using the explicit form of electromagnetic current (5), one has to use the spin-density matrix of the initial and final deuterons

$$U_{1\alpha}U^*_{1\beta} = -\frac{1}{3}\left(g_{\alpha\beta} - \frac{p_{1\alpha}p_{1\beta}}{M^2}\right) - \frac{i}{2M}(\alpha\beta W p_1) + Q_{\alpha\beta}, \quad U_{2\alpha}U^*_{2\beta} = -\left(g_{\alpha\beta} - \frac{p_{2\alpha}p_{2\beta}}{M^2}\right) \quad (8)$$

if the deuteron target is polarized and the polarization of the recoil deuteron is not measured. Here, W_α and $Q_{\alpha\beta}$ are the target-deuteron polarization 4-vector and the quadrupole tensor, respectively.

Taking Eqs. (4), (5), and (8) into account, we can write the hadronic tensor in the general case as

$$H_{\mu\nu} = H_{\mu\nu}(0) + H_{\mu\nu}(V) + H_{\mu\nu}(T), \quad (9)$$

where $H_{\mu\nu}(0)$ corresponds to the unpolarized case and $H_{\mu\nu}(V)$ ($H_{\mu\nu}(T)$) corresponds to the case of the vector (tensor) polarization of the deuteron target. The $H_{\mu\nu}(0)$ term has the form

$$H_{\mu\nu}(0) = -W_1\tilde{g}_{\mu\nu} + \frac{W_2}{M^2}\tilde{p}_{1\mu}\tilde{p}_{1\nu},$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad \tilde{p}_{1\mu} = p_{1\mu} - \frac{p_1 q}{q^2}q_\mu,$$

$$W_1 = -\frac{2q^2}{3}\left(1 + \frac{\rho}{4\tau}\right)G_M^2, \quad W_2 = 4M^2\left(\frac{\rho}{6\tau}G_M^2 + G_C^2 + \frac{\rho^2}{18\tau^2}G_Q^2\right). \quad (10)$$

In the case under consideration, the term $H_{\mu\nu}(V)$, responsible for the vector polarization of the deuteron target, can be written as

$$H_{\mu\nu}(V) = \frac{iG_M}{2M}\left[(G_M - G)(Wp_2)(\mu\nu q p_1) + 2M^2\left(1 + \frac{\rho}{4\tau}\right)G(\mu\nu q W)\right], \quad G = 2G_C + \frac{\rho}{6\tau}G_Q, \quad (11)$$

where the 4-vector of the target deuteron polarization satisfies the conditions

$$W^2 = -1, \quad Wp_1 = 0.$$

For the tensor-polarized deuteron target, the $H_{\mu\nu}(T)$ term can be written in terms of the electromagnetic form factors as

$$\begin{aligned} H_{\mu\nu}(T) = -\bar{Q}G_M^2\tilde{g}_{\mu\nu} + \frac{\bar{Q}}{M^2} \times \\ \times \left[G_M^2 + 4(1+\eta)^{-1}G_Q\left(G_C + \frac{\eta}{3}G_Q + \eta G_M\right)\right] \times \\ \times \tilde{p}_{1\mu}\tilde{p}_{1\nu} - 2\eta G_M(G_M + 2G_Q)(\tilde{p}_{1\mu}\widetilde{Q}_\nu + \tilde{p}_{1\nu}\widetilde{Q}_\mu) - \\ - q^2(1+\eta)G_M^2\widetilde{Q}_{\mu\nu}, \end{aligned} \quad (12)$$

where

$$\begin{aligned} &\widetilde{Q}_\mu = Q_{\mu\nu}q_\nu - \frac{q_\mu}{q^2}\bar{Q}, \quad \widetilde{Q}_\mu q_\mu = 0, \\ &\widetilde{Q}_{\mu\nu} = Q_{\mu\nu} + \frac{q_\mu q_\nu}{q^4}\bar{Q} - \frac{q_\nu q_\alpha}{q^2}Q_{\mu\alpha} - \frac{q_\mu q_\alpha}{q^2}Q_{\nu\alpha}, \\ &\widetilde{Q}_{\mu\nu}q_\nu = 0, \quad \bar{Q} = Q_{\alpha\beta}q_\alpha q_\beta. \end{aligned} \quad (13)$$

The target deuteron quadrupole polarization tensor $Q_{\mu\nu}$ satisfies the conditions

$$Q_{\mu\nu} = Q_{\nu\mu}, \quad Q_{\mu\mu} = 0, \quad p_{1\mu}Q_{\mu\nu} = 0. \quad (14)$$

Using the definitions of cross section (2) and leptonic (3) and hadronic (9) tensors, we can easily derive the expression for the unpolarized differential cross section in terms of the invariant variables suitable for the calculation of the radiative corrections,

$$\begin{aligned} &\frac{d\sigma_b^{un}}{dQ^2} = \frac{\pi\alpha^2}{Q^4}\left\{\frac{2\rho}{V}W_1 + \frac{W_2}{V\tau}[1 - \rho(1+\tau)]\right\}, \\ &\qquad Q^2 = -q^2 = 2k_1k_2. \end{aligned} \quad (15)$$

In the laboratory system, this expression can be written in a more familiar form,

$$\frac{d\sigma_b^{un}}{d\Omega} = \sigma_{NS}\left\{A(Q^2) + B(Q^2)\,\mathrm{tg}^2\,\frac{\theta_e}{2}\right\}, \quad (16)$$

where θ_e is the electron scattering angle, σ_{NS} is the Mott cross section multiplied by the deuteron recoil factor

$$\left(1 + 2(\varepsilon_1/M)\sin^2\frac{\theta_e}{2}\right)^{-1},$$

and ε_1 is the electron beam energy. The two structure functions $A(Q^2)$ and $B(Q^2)$ are quadratic combinations of the three electromagnetic form factors describing the deuteron structure,

$$A(Q^2) = G_C^2(Q^2) + \frac{8}{9}\eta^2G_Q^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2),$$

$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_M^2(Q^2), \quad \eta = \frac{Q^2}{4M^2}.$$

Before writing similar distributions for the scattering of polarized particles, we note that in this case, there may exist, in general, an azimuthal correlation between the reaction plane and the plane $(\mathbf{p}_2, \mathbf{W})$ if the recoil deuteron is detected (here, $\mathbf{W}$ is 3-vector of the deuteron polarization). But in the Born approximation, with the P-invariance of the electromagnetic interaction taken into account, such a correlation is absent. In what follows, we consider the situation where the vector $\mathbf{W}$ belongs to the reaction plane and the corresponding azimuthal angle equals to zero. Therefore, there exist only two independent components of $\mathbf{W}$, which we call the longitudinal and transverse ones. It is convenient to use the covariant parameterization of the deuteron polarization 4-vector in terms of the 4-momenta of the particles in the reaction. This parameterization is ambiguous and depends on the directions along which the longitudinal and transverse components of the deuteron polarization in its rest frame are defined.

As mentioned above, we have to define the longitudinal W^L and transverse W^T 4-vectors. In our case, it is natural to choose the longitudinal direction in the laboratory system along the 3-momentum $\mathbf{q}$ and the transverse direction perpendicular to the longitudinal one in the reaction plane. The corresponding 4-vectors can be written as [5]

$$\begin{aligned} &W_\mu^{(L)} = \frac{2\tau q_\mu - \rho p_{1\mu}}{M\sqrt{\rho(4\tau+\rho)}}, \\ &W_\mu^{(T)} = \frac{(4\tau+\rho)k_{1\mu} - (1+2\tau)q_\mu - (2-\rho)p_{1\mu}}{\sqrt{V(4\tau+\rho)(1-\rho-\rho\tau)}}. \end{aligned} \quad (17)$$

This leads to simple expressions for the corresponding part of the hadronic tensor,

$$\begin{aligned} &H_{\mu\nu}^L(V) = -\frac{iG_M^2}{4\tau}(\mu\nu qp_1)\sqrt{\rho(4\tau+\rho)}, \\ &H_{\mu\nu}^T(V) = \frac{iG_MG}{4}\left[(4\tau+\rho)(\mu\nu qk_1) - \right. \\ &\left. - (2-\rho)(\mu\nu qp_1)\right]\sqrt{\frac{(4\tau+\rho)}{\tau(1-\rho-\rho\tau)}}. \end{aligned} \quad (18)$$

The polarization-dependent parts of the cross section, due to the vector polarization of the deuteron target, are given by

$$\frac{d\sigma_b^L}{dQ^2} = \frac{\pi\alpha^2}{4\tau V^2}\frac{2-\rho}{\rho}\sqrt{\rho(4\tau+\rho)}G_M^2, \qquad (19)$$

$$\frac{d\sigma_b^T}{dQ^2} = \frac{\pi\alpha^2}{VQ^2}\sqrt{\frac{(4\tau+\rho)(1-\rho-\rho\tau)}{\tau}}G_M G, \qquad (20)$$

where we assumed that λ in Eq. (3) is equal to one and the deuteron-target polarization degree (longitudinal or transverse) is 100 percent.

In the laboratory system, these parts of the cross section can be written as

$$\frac{d\sigma_b^L}{dQ^2} = \frac{\pi}{\varepsilon_2^2}\eta\sigma_{NS}\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta_e}{2}\right)} \times \times \operatorname{tg}\frac{\theta_e}{2}\sec\frac{\theta_e}{2}G_M^2, \qquad (21)$$

$$\frac{d\sigma_b^T}{dQ^2} = = 2\frac{\pi}{\varepsilon_2^2}\sigma_{NS}\operatorname{tg}\frac{\theta_e}{2}\sqrt{\eta(1+\eta)}G_M\left(G_C+\frac{\eta}{3}G_Q\right), \qquad (22)$$

where ε_2 is the scattered electron energy.

It is worth noting that the ratio of the longitudinal polarization asymmetry $A^L = d\sigma_b^L/d\sigma_b$ to the transverse one, $A^T = d\sigma_b^T/d\sigma_b$,

$$\frac{A^L}{A^T} = \frac{2-\rho}{4}\sqrt{\frac{\rho}{\tau(1-\rho-\rho\tau)}}\frac{G_M}{G} \qquad (23)$$

is expressed in terms of the deuteron form factors G_M and G in the same way as the corresponding ratio in the case of elastic electron–proton scattering is expressed via the proton electromagnetic form factors G_{Mp} and G_{Ep} [17, 18]. This is a direct consequence of the relation between the proton $H^p_{\mu\nu}(V)$ and deuteron $H_{\mu\nu}(V)$ hadronic tensors, which depend on the proton and deuteron vector polarization, respectively,

$$H_{\mu\nu}(V)(G_M,G) = = -\frac{4\tau+\rho}{8\tau}H^p_{\mu\nu}(V)(G_{Mp},G_{Ep}). \qquad (24)$$

We now consider the tensor-polarized deuteron target. For completeness, we introduce the 4-vector

$$W_\mu^{(N)} = \frac{2\varepsilon_{\mu\lambda\rho\sigma}p_{1\lambda}k_{1\rho}k_{2\sigma}}{V\sqrt{V\rho(1-\rho-\rho\tau)}}, \qquad (25)$$

which is orthogonal to the reaction plane. It can then be verified that the set of the 4-vectros $W_\mu^{(I)}$, $I = L, T, N$, satisfies the conditions

$$W_\mu^{(\alpha)}W_\mu^{(\beta)} = -\delta_{\alpha\beta}, \quad W_\mu^{(\alpha)}p_{1\mu} = 0, \quad \alpha,\beta = L,T,N.$$

If one more 4-vector $W_\mu^{(0)} = p_{1\mu}/M$ is added to the set of the 4-vectors defined by the Eqs. (17) and (25), we obtain the complete set of orthogonal 4-vectors with the properties

$$W_\mu^{(m)}W_\nu^{(m)} = g_{\mu\nu}, \quad W_\mu^{(m)}W_\mu^{(n)} = g_{mn}, \quad m,n = 0,L,T,N. \qquad (26)$$

This allows us to express the deuteron quadrupole polarization tensor in the general case as

$$Q_{\mu\nu} = W_\mu^{(m)}W_\nu^{(n)}R_{mn} \equiv W_\mu^{(\alpha)}W_\nu^{(\beta)}R_{\alpha\beta}, \quad R_{\alpha\beta} = R_{\beta\alpha}, \quad R_{\alpha\alpha} = 0, \qquad (27)$$

because the components R_{00}, $R_{0\alpha}$, and $R_{\alpha 0}$ are identically equal to zero due to the condition $Q_{\mu\nu}p_{1\nu} = 0$. The quantities $R_{\alpha\beta}$ are in fact the degrees of the tensor polarization of the deuteron target. In the Born approximation, the components R_{NL} and R_{NT} do not contribute and expansion (27) can be rewritten in the standard form

$$Q_{\mu\nu} = \left[W_\mu^{(L)}W_\nu^{(L)} - \frac{1}{2}W_\mu^{(T)}W_\nu^{(T)}\right]R_{LL} + + \frac{1}{2}W_\mu^{(T)}W_\nu^{(T)}(R_{TT}-R_{NN}) + + \left(W_\mu^{(L)}W_\nu^{(T)} + W_\mu^{(T)}W_\nu^{(L)}\right)R_{LT}, \qquad (28)$$

where we took into account that

$$R_{LL} + R_{TT} + R_{NN} = 0.$$

The part of the cross section that depends on the tensor polarization of the deuteron target can be written as

$$\frac{d\sigma_b^Q}{dQ^2} = \frac{d\sigma_b^{LL}}{dQ^2}R_{LL} + \frac{d\sigma_b^{TT}}{dQ^2}(R_{TT}-R_{NN}) + + \frac{d\sigma_b^{LT}}{dQ^2}R_{LT}, \qquad (29)$$

where

$$\frac{d\sigma_b^{LL}}{dQ^2} = \frac{\pi\alpha^2}{Q^4}2(1-\rho-\tau\rho) \times \times\eta\left\{8G_CG_Q+\frac{8}{3}\eta G_Q^2+\frac{2-2\rho+2\tau\rho+\rho^2}{2(1-\rho-\tau\rho)}G_M^2\right\}, \qquad (30)$$

$$\frac{d\sigma_b^{TT}}{dQ^2} = \frac{\pi\alpha^2}{Q^4} 2\eta(1-\rho-\tau\rho)G_M^2, \quad (31)$$

$$\frac{d\sigma_b^{LT}}{dQ^2} = -\frac{\pi\alpha^2}{Q^4} 4\eta(2-\rho)\sqrt{\frac{\rho(1-\rho-\tau\rho)}{\tau}} G_Q G_M. \quad (32)$$

In the laboratory frame, this part of the cross section can be written as

$$\frac{d\sigma_b^Q}{dQ^2} = \frac{\pi}{\varepsilon_2^2} \times \times \sigma_{NS}[S_{LL}R_{LL} + S_{TT}(R_{TT} - R_{NN}) + S_{LT}R_{LT}], \quad (33)$$

where

$$S_{LL} = \frac{1}{2}\left\{8\eta G_C G_Q + \frac{8}{3}\eta^2 G_Q^2 + + \eta\left[1 + 2(1+\eta)\,\mathrm{tg}^2\frac{\theta_e}{2}\right] G_M^2\right\}, \quad (34)$$

$$S_{TT} = \frac{1}{2}\eta G_M^2, \quad (35)$$

$$S_{LT} = -4\eta\sqrt{\eta + \eta^2\sin^2\frac{\theta_e}{2}}\sec\frac{\theta_e}{2} G_Q G_M. \quad (36)$$

If the longitudinal direction is determined by the recoil deuteron 3-momentum, relations (18) and (21) are not affected by hard photon radiation in the lepton part of the interaction (this corresponds to the use of the so-called hadronic variables, see below) because

$$\mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1.$$

But when this direction is reconstructed using the 3-momentum of the scattered electron (lepton variables), these relations break down because

$$\mathbf{q} \neq \mathbf{k}_1 - \mathbf{k}_2$$

in this case. This means that in the leptonic variables, parameterization (17) is unstable and radiation of a hard photon by the electron leads to a mixture of the longitudinal and transverse polarizations.

This mixture can be eliminated by taking the longitudinal direction along the 3-momentum of the initial electron. The corresponding parameterization of the 4-vector polarizations is [19]

$$W_\mu^{(l)} = \frac{2\tau k_{1\mu} - p_{1\mu}}{M}, \quad W_\mu^{(t)} = \frac{k_{2\mu} - (1-\rho-2\rho\tau)k_{1\mu} - \rho p_{1\mu}}{\sqrt{V\rho(1-\rho-\rho\tau)}}. \quad (37)$$

The hadronic tensors $H_{\mu\nu}^{l,t}$ then have the form

$$H_{\mu\nu}^l = -i\frac{4\tau+\rho}{4\tau} \times \times \left\{G\left[-2\tau(\mu\nu q k_1) + \frac{2\tau(2-\rho)}{4\tau+\rho}(\mu\nu q p_1)\right] + + G_M\frac{\rho(1+2\tau)}{4\tau+\rho}(\mu\nu q p_1)\right\} G_M, \quad (38)$$

$$H_{\mu\nu}^t = -i\sqrt{\frac{\rho\tau}{1-\rho-\rho\tau}} \times \times \left\{G(1+2\tau)\left[\frac{2-\rho}{4\tau}(\mu\nu q p_1) - \frac{4\tau+\rho}{4\tau}(\mu\nu q k_1)\right] - - G_M\frac{1-\rho-\rho\tau}{2\tau}(\mu\nu q p_1)\right\} G_M. \quad (39)$$

In the case of scattering off a polarized target, the tensors $H_{\mu\nu}^{L,T}$ and $H_{\mu\nu}^{l,t}$ are connected by the trivial relations

$$\begin{aligned} H_{\mu\nu}^L &= \cos\theta H_{\mu\nu}^l + \sin\theta H_{\mu\nu}^t, \\ H_{\mu\nu}^T &= -\sin\theta H_{\mu\nu}^l + \cos\theta H_{\mu\nu}^t, \\ \cos\theta &= -(W^{(L)}W^{(l)}) = \frac{\rho(1+2\tau)}{\sqrt{\rho(4\tau+\rho)}}, \\ \sin\theta &= -(W^{(L)}W^{(t)}) = -2\sqrt{\frac{\tau(1-\rho-\rho\tau)}{4\tau+\rho}}. \end{aligned} \quad (40)$$

Using these relations, we can write the polarization-dependent parts of the Born cross section, which correspond to parameterization (37), as

$$\begin{aligned} \frac{d\sigma_b^l}{dQ^2} &= \cos\theta\frac{d\sigma_b^L}{dQ^2} - \sin\theta\frac{d\sigma_b^T}{dQ^2}, \\ \frac{d\sigma_b^t}{dQ^2} &= \sin\theta\frac{d\sigma_b^L}{dQ^2} + \cos\theta\frac{d\sigma_b^T}{dQ^2}, \end{aligned} \quad (41)$$

where $d\sigma_b^L/dQ^2$ and $d\sigma_b^T/dQ^2$ are defined by Eqs. (19) and (20). Therefore, we can write

$$\frac{d\sigma_b^l}{dQ^2} = = \frac{\pi\alpha^2}{V^2}\left[\frac{1+2\tau}{4\tau}(2-\rho)G_M + \frac{2}{\rho}(1-\rho-\rho\tau)G\right] G_M, \quad (42)$$

$$\frac{d\sigma_b^t}{dQ^2} = \frac{\pi\alpha^2}{VQ^2}\sqrt{\frac{\rho(1-\rho-\rho\tau)}{\tau}} \times \times\left[-\frac{1}{2}(2-\rho)G_M + (1+2\tau)G\right] G_M. \quad (43)$$

In the case of the tensor polarization, the relations that are an analogue of Eq. (41) become

$$\frac{d\sigma_b^Q}{dQ^2} = X_{IJ}\frac{d\sigma_b^{IJ}}{dQ^2}, \quad I,J = L,T, \tag{44}$$

where the partial cross sections $d\sigma_b^{IJ}/dQ^2$ are defined by Eq. (29) as the coefficients in front of the respective quantities R_{LL}, $R_{TT}-R_{NN}$, and R_{LN}, and the entries of the matrix X_{IJ} are

$$\begin{aligned} X_{LL} &= \frac{1}{4}(1+3\cos 2\theta)R_{ll} + \\ &+ \frac{1}{4}(1-\cos 2\theta)(R_{tt}-R_{nn}) + \sin 2\theta R_{lt}, \\ X_{TT} &= \frac{3}{4}(1-\cos 2\theta)R_{ll} + \\ &+ \frac{1}{4}(3+\cos 2\theta)(R_{tt}-R_{nn}) - \sin 2\theta R_{lt}, \\ X_{lt} &= -\frac{1}{4}\sin 2\theta\left[3R_{ll}-(R_{tt}-R_{nn})\right] + \cos 2\theta R_{lt}. \end{aligned} \tag{45}$$

As we can see, the polarization-dependent part of the cross section is now expressed in terms of the new polarization parameters R_{ll}, $R_{tt}-R_{nn}$, and R_{lt} defined in accordance with the directions given by Eq. (37), and the coefficients in front of these quantities in the right-hand side of Eq. (44) determine the corresponding partial cross sections $d\sigma_b^{ij}/dQ^2$.

We now consider the scattering off the unpolarized target in the case where the recoil deuteron polarization is measured. In this case, we can obtain both the vector and tensor polarizations of the recoil deuteron using the results given above. For this, we note that the longitudinal and transverse 4-vectors $S^{(L)}$ and $S^{(T)}$, which satisfy the relations $S^2=-1$ and $(Sp_2)=0$, are

$$S^{(L)} = \frac{2\tau q_\mu + \rho p_{2\mu}}{M\sqrt{\rho(4\tau+\rho)}}, \quad S^{(T)} = W^{(T)}. \tag{46}$$

The part $H_{\mu\nu}(V)$ of the hadronic tensor can be derived from Eq. (11) by the substitution $W \to S$, $p_1 \leftrightarrow -p_2$. This actually means that we have to replace (Wp_2) in the right side of Eq. (11) with (Sp_1). The vector polarization of the recoil deuteron (longitudinal P^L or transverse P^T) is defined as the ratio of the polarization-dependent part of the cross section to the unpolarized part. Taking into account that $(S^L p_1) = -(W^L p_2)$, we conclude that

$$P^L = -A^L, \quad P^T = A^T, \tag{47}$$

where A^L and A^T are the respective asymmetries for the scattering off the 100 %-polarized deuteron target.

Here, we want to draw the reader's attention to the fact that determination of G_M/G by measurement of the ratio A^L/A^T in the scattering off a polarized deuteron target is more attractive than by measuring the ratio P^L/P^T in the polarization transfer process because the second scattering is necessary in the latter case. This decreases the corresponding event number by about two orders [20], essentially increasing the statistical error. The problem with the depolarization effect that appears in the scattering of a high-intensity electron beam on the polarized solid target can be avoided using the polarized gas deuteron target [3].

By analogy, the components of the tensor polarization of the recoil deuteron are defined by the ratios of the corresponding partial cross sections to the unpolarized one,

$$\widetilde{R}_{LL} = \frac{d\sigma_b^{LL}}{d\sigma_b^{un}}, \quad \widetilde{R}_{LT} = \frac{d\sigma_b^{LT}}{d\sigma_b^{un}}, \quad \widetilde{R}_{TT} - \widetilde{R}_{NN} = \frac{d\sigma_b^{TT}}{d\sigma_b^{un}}. \tag{48}$$

The part $H_{\mu\nu}(T)$ of the hadronic tensor can be derived from Eq. (12) by changing the sign in the term proportional to $G_M(G_M+2G_Q)$. Straightforward calculations using this updated tensor and parameterization (46) leads to the following results. First, both diagonal partial cross sections in the right-hand side of Eq. (48) are the same as defined by Eq. (29) for the scattering off the polarized target, and second, the partial cross section $d\sigma_b^{LT}/dQ^2$ changes sign compared with the cross section in Eq. (29).

3. RADIATIVE CORRECTIONS

The total radiative correction can be divided into model-independent and model-dependent contributions. The model-independent radiative correction includes all QED corrections to the lepton part of the interaction and insertion of the vacuum polarization into the exchange photon propagator. The model-dependent radiative correction involves additional couplings of the photon with the off-mass-shell hadron and comes from box-type diagrams, hadronic vertex functions, hadron contribution to vacuum polarization, etc. It can be analyzed at the level allowed by the current knowledge of the hadronic structure; as a rule, the corresponding contribution is added to the systematic error.

The standard practice of the data analysis in *ep* and *ed* scatterings is that the model-independent radiative correction is taken into account with the accuracy allowed by theoretical calculations. The reason is that it gives the main contribution due to the smallness of the electron mass, and can be calculated without any additional assumptions. Therefore, the model-independent radiative correction is calculated theoretically and simply subtracted from the observed quantities or Monte Carlo generators constructed on the basis of these calculations are implemented into codes of the data analysis. In this paper, we calculate only the model-independent radiative correction; we bear this in mind in what follows.

There exist two sources of radiative corrections when the corrections of the order α are taken into account. The first is caused by virtual and soft photon emission that cannot affect the kinematics of process (1). The second arises due to the radiation of a hard photon,

$$e^-(k_1) + D(p_1) \to e^-(k_2) + \gamma(k) + D(p_2), \qquad (49)$$

because cuts on the event selection used in the current experiments allow photons to be radiated with the energy about 100 MeV and even more [6, 20]. Such photons cannot be interpreted as «soft» ones. The form of the radiative correction caused by the contribution due to the hard photon emission depends strongly on the choice of variables used to describe process (49) [21].

The hadronic variables were used formerly to compute the radiative correction in the elastic and deep-inelastic polarized electron–proton scattering [21, 22]. As noted in Ref. [21], the form and value of the radiative correction in the hadronic variables differ essentially from the radiative correction calculated in the leptonic variables. We want to point out that the results in Ref. [22] can be used for the elastic *ep* scattering and relations (10) and (22) can be used to calculate the radiative correction in the elastic unpolarized and polarized *ed* scattering in the case of the deuteron vector polarization. Here, we also calculate the radiative correction in the case of the deuteron tensor polarization, which is absent in Ref. [22] because the proton has spin 1/2. Our goal is to obtain physically transparent formulas for the radiatively corrected cross sections, which are absent in Ref. [22], and to generalize them with the higher orders of the coupling constant α taken into account by simple exponentiation of the leading contributions.

In contrast to Ref. [22], we assume at the very beginning that in reaction (49), the recoil deuteron is detected and the 4-momentum

$$q = p_2 - p_1 = k_1 - k_2 - k$$

is fixed. Because neither the scattered electron nor the hard photon is detected, the complete integration over the 3-momenta of these undetected particles must be performed.

In calculating the radiative correction using the hadronic variables, it is very convenient to use the method of invariant integration. In this method, integration of the leptonic tensor $L^{\gamma}_{\mu\nu}$ (with the emission of an additional photon taken into account) over the variables of the scattered electron and the emitted additional photon is performed before the contraction of the leptonic and hadronic tensors. At the beginning, we use the overall 4-dimension δ-function to eliminate the k_2 momentum and then perform analytic integration with respect to the photon 3-momentum in the special system where

$$\mathbf{k}_1 + \mathbf{p}_1 - \mathbf{p}_2 = 0.$$

It is convenient to introduce the dimensionless hadronic variables

$$x = \frac{q^2}{2k_1q}, \quad y = -\frac{2k_1q}{V}, \quad xy = \rho, \qquad (50)$$

which characterize inelasticity due to the hard-photon emission in the lepton block: if the photon is not radiated, then $x = 1$ and $y = \rho$. The quantity $1-x$ actually represents the energy fraction of the collinear photon radiated by the initial-state electron. It is easy to verify this statement because

$$q = xk_1 - k_2, \quad k_2^2 = k_1^2 \approx 0$$

in this case.

The use of these variables in the framework of our approach allows bypassing the complication that comes from the Gram determinant and appears in the standard method developed in Ref. [23] and used later in Refs. [22, 24]. This gives the possibility to simplify the calculations and write physically transparent expressions for both polarized and unpolarized cross sections.

Using the above strategy, we start from the following expression for the cross section of process (49) in the hadronic variables:

$$d\sigma = \frac{\alpha^2}{VQ^4} L^{\gamma}_{\mu\nu} H_{\mu\nu} \frac{d^3p_2}{E_2} \frac{\alpha}{4\pi^2} \frac{d^3k}{\omega} \delta(k_2^2 - m^2). \qquad (51)$$

Here, ω is the photon energy and m is the electron mass. The leptonic tensor corresponding to the hard-photon radiation is well known [25, 26]. It can be written as

$$L^{\gamma}_{\mu\nu} = L^{un}_{\mu\nu} + L^{p}_{\mu\nu}.$$

Its unpolarized symmetric part is

$$L_{\mu\nu}^{un} = 2\left[\frac{(q^2-t)^2+(q^2-s)^2}{st} - 2m^2q^2\left(\frac{1}{s^2}+\frac{1}{t^2}\right)\right] \times \\ \times\widetilde{g}_{\mu\nu} + 8\left(\frac{q^2}{st} - \frac{2m^2}{s^2}\right)\widetilde{k}_{1\mu}\widetilde{k}_{1\nu} + \\ + 8\left(\frac{q^2}{st} - \frac{2m^2}{t^2}\right)\widetilde{k}_{2\mu}\widetilde{k}_{2\nu}, \quad (52)$$

and the antisymmetric part, arising due to the longitudinal beam polarization, is

$$L_{\mu\nu}^{p} = 4i(\mu\nu q\rho)\left\{k_{1\rho}\left[\frac{q^2-s}{st} - 2m^2\left(\frac{1}{s^2}+\frac{1}{t^2}\right)\right] + \\ + k_{2\rho}\left[\frac{q^2-t}{st} - \frac{2m^2 s}{(q^2-s)t^2}\right]\right\}, \quad (53)$$

$$t = -2kk_1, \quad s = 2kk_2 = -Q^2+Vy, \quad \widetilde{a}_\mu = a_\mu - \frac{aq}{q^2}q_\mu.$$

After removing the overall δ-function, it is necessary to calculate the quantity

$$L_{\mu\nu}^{i} = \int L_{\mu\nu}^{\gamma}\frac{d^3k}{\omega}\delta(k_2^2 - m^2).$$

We calculate it using the method of invariant integration. We first consider the case of the unpolarized electron beam. With the P-invariance of the electromagnetic interaction and gauge invariance of the quantity $L_{\mu\nu}^i$ taken into account, we can represent this quantity in the general form as

$$L_{\mu\nu}^{iun} = A\tilde{g}_{\mu\nu} + B\tilde{k}_{1\mu}\tilde{k}_{1\nu}. \quad (54)$$

The two functions A and B can be obtained by contracting the left- and right-hand sides of this equation with the respective tensors $\tilde{g}_{\mu\nu}$ and $\tilde{k}_{1\mu}\tilde{k}_{1\nu}$. As a result, we obtain two equations for the two unknowns A and B,

$$I_1 = 3A - \frac{(q^2-s)^2}{4q^2}B, \\ I_2 = \frac{(q^2-s)^2}{4q^2}\left[-A + \frac{(q^2-s)^2}{4q^2}B\right], \quad (55)$$

where we introduce the notation

$$I_1 = \int L_{\mu\nu}^{un}\tilde{g}_{\mu\nu}\frac{d^3k}{\omega}\delta(k_2^2 - m^2),$$

$$I_2 = \int L_{\mu\nu}^{un}\tilde{k}_{1\mu}\tilde{k}_{1\nu}\frac{d^3k}{\omega}\delta(k_2^2 - m^2).$$

Next, we must integrate over the photon phase space in the integrals I_1 and I_2. Because the quantity $\delta(k_2^2 - m^2)d^3k/\omega$ that enters the integrands in I_1 and I_2 is Lorentz invariant, we can take any coordinate system to do this integration. The most convenient one is the coordinate system where

$$\mathbf{k}_1 + \mathbf{p}_1 - \mathbf{p}_2 = 0.$$

In fact, this is the center-of-mass system for the radiated photon and the scattered electron, and therefore the polar (ϑ) and azimuthal (ϕ) angles of the radiated photon cover the entire phase space. We therefore have

$$\frac{d^3k}{\omega}\delta(k_2^2 - m^2) = \frac{1-x}{4R_x}d\phi\, d\cos\vartheta, \\ 0 < \phi < 2\pi, \\ -1 < \cos\vartheta < 1, \quad R_x = 1 - x + \frac{m^2}{Q^2}, \quad (56)$$

where the z axis is chosen along the direction of the initial electron 3-momentum. In writing the quantity R_x, we set $x = 1$ in the coefficient in front of m^2/Q^2.

The energies of all particles and the polar angle of the initial deuteron in this system can be expressed in terms of the invariant variables as

$$\omega = \frac{Vy(1-x)}{2\sqrt{R}}, \quad \varepsilon_1 = \frac{Vy + 2m^2}{2\sqrt{R}}, \\ \varepsilon_2 = \frac{Vy(1-x) + 2m^2}{2\sqrt{R}}, \quad E_1 = \frac{V(1-\rho)}{2\sqrt{R}}, \\ E_2 = \frac{V(1-y+\rho)}{2\sqrt{R}}, \quad \cos\theta_1 = \frac{2E_1\varepsilon_1 - V}{2|\mathbf{p}_1||\mathbf{k}_1|}, \\ R = VyR_x, \quad (57)$$

where ε_1 and E_1 (ε_2 and E_2) are the respective energies of the initial (final) electron and deuteron.

The necessary angular integrals are given by

$$\int \frac{1}{-t} = \frac{2R_x}{Vy(1-x)}L, \\ \int \frac{2\chi}{-t} = \frac{2}{y}\left[R_x(L-2) + 1 - \rho\right], \\ \int t = -\frac{Vy(1-x)}{R_x}, \\ \int \frac{4\chi^2}{-t} = \frac{2V(1-x)}{y} \times \\ \times\left[R_x(L-3) + \frac{(1-\rho)^2}{2R_x} + 1 - \rho - y\tau\right], \\ \int 2\chi = \frac{V(1-x)(1-\rho)}{R_x}, \\ \int \frac{2m^2}{t^2} = \frac{4R_x}{Vy(1-x)^2}, \quad \chi = kp_1, \quad (58)$$

where we use the short notation for definite integral and the quantity L,

$$\int \equiv \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{-1}^{1} d\cos\vartheta, \quad L = \ln\frac{Q^2}{m^2 x R_x},$$

and neglect the terms of the order (m^2/Q^2) whenever possible.

Calculating the integrals I_1 and I_2 as described above and solving the system of two equations (55), we obtain that the functions A and B are given by

$$A = -2\pi[F^p(x, Q^2/m^2) + 3(1-x)],$$

$$B = 8\pi\frac{x^2}{Q^2}[F^p(x, Q^2/m^2) + 3 + x],$$

$$F^p(x, Q^2/m^2) = \frac{1+x^2}{1-x}\ln\frac{Q^2}{m^2 x R_x} - \frac{4}{1-x} + \\ + \frac{1}{2R_x} - \frac{m^2}{2Q^2R_x^2} + 1 + 4x. \quad (59)$$

The contraction of the unpolarized parts of the leptonic tensor (integrated over the photon phase space) and the hadronic tensor for radiative process (49) is given by

$$L_{\mu\nu}^{iun} H_{\mu\nu}(0) = -\left(3A + \frac{Vy^2}{4\rho}B\right) W_1 + \\ + \left[(1+\eta)A + \frac{V(2-y)^2}{16\tau}B\right] W_2. \quad (60)$$

To write the respective contractions in the polarized case, we have to take into account that the parameterization of the polarization 4-vectors $S^{(L,T)}$ in the radiative process differs somewhat from the Born expressions given by Eq. (46) (we here consider the polarized recoil deuteron for definiteness). Formally, they can be derived by the substitution

$$\rho \to y, \quad k_1 \to xk_1, \quad \tau \to \tau/x, \quad V \to xV \quad (61)$$

in Eq. (46).

The contraction of the unpolarized part of the leptonic tensor (integrated over the photon phase space) and the hadronic $H_{\mu\nu}(T)$ tensor for radiative process (49) is given by

$$L_{\mu\nu}^{iun} H_{\mu\nu}(T) = \left\{-A\bar{Q} + VB\left[\frac{(1-y)^2}{4\tau}\bar{Q} + \right.\right. \\ \left.\left. + (2\eta(1-\eta) - y)Q_1 + \rho(1+\eta)Q_{11}\right]\right\} G_M^2 + \\ + (2-y)VB\left[2\eta Q_1 - \frac{y - 2\eta(1-y)}{4\tau+\rho}\bar{Q}\right] G_M G_Q + \\ + \frac{\bar{Q}}{2}\left[4A + \frac{(2-y)^2}{4\tau+\rho}VB\right] G_Q G, \quad (62)$$

where

$$Q_1 = Q_{\mu\nu} q_\mu k_{1\nu}, \quad Q_{11} = Q_{\mu\nu} k_{1\mu} k_{1\nu}.$$

Taking into account the relation

$$\frac{d^3 p_2}{E_2} = \pi\rho dQ^2 \frac{dx}{x^2}$$

that holds for the recoil-deuteron phase space, after some algebra we derive the following representation for unpolarized cross section of reaction (49):

$$\frac{d\sigma_H^{un}}{dQ^2}(k_1) = \frac{\alpha}{2\pi}\int_{x_m}^{1-\Delta}\left[\frac{d\sigma_b^{un}}{dQ^2}(xk_1)F^{un}(x, Q^2/m^2) + \right. \\ \left. + \frac{\pi\alpha^2}{Q^4}\frac{W_2}{M^2} f(x, \rho, \tau)\right] dx, \quad (63)$$

$$F^{un}(x, Q^2/m^2) = F^p(x, Q^2/m^2) + 3 - 4x,$$

$$f(x, \rho, \tau) = 3(x-\rho) + \frac{\rho}{x}\left(\frac{\rho}{2} - \tau\right).$$

Here, Δ is the minimum-energy fraction of the hard photon and x_m depends on the experimental cuts for the photon energy.

For the partial cross sections in the case of tensor polarization of the recoil deuteron in radiative process (49), we obtain the expressions

$$\frac{d\sigma_H^{LL}}{dQ^2}(k_1) = \frac{\alpha^3}{V^2}(1+\eta)\ln x_m (G_M^2 - 2G_Q G) + \\ + \frac{\alpha}{2\pi}\int_{x_m}^{1-\Delta}\frac{d\sigma_b^{LL}}{dQ^2}(xk_1)F^T(x, Q^2/m^2)dx, \quad (64)$$

$$\frac{d\sigma_H^{TT}}{dQ^2}(k_1) = \frac{\alpha}{2\pi}\int_{x_m}^{1-\Delta}\frac{d\sigma_b^{TT}}{dQ^2}(xk_1)F^T(x, Q^2/m^2)dx,$$

$$\frac{d\sigma_H^{LT}}{dQ^2}(k_1) = \frac{\alpha}{2\pi}\int_{x_m}^{1-\Delta}\frac{d\sigma_b^{LT}}{dQ^2}(xk_1)F^T(x, Q^2/m^2)dx,$$

where

$$F^T(x, Q^2/m^2) = F^p(x, Q^2/m^2) + 3 - x.$$

We now consider the case of the longitudinally polarized electron beam and calculate the necessary integral, where $L^{\gamma}_{\mu\nu} = L^p_{\mu\nu}$, using the method of invariant integration. Taking the P-invariance of the electromagnetic interaction and gauge invariance of the quantity $L^i_{\mu\nu}$ into account, we can represent this quantity in the general form as

$$L^{ip}_{\mu\nu} = iC(\mu\nu q k_1). \qquad (65)$$

The unknown function C can be obtained by contracting the left- and right-hand sides of this equation with the tensor $(\mu\nu q k_1)$. As a result, we obtain the following expression for C:

$$\frac{i}{2}(q^2 - s)C = \int L^p_{\mu\nu}(\mu\nu q k_1)\frac{d^3k}{\omega}\delta(k_2^2 - m^2). \qquad (66)$$

Calculating this integral as explained above, we obtain

$$C = \frac{2\pi x}{Q^2}F^p(x, Q^2/m^2).$$

The contraction of the polarized part of the leptonic tensor (integrated over the photon phase space) and the hadronic $H_{\mu\nu}(V)$ tensor for radiative process (49) is given by

$$L^{ip}_{\mu\nu}H^L_{\mu\nu} = -\frac{1}{2}(2-y)\eta V^2 C\sqrt{\rho(4\tau+\rho)}G_M^2, \qquad (67)$$

$$L^{ip}_{\mu\nu}H^T_{\mu\nu} = -\frac{1}{4}V^2C[2\rho(1-y) + y^2(\rho+2\tau)] \times \times \sqrt{\frac{x(4\tau+\rho)}{\tau(x-\rho-y\tau)}}G_M G. \qquad (68)$$

After some algebra, we derive the following representation for the parts of the cross section that depend on the vector polarization of the recoil deuteron:

$$\frac{d\sigma_H^{L,T}}{dQ^2}(k_1) = = \frac{\alpha}{2\pi}\int\limits_{x_m}^{1-\Delta}\frac{d\sigma_b^{L,T}}{dQ^2}(xk_1)F^p(x, Q^2/m^2)dx. \qquad (69)$$

The infrared auxiliary parameter $\Delta \ll 1$ is related to the minimal energy of the hard photon in the chosen coordinate system and the lower integration limit is defined by its maximum value that depends on the experimental cuts on the event selection in the experimental measurement of the observables in elastic electron–deuteron scattering:

$$\Delta = \frac{\omega_{min}}{\varepsilon_1}, \quad x_m = 1 - \frac{\omega_{max}}{\varepsilon_1}.$$

For example, if the lost invariant mass M_{max} (of the scattered electron and the undetected additional hard photon) is allowed,

$$(k_1 + p_1 - p_2)^2 \le M^2_{max},$$

then

$$x_m = \frac{Q^2}{Q^2 + M^2_{max}}.$$

On the other hand, the quantity x_m cannot be arbitrary (but must of course be smaller than unity) even if no experimental constraints on the event selection are used. The restriction on x_m follows from the inequality

$$x^2 - x\rho - \rho\tau > 0,$$

which reflects the obvious relation

$$-q^2 < -q^2_{max} = \frac{x^2V^2}{xV + M^2}$$

for radiative process (49). In any case, we therefore have

$$x_m > \frac{\rho}{2}\left(1 + \sqrt{1 + \frac{4\tau}{\rho}}\right).$$

We note one interesting point regarding formula (69). It looks very similar to the corresponding result in the quasireal electron approximation [27] for the description of the collinear photon radiation ($\theta_\gamma < \theta_0$, $\theta_0 \ll 1$) by the longitudinally polarized electron, which is suitable for the leptonic variables,

$$d\sigma(k_1, k) = \frac{\alpha}{2\pi}\int d\sigma_b(xk_1)P(x, L_0)dx, \quad k = (1-x)k_1, \qquad (70)$$

$$P(x, L_0) = \frac{1+x^2}{1-x}L_0 - \frac{2(1-x+x^2)}{1-x}, \quad L_0 = \ln\frac{\varepsilon_1^2\theta_0^2}{m^2},$$

where $d\sigma_b$ is the cross section of the radiationless process. It is not surprising that the function F^p differs from P in Eq. (70) because it also has to contain traces from the final electron radiation.

Formulas (63), (64), and (69) describe the distribution over the momentum transfer squared in reaction (49) and define the respective radiative correction due to the hard photon emission. To compute the total radiative correction, we must also add the contribution due to emission of the virtual photon and the real soft photon (with the energy less than $\Delta\varepsilon_1$). This contribution is the same for the polarized and unpolarized scattering,

$$\frac{d\sigma^{S+V}}{dQ^2} = \frac{d\sigma_b}{dQ^2}\frac{\alpha}{2\pi}(\delta^V + \delta^S). \tag{71}$$

The virtual correction is standard [28],

$$\delta^V = 4(L_Q - 1)\ln\frac{\lambda}{m} - L_Q^2 + 3L_Q + \frac{\pi^2}{3} - 4, \quad L_Q = \ln\frac{Q^2}{m^2}, \tag{72}$$

where λ is the «photon mass», while the soft-photon correction has some specification in the hadronic variables

$$\delta^S = 4(L_Q - 1)\ln\frac{m\Delta}{\lambda} + 2L_Q^2 - 2L_Q - \frac{\pi^2}{3} + 2. \tag{73}$$

It can be seen that the terms proportional to L_Q^2 do not vanish in the sum $\delta^V + \delta^S$ (as they do for the leptonic variables) and the contribution of the hard photon emission has to be taken into account to cancel them (due to the terms with $\ln R_x/(1-x)$ in the functions F^p, F^T, and F^{un}).

The observed cross sections, which take the total radiative correction into account, do not depend on the auxiliary infrared parameter Δ and can be written in the form suitable for numerical integration as

$$\frac{d\sigma^{un}}{Q^2}(k_1) = \frac{d\sigma_b^{un}}{Q^2}(k_1)\left(1 + \frac{\alpha}{2\pi}\delta\right) + \frac{\alpha}{2\pi} \times \int\limits_{x_m}^{1}\left[\frac{d\Delta\sigma_b^{un}}{dQ^2}\frac{f_\Delta}{1-x} + \frac{d\sigma_b^{un}}{dQ^2}(xk_1)\widetilde{F}^{un} + \frac{\pi\alpha^2}{Q^4}\frac{W_2}{M^2}f(x,\rho,\tau)\right]dx, \tag{74}$$

$$\frac{d\sigma^{L,T}}{Q^2}(k_1) = \frac{d\sigma_b^{L,T}}{Q^2}(k_1)\left(1 + \frac{\alpha}{2\pi}\delta\right) + \frac{\alpha}{2\pi}\int\limits_{x_m}^{1}\left[\frac{d\Delta\sigma_b^{L,T}}{dQ^2}\frac{f_\Delta}{1-x} + \frac{d\sigma_b^{L,T}}{dQ^2}(xk_1)\widetilde{F}^p\right]dx. \tag{75}$$

The partial cross sections in the case of the tensor polarization of the recoil deuteron are defined by the formula

$$\frac{d\sigma^{IJ}}{Q^2}(k_1) = \frac{d\sigma_b^{IJ}}{Q^2}(k_1)\left(1 + \frac{\alpha}{2\pi}\delta\right) + \frac{\alpha}{2\pi}\int\limits_{x_m}^{1}\left[\frac{d\Delta\sigma_b^{IJ}}{dQ^2}\frac{f_\Delta}{1-x} + \frac{d\sigma_b^{IJ}}{dQ^2}(xk_1)\widetilde{F}^T\right]dx + \frac{\alpha^3}{V^2}(1+\eta)\ln x_m(G_M^2 - 2G_QG)\delta_{IL}\delta_{JL}, \quad I,J = L,T, \tag{76}$$

where

$$d\Delta\sigma_b = d\sigma_b(xk_1) - d\sigma_b(k_1)$$

for both polarized and unpolarized cases and

$$\delta = \left(\frac{3}{2} + 2\ln(1-x_m)\right)L_Q - \frac{7}{2}\ln(1-x_m) - \ln^2(1-x_m) - \frac{5}{2} - \frac{\pi^2}{3},$$

$$f_\Delta = 2L_Q - 2\ln(1-x) - \frac{7}{2},$$

$$\widetilde{F}^p = -(1+x)L_Q - \frac{1+x^2}{1-x}\ln x + (1+x)\ln(1-x) + 1 + 4x, \quad \widetilde{F}^{un} = \widetilde{F}^p + 3 - 4x, \quad \widetilde{F}^T = \widetilde{F}^p + 3 - x. \tag{77}$$

The singularity at $x = 1$ in the integrands of Eqs. (74)–(76) cancels by the corresponding quantity $d\Delta\sigma_b/dQ^2$. For example, in the simplest unpolarized case, we have

$$\frac{d\Delta\sigma_b^{un}}{dQ^2} = \frac{\pi\alpha^2}{Q^2V^2x^2} \times \left[2(1+x)W_1 - \frac{W_2}{\tau}\big(x + \tau(1+x)\big)\right](1-x).$$

It is well known that the leading logarithmic contributions to the radiative correction of the order $(\alpha L_Q)^n$, $n = 1, 2, \ldots$, are controlled by the electron structure function $D(x, L_Q)$,

$$\frac{d\sigma_{lead}}{dQ^2} = \int\limits_{x_m}^{1} D(x,L_Q)\frac{d\sigma_b}{dQ^2}(xk_1)dx, \quad D(x,L_Q) = \delta(1-x) + \frac{\alpha L_Q}{2\pi}P_1(x) + \frac{1}{2}\left(\frac{\alpha L_Q}{2\pi}\right)^2 P_2(x) + \ldots \tag{78}$$

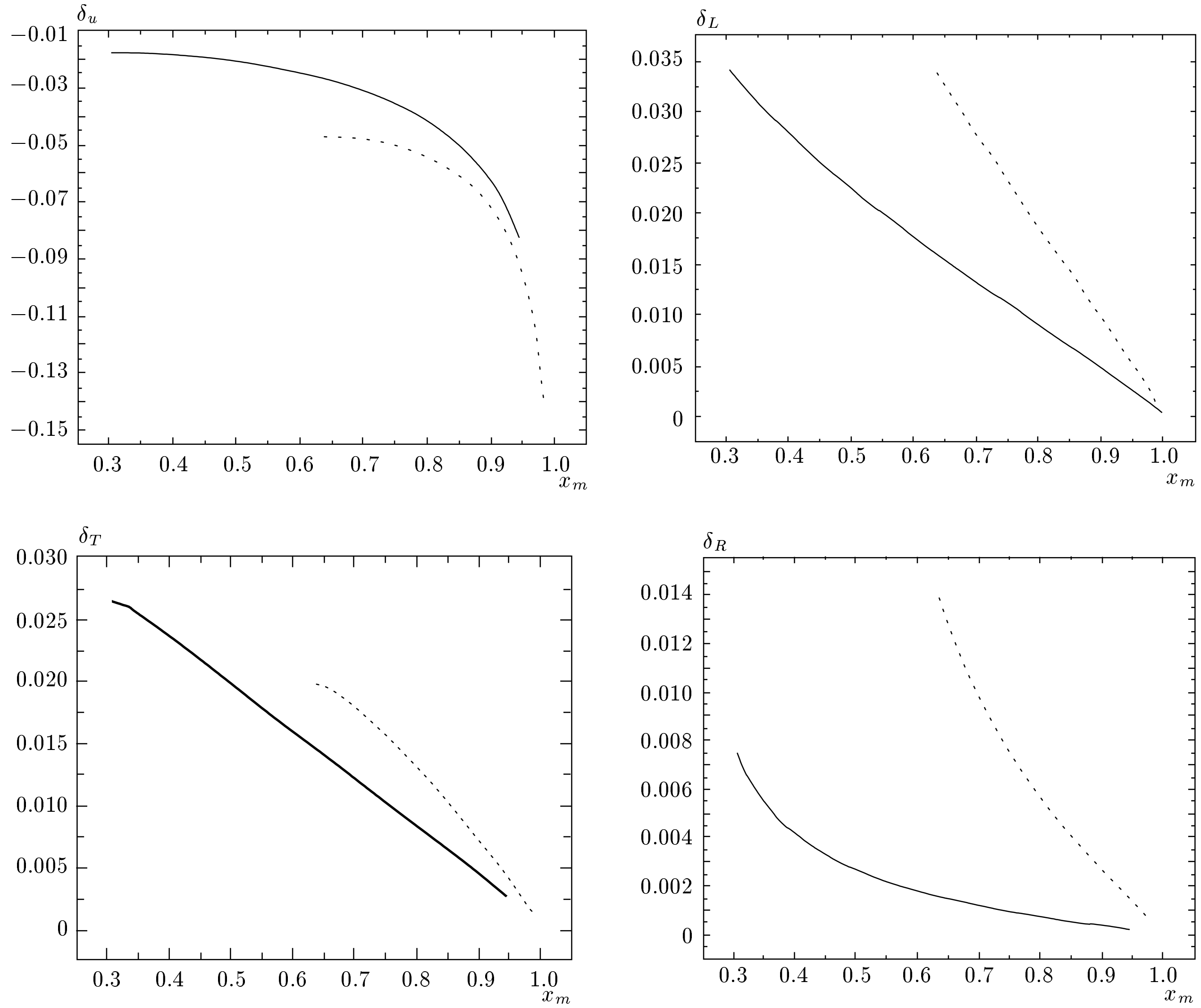

Fig. 1. The effect of the radiative correction on the unpolarized cross section and vector polarizations of the recoil deuteron calculated via Eqs. (74), (75) and Eq. (82) at $V = 8$ GeV^2. The solid curves correspond to $Q^2 = 1$ GeV^2 and the dashed ones to $Q^2 = 3$ GeV^2. Parameterization I is used for the deuteron electromagnetic form factors [32]

It can be verified that the leading part of the first-order correction defined by Eqs. (74)–(76) can be derived using representation (78) at

$$D(x, L_Q) = \frac{\alpha L_Q}{2\pi} P_1(x),$$

$$P_1(x) = \frac{1+x^2}{1-x}\Theta(1-x-\Delta) + \delta(1-x)\left(2\ln\Delta + \frac{3}{2}\right).$$

Thus, we can improve our result by insertion of the higher-order leading contributions using Eq. (78) and the known expressions for the functions $P_n(x)$ [29]. This improvement results in modification of the quantities δ, f_Δ, and $\widetilde{F}^p$ in the right-hand sides of Eqs. (74)–(76). For example, to account for the corresponding second-order terms, we must use the substitutions

$$\delta \to \delta + \gamma\delta_1, \quad f_\Delta \to f_\Delta + \gamma f_{\Delta 1},$$
$$\widetilde{F}^p \to \widetilde{F}^p + \gamma \widetilde{F}^p_1, \quad \gamma = \frac{\alpha L_Q^2}{2\pi}, \tag{79}$$

$$\delta_1 = \frac{9}{8} - \frac{\pi^2}{3} + 3\ln(1-x_m) + 2\ln^2(1-x_m)\ ,$$
$$f_{\Delta 1} = 3 + 4\ln(1-x),$$
$$\widetilde{F}^p_1 = -2(1+x)\ln(1-x) - \frac{1+3x^2}{2(1-x)}\ln x - \frac{5+x}{2}.$$

On the other hand, there exists a simple method of summation of the all singularities at $x = 1$ in the to-

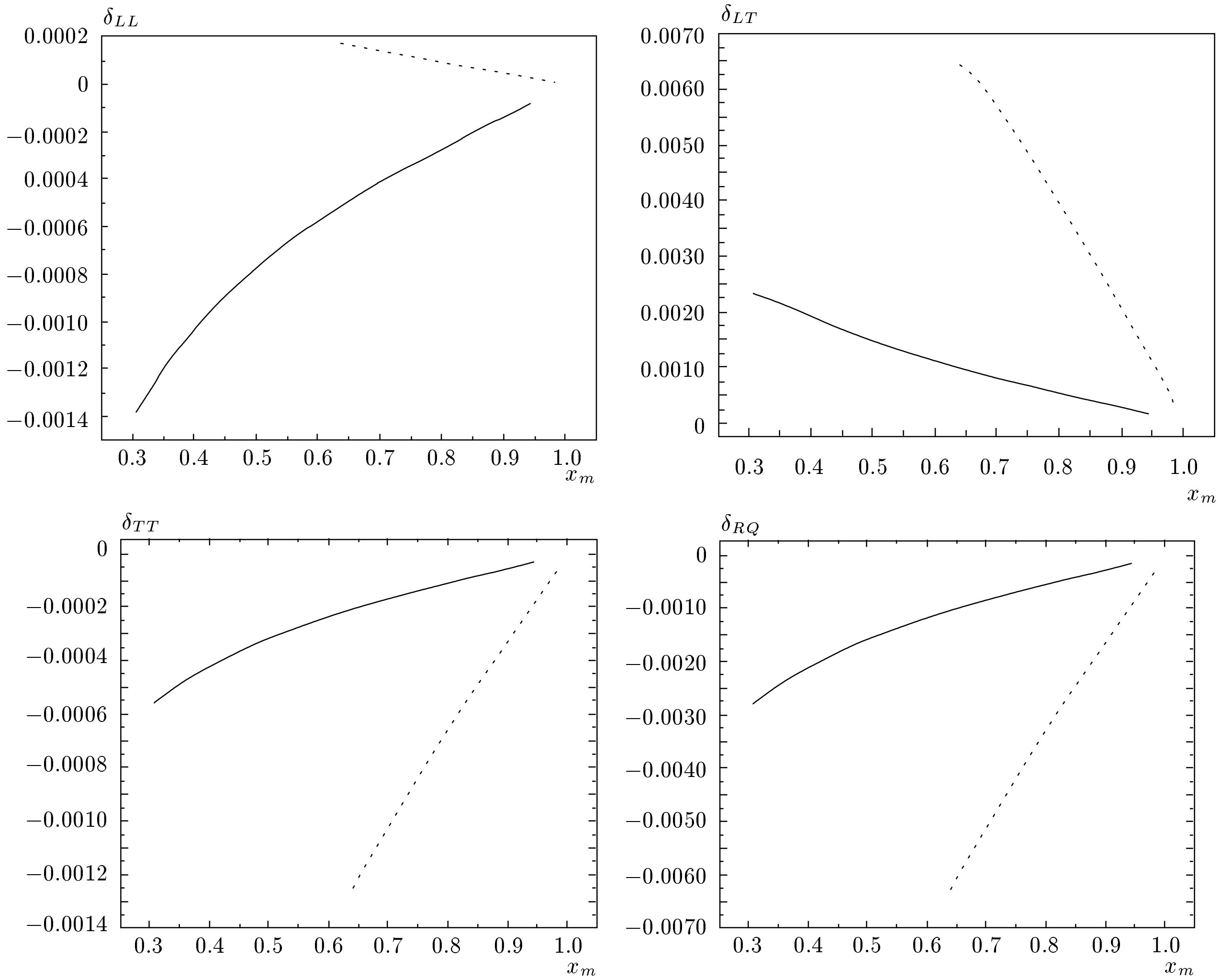

Fig. 2. The effect of the radiative correction in the case of tensor polarization of the recoil deuteron. The partial cross sections in Eq. (83) are calculated using Eq. (76). Kinematical conditions and parameterization of the form factors are the same as in Fig. 1

tal radiative correction, which goes beyond the leading logarithmic approximation (see, e.g., Ref. [30]). It consists in the use of the exponential form of the electron structure function and in our case can be introduced as

$$\delta(1-x)+\beta\left[\frac{1}{1-x}\theta(1-\Delta-x)+\delta(1-x)\ln\Delta\right]\to$$
$$\to\frac{\beta(1-x)^{\beta-1}}{\Gamma(1+\beta)}\exp\left[\beta\left(\frac{3}{4}-C\right)\right], \quad (80)$$

where

$$\beta=\frac{\alpha}{\pi}(L_Q-1),$$

C is the Euler constant, and $\Gamma(x)$ is the Gamma function. This procedure leads to a redefinition of δ and f_Δ in Eqs. (74)–(76),

$$\delta\to\delta^{exp}=\frac{3}{2}(L_Q-\ln(1-x_m))-\ln^2(1-x_m)-\frac{5}{2}-\frac{\pi^2}{3},$$

$$f_\Delta\to f_\Delta^{exp}=-\frac{3}{2}-2\ln(1-x),$$

and to the appearance of the additional term

$$\int\limits_{x_m}^{1}\frac{d\sigma_b}{dQ^2}(xk_1)\beta(1-x)^{\beta-1}\frac{\exp\left[\beta\left(\frac{3}{4}-C\right)\right]}{\Gamma(1+\beta)}dx$$

that absorbs the purely Born cross section and a part

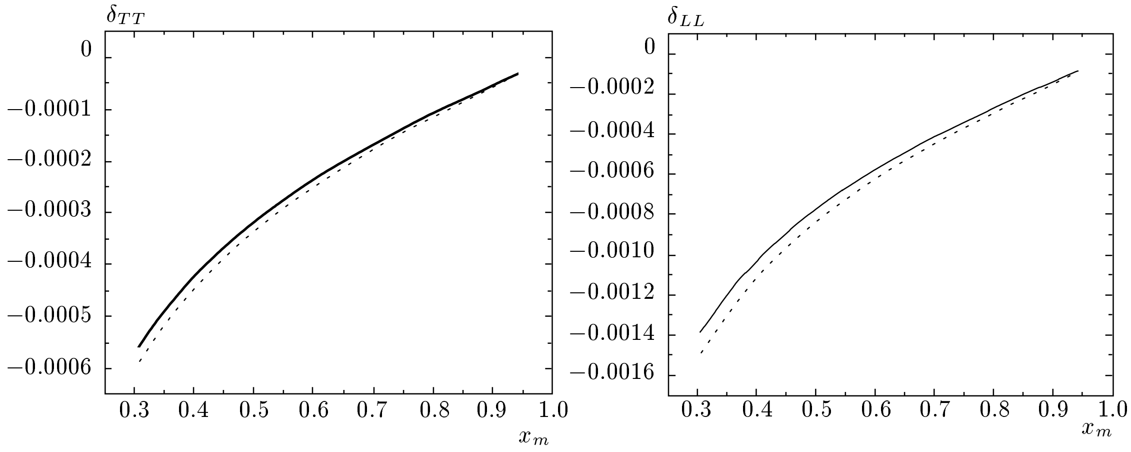

Fig. 3. The influence of different parameterizations of the deuteron electromagnetic form factors on the radiative correction. The solid (dashed) curve corresponds to parameterization I (II) [32]. The kinematical conditions are $V = 8$ GeV^2 and $Q^2 = 1$ GeV^2

of the radiative corrections. For example, partial cross sections (76) becomes

$$\frac{d\sigma^{IJ}}{Q^2}(k_1) = \int\limits_{x_m}^{1} \left[\frac{d\sigma_b^{IJ}}{dQ^2}(xk_1) \times \right. \times \left(\frac{\beta(1-x)^{\beta-1}}{\Gamma(1+\beta)} \exp\left[\beta\left(\frac{3}{4}-C\right)\right] + \frac{\alpha}{2\pi}\widetilde{F}^T\right) + \left. + \frac{\alpha}{2\pi}\frac{d\Delta\sigma_b^{IJ}}{dQ^2}\frac{f_\Delta^{exp}}{1-x}\right] dx + + \frac{\alpha}{2\pi}\delta^{exp}\frac{d\sigma_b^{IJ}}{Q^2}(k_1) + \frac{\alpha^3}{V^2}(1+\eta) \times \times \ln x_m (G_M^2 - 2G_Q G)\delta_{IL}\delta_{JL}. \quad (81)$$

4. NUMERICAL ESTIMATIONS

There are different approaches to the analysis of polarization observables. If the experimental information is extracted directly from the spin-dependent part of the cross section (see Ref. [31] for the corresponding experimental method), the radiative correction can be large due to the contribution of factored virtual and soft corrections. The nonfactored contribution to the radiative correction, caused by the hard photon emission, cannot be large in elastic scattering because the phase space of such a photon is strongly suppressed by restrictions on the event selection. The effect of the radiative correction in this case is demonstrated in Figs. 1–4 for the ratios

$$\delta_u = \frac{d\sigma_{obs}^{un}}{d\sigma_b^{un}} - 1, \quad \delta_{L,T} = \frac{P_{obs}^{L,T}}{P_b^{L,T}} - 1, \quad \delta_R = \frac{P_{obs}^T P_b^L}{P_{obs}^L P_b^T} - 1 \quad (82)$$

in the unpolarized case and for vector polarization of the recoil deuteron, and for the ratios

$$\delta_{IJ} = \frac{d\sigma_{obs}^{IJ} d\sigma_b^{un}}{d\sigma_{obs}^{un} d\sigma_b^{IJ}} - 1, \quad \delta_{RQ} = \frac{d\sigma_{obs}^{TT} d\sigma_b^{LT}}{d\sigma_{obs}^{LT} d\sigma_b^{TT}} - 1 \quad (83)$$

in the case of tensor polarization. We note that if the radiative correction is ignored, all the quantities defined by Eqs. (82) and (83) are equal to zero. The quantities δ_R and δ_{RQ} are very important physical values because they can be used for an independent determination of the ratios of form factors such as G_M/G and G_M/G_Q (see Eqs. (19), (20), (31), (32), and (48)).

The observed cross sections in Eqs. (82) and (83) are defined by Eqs. (74)–(76) or their exponential modification (as in Eq. (81)). We consider two different parameterizations of the deuteron electromagnetic form factors given in Ref. [32] and label them as I and II.

As can be seen from Fig. 1, the radiative corrections to the unpolarized cross section depends strongly on the value x_m that is connected with the energy of the hard photon in process (49). If x_m is close to unity ($x_m \approx 1$), the total radiative correction, being negative, can reach 10 % and even more. As x_m decreases, the total radiative correction becomes much smaller. Such behavior of the radiative correction has a simple

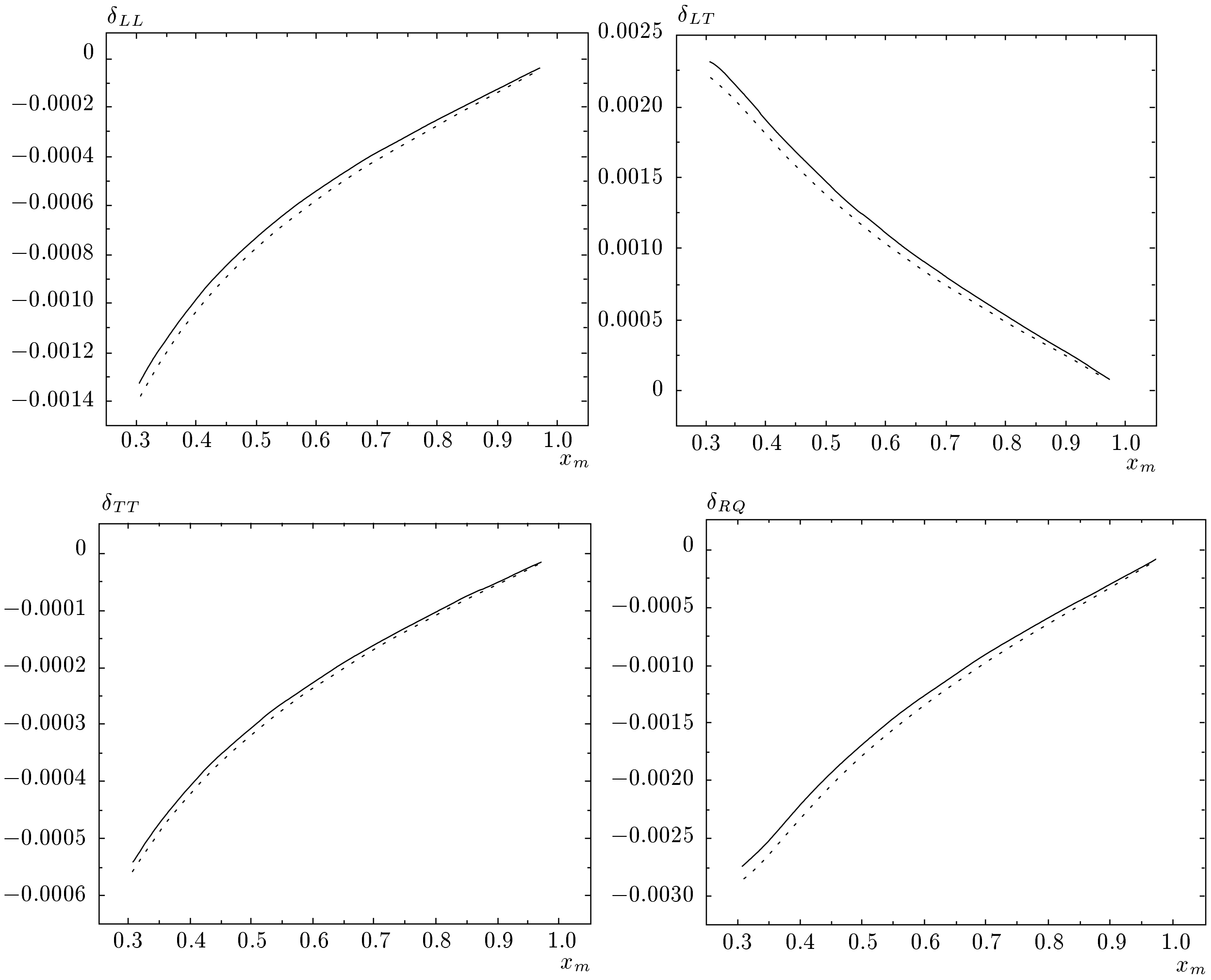

Fig. 4. Comparison of the total radiative corrections in the case of tensor polarization calculated by Eq. (76) (solid curve) and Eq. (81) (dashed curve) at $V = 8$ GeV^2 and $Q^2 = 1$ GeV^2

physical interpretation. If $x_m \approx 1$, the energy of the photon in process (49) is sufficiently small, and the positive contribution into the radiative correction due to the hard photon emission cannot compensate the factored negative contribution caused by virtual and soft photon corrections that accompany process (1). As the hard photon energy increases, such compensation occurs and the absolute value of the total radiative correction decreases. The same behavior is also exhibited by polarization-dependent parts of the cross section in the case of vector polarization of the recoil deuteron and by partial cross sections in the case of tensor polarization.

But the effect of the radiative corrections is just opposite for the ratios defined by Eqs. (82) and (83). At $x_m \approx 1$, the total radiative correction is defined mainly by its factored part, which is the same for the polarization-dependent and unpolarized cross sections. Therefore, the radiative correction in fact cancels in this region for such ratios. On the contrary, at smaller values of x_m, the nonfactored part of the radiative correction becomes significant and the total radiative correction increases. An unexpected fact is that the ratios δ_{IJ} in (83) are approximately one order smaller than $\delta_{L,T}$ in (82).

As our calculations show, the sensitivity of the radiative correction to two different parameterizations of the deuteron electromagnetic form factors [32] at relevant values of energies and momentum transfers is practically negligible. In fact, the respective curves coincide in the entire range of x_m (see Fig. 3).

The influence of the higher-order corrections, calculated by summing the leading contributions by the exponentiation procedure, is demonstrated in Fig. 4 for the tensor polarization ratios. The corresponding curves for the vector ones are very similar. We see that the effect is small and cannot even exhibit itself at small x_m, where the nonfactored radiative correction contributes. As usual, the large correction factor caused by exponentiation of the higher-order leading radiative corrections at $x_m \approx 1$ to the unpolarized and polarized parts of the cross section cancels in their ratios.

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