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RADIATIVE CORRECTIONS TO CHIRAL AMPLITUDES IN QUASIPERIPHERAL KINEMATICS

V. Bytev[a], *E. Bartoš*[b], *M. V. Galynskii*[c], *E. A. Kuraev*[a*]

[a] *Joint Institute for Nuclear Research*
141980, Dubna, Russia

[b] *Institute of Physics, Slovakia Academy of Sciences*
84511, Bratislava, Slovakia

[c] *Institute of Physics, Belarus Academy of Sciences*
220072, Minsk, Belarus

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Chiral amplitudes for two-jet processes in the quasiperipheral kinematics are calculated at the Born and one-loop correction levels. The amplitudes of subprocesses describing the interaction of a virtual and a real photon with creation of a charged fermion pair for various chiral states are considered in detail. Similar results are presented for the Compton subprocess with a virtual photon. Contributions of the emission of virtual, soft, and hard real additional photons are taken into account explicitly. The relevant cross sections expressed in terms of impact factors are in agreement with the structure-function approach in the leading logarithmic approximation. Contributions of the next-to-leading terms are presented in analytic form. Accuracy estimation is discussed.

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1. INTRODUCTION

Much attention was paid during the last decades (see [1] and the references therein) to different processes of the kind

$$a_1(p_1,\delta_1) + a_2(p_2,\delta_2) \to \mathrm{jet}_1^{(\lambda_1)} + \mathrm{jet}_2^{(\lambda_2)}, \qquad (1)$$

where

$$a_{1,2} = e^{\pm}, \gamma, \quad (p_1+p_2)^2 = s \gg m_i^2,$$

and $\delta_i(\lambda_i)$ describe the polarization states of the initial and jet particles. Below, we choose

$$\delta_1 = \delta_2 = +1$$

without a loss of generality (see Fig. 1). These processes can be studied at high-energy collisions of the initial particles in peripheral kinematics, i.e., small angles θ of the emission of jet particles to the direction of their parent particle (the center-of-mass (cms) frame of the initial particles is implied), see Fig. 2. A remarkable property of nondecreasing of the differential and total cross sections as functions of the cms total energy $\sqrt{s}$ in this kinematics is commonly known [2]. This property is a consequence of the presence of a massless vector particle (photon) in the scattering channel state. The contributions of Feynman diagrams with fermions and the interference of amplitudes of these types are suppressed compared with the photon exchange ones.

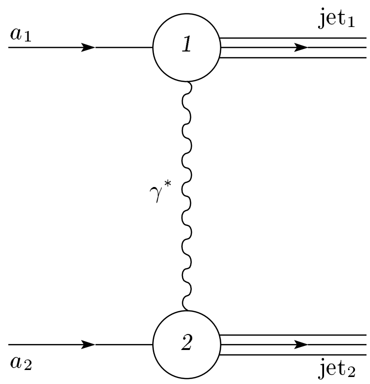

Fig. 1. General diagram for the process

Because the corresponding cross sections of the rele-

*E-mail: kuraev@theor.jinr.ru

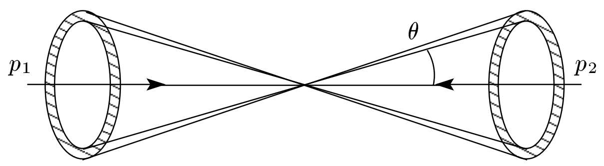

Fig. 2. Kinematics of a quasiperipheral process

vant QED processes are numerically large, they provide an essential background in the study of the effects of weak and strong interactions. In addition, these processes can be used for monitoring and calibration purposes.

Unfortunately, very small emission angles cannot be measured in practice. We therefore suggest considering processes (1) in the so-called quasiperipheral kinematics, which implies the values of emission angles to be small compared with unity but much larger than

$$\frac{m}{E} = \frac{2m}{\sqrt{s}},$$

where m is a characteristic mass of the jet constituents:

$$\begin{gathered} \frac{2m_i}{\sqrt{s}} \ll \theta_i \ll 1, \quad m_i^2 \ll -q^2 \ll s, \\ q = -p_1 + \sum_i p_{i1} = -\sum_i p_{i2} + p_2, \end{gathered} \tag{2}$$

with p_{i1}, p_{i2} being the 4-momenta of particles from $\text{jet}_{1,2}$ and q the momentum of the t-channel virtual photon. The quasiperipheral kinematics provides the independence from the energy of differential cross sections but has accuracy of the order of θ^2 — the order of contributions of neglected terms compared to those considered.

Another important property of the quasiperipheral kinematics is the independence of spin states from the $a_1 - \text{jet}_1$ and $a_2 - \text{jet}_2$ blocks of a process. We can therefore set $\delta_{1,2} = +1$. This fact can be seen by using Gribov's form of the Green's function of an exchanged photon with momentum q:

$$\frac{g_{\mu\nu}}{q^2} = \frac{1}{q^2}\left[g_{\mu\nu\perp} + \frac{2}{s}[p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu}]\right], \tag{3}$$

which results in the amplitude

$$\begin{aligned} M^{(12)} = -i\frac{4\pi\alpha}{q^2} g_{\mu\nu}(q) J^{1\mu}(q) J^{2\nu}(-q) = \\ = \frac{8\pi\alpha i s}{-q^2}\Phi_1\Phi_2, \end{aligned} \tag{4}$$

where $J^{1,2}$ are the currents associated with blocks *1* and *2* of the Feynman diagram (Fig. 1) and their light-cone projections (LP) are defined as

$$\Phi_1 = \frac{1}{s}J^{1,\mu}p_{2\mu}, \quad \Phi_2 = \frac{1}{s}J^{2,\nu}p_{1\nu}. \tag{5}$$

The LP factors Φ^i are independent of s in the limit as $s \to \infty$.

At this stage, we introduce Sudakov's parameterization of 4-momenta,

$$\begin{aligned} & p_{i1} = \alpha_i p_2 + x_i p_1 + p_{\perp i1}, \\ & \sum_i x_i = 1, \quad \alpha_i = \frac{\mathbf{p}_{i1}^2}{s x_i}, \\ & p_{j2} = y_j p_2 + \beta_j p_1 + p_{\perp j2}, \\ & \sum_i y_j = 1, \quad \beta_j = \frac{\mathbf{p}_{j2}^2}{s y_j}, \\ & q = \alpha p_2 + \beta p_1 + q_\perp, \quad q^2 \approx -\mathbf{q}^2, \quad p_{\perp j}^2 = -\mathbf{p}_j^2, \\ & \sum_i \mathbf{p}_{i1} = \mathbf{q}, \quad \sum_i \mathbf{p}_{i2} = -\mathbf{q}. \end{aligned} \tag{6}$$

Sudakov's longitudinal parameters α and β of the exchanged photon with momentum q are related to the jet invariant masses squared,

$$\begin{aligned} s_1 = (q + p_1)^2 \approx -\mathbf{q}^2 + s\alpha, \\ s_2 = (-q + p_2)^2 \approx -\mathbf{q}^2 - s\beta. \end{aligned} \tag{7}$$

Here, we use the on-shell condition

$$p_{i1}^2 = p_{j2}^2 = 0,$$

the conservation law, and introduce the Euclidean two-dimensional vectors

$$(p_{i\perp}p_{1,2}) = 0.$$

We note that the current conservation condition

$$J^{1\mu}q_\mu = J^{2\nu}q_\nu = 0$$

leads to

$$\frac{1}{s}p_2^\mu J_\mu^1 = -\frac{1}{s\alpha}q_\perp^\mu J^{1\mu}, \quad \frac{1}{s}p_1^\nu J_\nu^2 = -\frac{1}{s\beta}q_\perp^\mu J^{2\mu}. \tag{8}$$

We use the property that the matrix elements vanish at small $\mathbf{q}$ as an important check of the calculations (see (4)).

The differential cross section can be written in terms of the Cheng–Wu impact factors [3]:

$$\begin{gathered} d\sigma^{(12)} = \frac{\alpha^2}{\pi^2}\frac{d^2\mathbf{q}}{(\mathbf{q}^2)^2}\int d\tau_1^{(\lambda_1)}\int d\tau_2^{(\lambda_2)}, \\ \int d\tau_i^{(\lambda_i)} = \int |\Phi_i^{(\lambda_i)}(q)|^2 d\Gamma_i, \quad i = 1, 2, \end{gathered} \tag{9}$$

with

$$d\Gamma_1 = (2\pi)^4 \int ds_1 \delta\left(p_1 + q - \sum_i p_{i1}\right) \times \prod_i \frac{d^3 p_{i1}}{2\varepsilon_{i1}(2\pi)^3},$$
$$d\Gamma_2 = (2\pi)^4 \int ds_2 \delta\left(p_2 - q - \sum_i p_{i2}\right) \times \prod_i \frac{d^3 p_{i2}}{2\varepsilon_{i2}(2\pi)^3}. \quad (10)$$

The impact factors $\int d\tau_i$ are independent of s. In the cases where a jet consists of one, two or three particles, we have

$$\int d\Gamma_1^{(1)} = 2\pi,$$
$$d\Gamma_1^{(2)} = \frac{d^2\mathbf{p}_{11}dx_1}{2(2\pi)^2 x_1 x_2},$$
$$\mathbf{p}_{11} + \mathbf{p}_{21} = \mathbf{q}, \quad x_1 + x_2 = 1, \quad (11)$$
$$d\Gamma_1^{(3)} = \frac{d^2\mathbf{p}_{11}d^2\mathbf{p}_{21}dx_1 dx_2}{4(2\pi)^5 x_1 x_2 x_3},$$
$$x_1 + x_2 + x_3 = 1,$$
$$\mathbf{p}_{11} + \mathbf{p}_{21} + \mathbf{p}_{31} = \mathbf{q}.$$

For conversion of the initial photon with momentum p_1 and chirality λ to the charged fermion–antifermion pair,

$$\gamma^*(q) + \gamma(p_1, \lambda) \to e^+(q_+) + e^-(q_-, \sigma), \quad (12)$$

we accept the description of chiral states of the photon and lepton developed in [4]:

$$\hat{e}^\lambda(p_1) = N_\gamma(\hat{q}_-\hat{q}_+\hat{p}_1\omega_{-\lambda} - \hat{p}_1\hat{q}_-\hat{q}_+\omega_\lambda),$$
$$(e^\lambda)^2 = 0, \quad (e^\lambda e^{-\lambda}) = -1,$$
$$N_\gamma^2 = \frac{2}{s_1\chi_+\chi_-}, \quad s_1 = 2q_+q_-, \quad \chi_\pm = 2p_1q_\pm, \quad (13)$$
$$\omega_\sigma = \frac{1+\sigma\gamma_5}{2}, \quad \sigma = \pm 1.$$

Chiral states of fermions are defined as

$$u^\sigma = \omega_{-\sigma}u, \quad v^\sigma = \omega_\sigma v.$$

Hereafter, we imply that chiral states of subprocess (12) are defined as amplitudes with a definite chiral state $\Phi^{\lambda\sigma}$ of the initial photon (λ) and one electron (σ) from the pair. The LP factor of the photon $\Phi^{\lambda\sigma}$ in the Born approximation has the form

$$\Phi_B^{\gamma,++} = N_\gamma f_0 \bar{u}(q_-)\omega_-\hat{q}_+\hat{q}\frac{\hat{p}_2}{s}\omega_+ v(q_+), \quad (14)$$

$$\Phi_B^{\gamma,+-} = N_\gamma f_0 \bar{u}(q_-)\frac{\hat{p}_2}{s}\hat{q}\hat{q}_-\omega_- v(q_+),$$
$$f_0 = -i\sqrt{4\pi\alpha}. \quad (15)$$

We note that in the combination

$$\hat{p}_2\hat{q} = \hat{p}_2\hat{q}_\perp,$$

we can regard the 4-vector q as a two-dimensional vector $q_\mu = q_{\perp\mu}$.

The property of the LP factors

$$\Phi_{1,2}(\mathbf{q}) \to 0 \quad \text{as} \quad |\mathbf{q}| \to 0$$

is a consequence of gauge invariance, as we have noted above.

The relevant impact factors are

$$\int d\tau_B^{\gamma,+\pm} = \frac{\alpha}{\pi}\int \frac{d^2\mathbf{q}_- dx_-}{x_+x_-}\frac{\mathbf{q}^2 x_\pm^2}{\chi_+\chi_-},$$
$$\chi_\pm = \frac{\mathbf{q}_\pm^2}{x_\pm}, \quad x_+ + x_- = 1, \quad \mathbf{q}_+ + \mathbf{q}_- = \mathbf{q}. \quad (16)$$

The LP amplitudes $\Phi_B^{\gamma,-\pm}$ and the corresponding impact factors can be obtained by applying of the space reflection operator.

We consider the pair production process by a photon on an electron in the case of definite chiral states of all the particles,

$$\gamma(p_1, \lambda = +) + e^-(p_2, \eta) \to \to e^+(q_+, \mp) + e^-(q_-, \pm) + e^-(p_2', \eta). \quad (17)$$

Using the impact factor of a spectator electron in the lowest order of the perturbation theory with a single-particle jet $e^-(p_2, \eta) + \gamma^*(-q) \to e^-(p_2')$,

$$\Phi_2^\eta = \bar{u}(p_2')\frac{\hat{p}_1}{s}\omega_\eta u(p_2),$$
$$|\Phi_2^\eta|^2 = 1, \quad \int d\tau_2 = 2\pi, \quad (18)$$

we obtain the cross section of pair photoproduction on the electron with definite chiral states of the initial photon and positron from the pair (it is independent of the chiral state of the spectator)

$$\frac{d\sigma_{B,\eta}^{\gamma,++}}{d\Gamma} = \frac{d\sigma_{B,\eta}^{\gamma,--}}{d\Gamma} = \frac{2x_+^2\alpha^3}{\pi^2\mathbf{q}^2\chi_+\chi_-},$$
$$\frac{d\sigma_{B,\eta}^{\gamma,+-}}{d\Gamma} = \frac{d\sigma_{B,\eta}^{\gamma,-+}}{d\Gamma} = \frac{2x_-^2\alpha^3}{\pi^2\mathbf{q}^2\chi_+\chi_-}, \quad (19)$$
$$d\Gamma = \frac{d^2\mathbf{q}\, d^2\mathbf{q}_- dx_-}{x_+x_-}.$$

In considering the process of single photon emission at peripherical scattering of high-energy electrons, we also set

$$e^-(p_2, \eta) + e^-(p_1, \sigma = +) \to \to e^-(p_1', +) + \gamma(k_1, \lambda = \pm) + e^-(p_2', \eta), \quad (20)$$

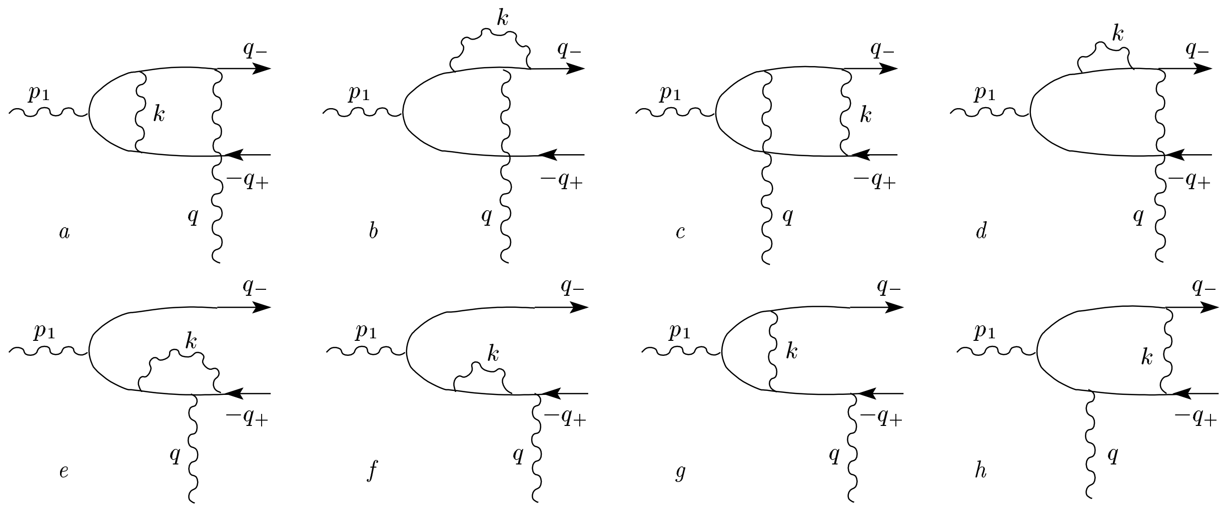

Fig. 3. Photon impact factor diagram

with definite chiral states

$$\hat{e}^{\lambda}(k_1) = N_1(\hat{p}_1'\hat{p}_1\hat{k}_1\omega_{-\lambda} - \hat{k}_1\hat{p}_1'\hat{p}_1\omega_{\lambda}),$$
$$N_1^2 = \frac{2}{u\chi\chi'}, \quad u = 2p_1p_1',$$
$$\chi = 2p_1k_1, \quad \chi' = 2p_1'k_1. \tag{21}$$

The LP factor of the electron $\Phi^{e,\sigma\lambda}$ is given by

$$\Phi_B^{e,++} = N_1 f_0 \bar{u}(p_1')\omega_-\hat{p}_1\hat{q}\frac{\hat{p}_2}{s}\omega_+ u(p_1)\,,$$
$$\Phi_B^{e,+-} = -N_1 f_0 \bar{u}(p_1')\frac{\hat{p}_2}{s}\hat{q}\hat{p}_1'\omega_+ u(p_1)\,. \tag{22}$$

The corresponding impact factors are

$$\int d\tau_B^{e,++} = \frac{\alpha}{\pi}\int \frac{d^2\mathbf{k}_1\,dx_1}{x_1x'}\frac{\mathbf{q}^2}{\chi\chi'},$$
$$\int d\tau_B^{e,+-} = \frac{\alpha}{\pi}\int \frac{d^2\mathbf{k}_1\,dx_1}{x_1x'}\frac{\mathbf{q}^2(x')^2}{\chi\chi'}\,, \tag{23}$$

where x_1 and x' are the energy fractions of the photon and the scattered electron from the jet. From the conservation law and the on-shell conditions, we have

$$x_1 + x' = 1, \quad \mathbf{p}_1{}' + \mathbf{k}_1 = \mathbf{q},$$
$$\chi = \frac{\mathbf{k}_1^2}{x_1}, \quad \chi' = \frac{1}{x_1x'}(x_1\mathbf{p} - x'\mathbf{k}_1)^2. \tag{24}$$

The differential cross sections have the form

$$\frac{d\sigma_{B,\eta}^{e,++}}{d\Gamma} = \frac{d\sigma_{B,\eta}^{e,--}}{d\Gamma} = \frac{2\alpha^3}{\mathbf{q}^2\pi^2\chi\chi'}\,,$$
$$\frac{d\sigma_{B,\eta}^{e,+-}}{d\Gamma} = \frac{d\sigma_{B,\eta}^{e,-+}}{d\Gamma} = \frac{2\alpha^3(x')^2}{\mathbf{q}^2\pi^2\chi\chi'},$$
$$d\Gamma = \frac{d^2\mathbf{k}_1\,dx_1 d^2\mathbf{q}}{x_1x'}\,. \tag{25}$$

This paper is organized as follows. In Sec. 2, we consider the virtual (in the one-loop approximation) and soft real photon emission contribution to the photon impact factor. In Sec. 3, similar calculations are presented for the electron impact factor. In Secs. 4 and 5, we consider the emission of an additional hard photon in collinear and noncollinear kinematics. Some general remarks are given in the Conclusions. In particular, we discuss the validity of the structure-function approach in the leading and next-to-leading approximations. The relevant one-loop integrals are listed in Appendix A. Appendix B contains explicit expressions for nonleading contributions arising from virtual and soft real photon emission. These nonleading contributions expressed in terms of a K-factor turn out to be quantities of the order of unity for typical experimental conditions.

2. PHOTON IMPACT FACTOR: VIRTUAL AND SOFT PHOTON CONTRIBUTIONS

We can divide all diagrams (see Fig. 3) into several types, some of which (Fig. 3*a,d* and *e,h*) can be obtained by simple exchanges of chiralities and 4-momenta of particles:

$$\mathrm{Re}[\Phi_{+,\,i}^{\gamma,+\pm}(\Phi_{Born}^{\gamma,+\pm})^*] =$$
$$= \mathrm{Re}[\Phi_{-,\,i}^{\gamma,+\mp}(\Phi_{Born}^{\gamma,+\mp})^*(q_- \to q_+,\, q_+ \to q_-)], \tag{26}$$

with $i = \Sigma V, V, B$ for the self-energy, vertex, and box-type Feynman diagram contribution, respectively. Here, the subscript describes the absorption of a virtual photon by an electron $(-)$ (Fig. 3*a–d*) or positron $(+)$ line (Fig. 3*e–h*).

One class of radiative corrections to the electron impact factor consists of the renormalized electron mass operator and the vertex function with only one off-shell electron or positron (see Figs. 3*a,d* and *f,g*). Its contribution can be written as [5]

$$\Phi^{\gamma,\lambda\sigma}_{-,\Sigma V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\bar u(q_-)\,\frac{\hat p_2}{s}\,\frac{\hat p_1-\hat q_+}{-\chi_+}\times \\ \times\left[2\left(\frac{3}{2}-l_l+\frac{1}{2}l_+\right)\hat e^\lambda + \right. \\ \left. +\int\frac{d^4k}{i\pi^2}\frac{\gamma^\mu(-\hat q_+ +\hat p_1-\hat k)\hat e^\lambda(-\hat q_+-\hat k)\gamma^\mu}{(0)(\bar 2)(q)}\right]\times \\ \times\,\omega^\sigma v(q_+), \\ \Phi^{\gamma,\lambda\sigma}_{+,\Sigma V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\bar u(q_-)\times \\ \times\left[2\left(\frac{3}{2}-l_l+\frac{1}{2}l_-\right)\hat e^\lambda + \right. \\ \left. +\int\frac{d^4k}{i\pi^2}\frac{\gamma^\mu(\hat q_- -\hat k)\hat e^\lambda(\hat q_- -\hat p_1-\hat k)\gamma^\mu}{(0)(2)(\bar q)}\right]\times \\ \times\,\frac{\hat q_- -\hat p_1}{-\chi_-}\frac{\hat p_2}{s}\omega^\sigma v(q_+), \\ l_l=\ln\frac{m^2}{\lambda^2},\quad l_\pm=\ln\frac{\chi_\pm}{m^2}, \tag{27}$$

with the denominators $(0),(2),(\bar 2),(q),(\bar q)$ defined below (see Appendix A). After integration, we obtain

$$\Phi^{\gamma,+-}_{-,\Sigma V} = \\ = -f_0N_\gamma\frac{\alpha}{2\pi}\left(l_+-\frac{1}{2}\right)\bar u(q_-)\frac{\hat p_2}{s}\hat p_1\hat q_-\omega_+v(q_+), \\ \Phi^{\gamma,++}_{+,\Sigma V} = \\ = f_0N_\gamma\frac{\alpha}{2\pi}\left(l_--\frac{1}{2}\right)\bar u(q_-)\hat q_+\hat p_1\frac{\hat p_2}{s}\omega_-v(q_+). \tag{28}$$

After multiplying these with the relevant Born amplitude, we obtain [6]

$$2\Phi^{\gamma,+\pm}_{\mp,\Sigma V}(\Phi^{\gamma,+\pm}_{Born})^* = \frac{8\alpha^2}{\chi_+\chi_-}x_\pm(\mathbf{q}\mathbf{q}_\pm)\left(l_\mp-\frac{1}{2}\right). \tag{29}$$

We note that the ΣV contribution does not satisfy the gauge condition (vanishing as $\mathbf{q}^2$ at small $\mathbf{q}$). But we see in what follows that the total sum does satisfy the gauge condition.

The contribution of the vertex functions with a virtual photon can be written as

$$\Phi^{\gamma,\lambda\sigma}_{-,V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\int\frac{d^4k}{i\pi^2}\frac{\bar u(q_-)\gamma_\mu(\hat q_- -\hat k)\frac{\hat p_2}{s}(\hat q_- -\hat q-\hat k)\gamma_\mu(\hat p_1-\hat q_+)\hat e^\lambda\omega^\sigma v(q_+)}{(0)(2)(q)(-\chi_+)}, \\ \Phi^{\gamma,\lambda\sigma}_{+,V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\int\frac{d^4k}{i\pi^2}\frac{\bar u(q_-)\hat e^\lambda(\hat q_- -\hat p_1)\gamma_\mu(-\hat q_+ +\hat q-\hat k)\frac{\hat p_2}{s}(-\hat q_+ -\hat k)\gamma_\mu\omega^\sigma v(q_+)}{(0)(\bar 2)(\bar q)(-\chi_-)}. \tag{30}$$

Using the list of integrals (see Appendix A for the notation), we obtain

$$2\Phi^{+-}_{-,V}(\Phi^{+-}_B)^* = -2|\Phi^{+-}_B|^2\frac{\alpha}{2\pi}\times \\ \times\left[-\frac{1}{2}L-\frac{1}{4}-\frac{\chi_++2\mathbf{q}^2}{2a}l_++\frac{3\mathbf{q}^2}{2a}l_q-\mathbf{q}^2J_{02q}\right], \\ 2\Phi^{++}_{+,V}(\Phi^{++}_B)^* = -2|\Phi^{++}_B|^2\frac{\alpha}{2\pi}\times \\ \times\left[-\frac{1}{2}L-\frac{1}{4}-\frac{\chi_-+2\mathbf{q}^2}{2\tilde a}l_-+\frac{3\mathbf{q}^2}{2\tilde a}l_q-\mathbf{q}^2J_{0\bar 2\bar q}\right], \\ a=\chi_+-\mathbf{q}^2,\quad \tilde a=\chi_--\mathbf{q}^2. \tag{31}$$

The other contributions

$$\Phi^{++}_{-,V}(\Phi^{++}_B)^*,\Phi^{+-}_{+,V}(\Phi^{+-}_B)^*,\Phi^{\gamma,+-}_{+,\Sigma V}(\Phi^{+-}_B)^*,$$

and

$$\Phi^{\gamma,++}_{-,\Sigma V}(\Phi^{++}_B)^*$$

are equal to zero.

We recall that we work in the framework of the unrenormalized field theory. The regularization procedure consists in replacing the ultraviolet cut-off logarithm

$$L=\ln\frac{\Lambda^2}{m^2}$$

as

$$L\to 2l_l-\frac{9}{2}$$

(see [5]).

The most complicated case is the calculation of the box-type contribution. It can be written as (see Fig. 3*c,h*,)

$$\Phi^{\gamma,\lambda\sigma}_{-,box} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\int\frac{d^4k}{i\pi^2}\frac{\bar u(q_-)\gamma^\mu(\hat q_- - \hat k)\frac{\hat p_2}{s}(\hat q_- - \hat q - \hat k)\hat e^\lambda(-\hat q_+ - \hat k)\gamma^\mu\omega^\sigma v(q_+)}{(0)(\bar 2)(2)(q)},$$
$$\Phi^{\gamma,\lambda\sigma}_{+,box} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2}\int\frac{d^4k}{i\pi^2}\frac{\bar u(q_-)\gamma^\mu(\hat q_- - \hat k)\hat e^\lambda(\hat q_- - \hat p_1 - \hat k)\frac{\hat p_2}{s}(-\hat q_+ - \hat k)\gamma^\mu\omega^\sigma v(q_+)}{(0)(\bar 2)(2)(\bar q)}. \quad (32)$$

All the details about loop calculations and relevant integrals can be found in Appendix A. It is worth mentioning that in the case of the box-type contribution, both chiral amplitudes $\lambda = +1$ and $\sigma = \pm 1$ are nonzero.

An additional real soft photon emission contribution to the LP factor has the standard form

$$\Phi^{\gamma,\lambda\sigma\eta}_{soft} = \Phi^{\gamma,\lambda\sigma}_{B}\sqrt{4\pi\alpha}\left(\frac{q_-}{q_-k} - \frac{q_+}{q_+k}\right)e(k)^\eta. \quad (33)$$

The corresponding contribution to the impact factor is

$$\int\frac{d^3k}{16\omega\pi^3}\sum_\eta|\Phi^{\gamma,\lambda\sigma\eta}_{soft}|^2|_{\omega<\Delta\varepsilon\ll\varepsilon_\gamma}, \quad (34)$$

where ε_γ is the energy of the initial electron in the cms frame. The result is

$$d\tau^{\gamma,\lambda\sigma}_{soft} = \frac{\alpha}{\pi}d\tau^{\gamma,\lambda\sigma}_{B}\left[(l_s-1)\left(l_l + \ln\frac{(\Delta)^2}{x_+x_-}\right) + \right.$$
$$\left. + \frac{1}{2}l_s^2 - \frac{1}{2}\ln^2\frac{x_+}{x_-} - \frac{\pi^2}{6}\right], \quad (35)$$

$$\Delta = \frac{\Delta\varepsilon}{\varepsilon_\gamma}, \quad l_s = \ln\frac{s_1}{m^2}.$$

We here use the smallness of the angle between 3-momenta of pair components in beams in the cms frame. The smallness of the emission angles allows performing the angular integration in (34) in the frame S_0 coinciding with the cms frame [7].

After summing all contributions (27), (30), and (32) and adding the soft photon contribution, we explicitly see the cancellation of an auxiliary parameter λ and the squared large logarithm:

$$2\left[d\tau^{\gamma,+\pm}_{\pm,box} + d\tau^{\gamma,+\pm}_{\mp,\Sigma V} + d\tau^{\gamma,+\pm}_{\mp,V}\right] + d\tau^{\gamma,+\pm}_{soft} = \frac{\alpha}{2\pi}\times$$
$$\times d\tau^{\gamma\pm}_{B}\left[(l_s-1)(4\ln\Delta+3-2\ln(x_+x_-))+K^{\gamma;\pm}_{SV}\right]. \quad (36)$$

We see that the leading-logarithm contribution (containing the factor $(l_s - 1)$) is proportional to the Born cross section, and therefore our calculation is in agreement with predictions of the structure-function approach that the leading-logarithm contribution is exactly the Δ part of the evolution equation kernel (see Sec. 6). All nonleading terms are gathered in the so-called K-factor.

Due to gauge invariance, the right-hand side of (36) including the K-factor tends to zero as $\mathbf{q}^2 \to 0$. This fact provides an important check of our calculation.

The $K^{\gamma;\pm}_{SV}$ factors are presented in the analytic form in Appendix B.

The contribution from the emission of a hard photon, which eliminates the Δ-dependence, can be written as a sum of two parts, one from collinear and the other from noncollinear kinematics. It is considered below.

3. ELECTRON IMPACT FACTOR: VIRTUAL AND SOFT PHOTON CONTRIBUTIONS

In the same way, we calculate the electron impact factor. All diagrams (see Fig. 4) are divided into six types, the contribution of three of them to LP (Fig. 4*c,d* and *f,h*) can be obtained by a simple exchange

$$\mathrm{Re}[\Phi^{e,+\mp}_{i,contr}(\Phi^{e,+\mp}_{Born})^*] =$$
$$= \mathrm{Re}[\Phi^{e,+\pm}_{f,contr}(\Phi^{e,+\pm}_{Born})^*(p_1\to -p_1', p_1'\to -p_1)], \quad (37)$$

where the subscripts correspond to the interaction of the virtual photon with the initial (i) or scattered (f) electron, and *contr* $= \Sigma V, V, box$ for the self-energy, vertex, and box-type Feynman diagram contributions, respectively.

We evaluate the contributions of the self-energy (Fig. 4*a,b*), vertex (Fig. 4*e*) and box-type (Fig. 4*g*) Feynman diagrams amplitudes as

$$\Phi^{e,+\lambda}_{i,\Sigma V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2(-\chi)} \bar u(p_1') \frac{\hat p_2}{s} (\hat p_1 - \hat k_1) \times$$
$$\times \left[\int \frac{d^4k}{i\pi^2} \frac{\gamma_\mu(\hat p_1 - \hat k_1 - \hat k)\hat e^\lambda(\hat p_1 - \hat k)\gamma_\mu}{(0)_e(1)_e(q)_e} + 2\left(\frac{1}{2}\ln\frac{\chi}{m^2} - 2\ln\frac{m}{\lambda} + \frac{3}{2}\right)\hat e^\lambda \right] \omega_+ u(p_1),$$
$$\Phi^{e,+\lambda}_{i,V} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2(-\chi)} \int \frac{d^4k}{i\pi^2} \frac{\bar u(p_1')\gamma_\mu(\hat p_1' - \hat k)\frac{\hat p_2}{s}(\hat p_1' - \hat k - \hat q)\gamma_\mu(\hat p_1 - \hat k_1) e^\lambda \omega_+ u(p_1)}{(0)_e(2)_e(q)_e}, \tag{38}$$
$$\Phi^{e,+\lambda}_{i,box} = \frac{(4\pi\alpha)^{3/2}}{16\pi^2} \int \frac{d^4k}{i\pi^2} \frac{\bar u(p_1')\gamma_\mu(\hat p_1' - \hat k)\frac{\hat p_2}{s}(\hat p_1' - \hat k - \hat q)\hat e^\lambda \gamma_\mu \omega_+ u(p_1)}{(0)_e(1)_e(2)_e(q)_e}.$$

The first two contribute (see the details in Appendix A):

$$2\Phi^{e,+-}_{i,V}(\Phi^{e,+-}_B)^* = -2|\Phi^{e,+-}_B|^2 \frac{\alpha}{2\pi} \times$$
$$\times \left[-\frac{1}{2}L - \frac{1}{4} - \mathbf{q}^2 I_{(0)_e(2)_e(q)_e} + \frac{3\mathbf{q}^2}{2d}\ln\frac{\mathbf{q}^2}{m^2} - \right.$$
$$\left. - \frac{\chi + 2\mathbf{q}^2}{2d}\ln\frac{\chi}{m^2} \right],$$
$$2\Phi^{e,+-}_{i,\Sigma V}(\Phi^{e,+-}_B)^* = \frac{8\alpha^2}{\chi\chi'}\left(\ln\frac{\chi}{m^2} - \frac{1}{2}\right) \times \tag{39}$$
$$\times x'[x'\mathbf{k}_1 - x\mathbf{p}_1{}']\mathbf{q},$$
$$d = \chi - \mathbf{q}^2, \quad l_{\chi'} = \ln\frac{\chi'}{m^2} - i\pi.$$

The contribution for the other polarization can be obtained by substitution (37).

The soft photon contribution has the standard form (the soft photon energy does not exceed $\Delta\varepsilon$)

$$d\tau^{e,+\pm}_{soft} = d\tau^{e,+\pm}_B \frac{\alpha}{\pi} \times$$
$$\times \left[\left(\ln\frac{u}{m^2} - 1\right)\left(2\ln\frac{m}{\lambda} + 2\ln\Delta - \ln x'\right) + \right.$$
$$\left. + \frac{1}{2}\ln^2\frac{u}{m^2} - \frac{1}{2}\ln^2 x' - \frac{\pi^2}{6} \right], \tag{40}$$

where $\Delta = \Delta\varepsilon/\varepsilon$, ε is the energy of the initial electrons in the cms frame. We can express the contribution to the electron impact factors with a definite chiral state as

$$2(d\tau^{e,+\pm}_{i,\Sigma V} + d\tau^{e,+\pm}_{i,V} + d\tau^{e,+\pm}_{i,box} + d\tau^{e,+\pm}_{f,box}) + d\tau^{+\pm}_{e,soft} =$$
$$= d\tau^{e,+\pm}_B \frac{\alpha}{2\pi} \times$$
$$\times \left[\left(\ln\frac{u}{m^2} - 1\right)(4\ln\Delta + 3 - 2\ln x') + K^{e,+\pm}_{SV} \right]. \tag{41}$$

We again see the cancellation of the auxiliary «photon mass» parameter λ and agreement with the prediction of the structure-function approach. The $K^{e,+\pm}_{SV}$ term is presented in the analytic form in Appendix B.

4. COLLINEAR KINEMATICS OF THE ADDITIONAL HARD PHOTON EMISSION CONTRIBUTION

For appropriate consideration of radiation corrections to impact factors, we have to consider an additional hard collinear photon emission. It is convenient to distinguish the collinear and noncollinear kinematics of the emission of a hard photon. For this, we introduce an auxiliary small parameter $\theta_0 \ll 1$. Collinear kinematics corresponds to the case where the photon emission angle θ to the direction of motion of some charged particle (initial or final) does not exceed θ_0. Noncollinear kinematics corresponds to large emission angles $\theta > \theta_0$. Chiral amplitudes in the noncollinear kinematics can be calculated using the methods developed by the CALCUL collaboration [4]. The contribution from collinear kinematics can be obtained using the quasi-real electron method developed in [8]. The total sum is independent of the parameter θ_0. Cancellation of the θ_0 dependence is a check of our calculations. The nonleading contributions from additional hard photon emission essentially depend on the experimental set-up. We do include it in the K-factors in the structure-function picture of impact factors.

Using the quasi-real electron method [8] for the contribution to the photon impact factor in collinear kinematics, we obtain

$$d\tau_{coll}^{\gamma,+\lambda} = \frac{\alpha}{2\pi} \int\limits_{x_-(1+\Delta)}^{1} \frac{dz_-}{z_-} \times \left[\frac{1+\tilde{x}_-^2}{1-\tilde{x}_-}(l_s + r_- + \ln\theta_0^2 - 1) + 1 - \tilde{x}_-\right] \times d\tau_B^{\gamma,+\lambda}\left(\frac{q_-}{z_-}, q_+\right) + \frac{\alpha}{2\pi}\int\limits_{x_+(1+\Delta)}^{1} \frac{dz_+}{z_+} \times \left[\frac{1+\tilde{x}_+^2}{1-\tilde{x}_+}(l_s + r_+ + \ln\theta_0^2 - 1) + 1 - \tilde{x}_+\right] \times d\tau_B^{\gamma,+\lambda}\left(q_-, \frac{q_+}{z_+}\right), \quad (42)$$

where the first term in square brackets corresponds to the emission of a hard photon along the electron and the second one along positron from the pair created. We use the notation

$$\tilde{x}_\pm = \frac{x_\pm}{z_\pm}$$

and

$$l_s = \ln\frac{2q_+q_-}{m^2} = \ln\frac{2E^2x_+x_-(1-c_\gamma)}{m^2}, \quad r_\pm = \ln\frac{x_\pm}{2x_\mp(1-c_\gamma)}, \quad (43)$$

where c_γ is the cosine of the angle between the pair momenta (the cms frame of colliding beams is implied). The «shifted» photon impact factor is given by

$$d\tau_B^{\gamma,+\pm}\left(\frac{q_+}{z_+}, \frac{q_-}{z_-}, x_\pm\right) = \frac{\alpha}{\pi}\frac{\tilde{x}_\pm^2\mathbf{q}^2}{\mathbf{q}_+^2\mathbf{q}_-^2}d^2q_-d\tilde{x}_-, \quad \tilde{x}_+ + \tilde{x}_- = 1, \quad \mathbf{q} = \frac{1}{z_-}\mathbf{q}_- + \frac{1}{z_+}\mathbf{q}_+, \quad (44)$$

and the conservation law is

$$p_1 + q = \frac{1}{z_-}q_- + \frac{1}{z_+}q_+.$$

A similar method can be applied to the problem of calculating the contribution from the collinear kinematics of photon emission for the impact factor of the electron. The result is

$$d\tau_{coll}^{e,+\lambda} = \frac{\alpha}{2\pi}\int\limits_0^{1-\Delta} dz_1 \times \left[\frac{1+z_1^2}{1-z_1}(l_u + l_1 + \ln\theta_0^2 - 1) + 1 - z_1\right] \times d\tau_B^{e,+\lambda}(p_1z_1, p_1') + \frac{\alpha}{2\pi}\int\limits_{x'(1+\Delta)}^{1}\frac{dz_2}{z_2} \times \left[\frac{1+(x'/z_2)^2}{1-x'/z_2}(l_u + l_2 + \ln\theta_0^2 - 1) + 1 - \frac{x'}{z_2}\right] \times d\tau_B^{e,+\lambda}\left(p_1, \frac{1}{z_2}p_1'\right), \quad (45)$$

where the first term in the square brackets describes the emission from the initial electron and the second one the emission from the scattered electron. We use the notation

$$l_u = \ln\frac{2p_1p_1'}{m^2} = \ln\frac{2E^2x'(1-c_e)}{m^2}, \quad l_1 = \ln\frac{z_1^2}{2x'(1-c_e)}, \quad l_2 = \ln\frac{x'}{2z_2^2(1-c_e)}, \quad (46)$$

and c_e is the cosine of the angle between the momenta of the initial and scattered electrons. The «shifted» electron impact factor in the Born approximation is given by

$$d\tau_B^{e,+\pm}\left(p_1z_1, \frac{1}{z_2}p_1'\right) = \frac{\alpha}{\pi}\frac{\mathbf{q}^2}{\chi\chi'}\eta^\pm\frac{z_2d^2kdx_1}{x_1x'}, \quad \eta^+ = z_1^2, \quad \eta^- = \left(\frac{x'}{z_2}\right)^2, \quad \chi = \frac{z_1}{x}\mathbf{k}_1^2, \quad \chi' = \frac{z_2\left(\mathbf{p}_1'x_1 - \mathbf{k}_1\frac{x'}{z_2}\right)^2}{x_1x'}, \quad x_1 + \frac{x'}{z_2} = 1, \quad \mathbf{q} = \mathbf{k}_1 + \frac{1}{z_2}\mathbf{p}_1', \quad (47)$$

and the conservation law is

$$z_1p_1 + q = \frac{1}{z_2}p_1' + k_1.$$

The terms containing «large» logarithms $l_s - 1$ and $l_u - 1$ are to be included in the lepton nonsinglet structure functions in the Drell–Yan form of impact factors, whereas the other terms contribute to the relevant K-factors. We can therefore rewrite these formulas in

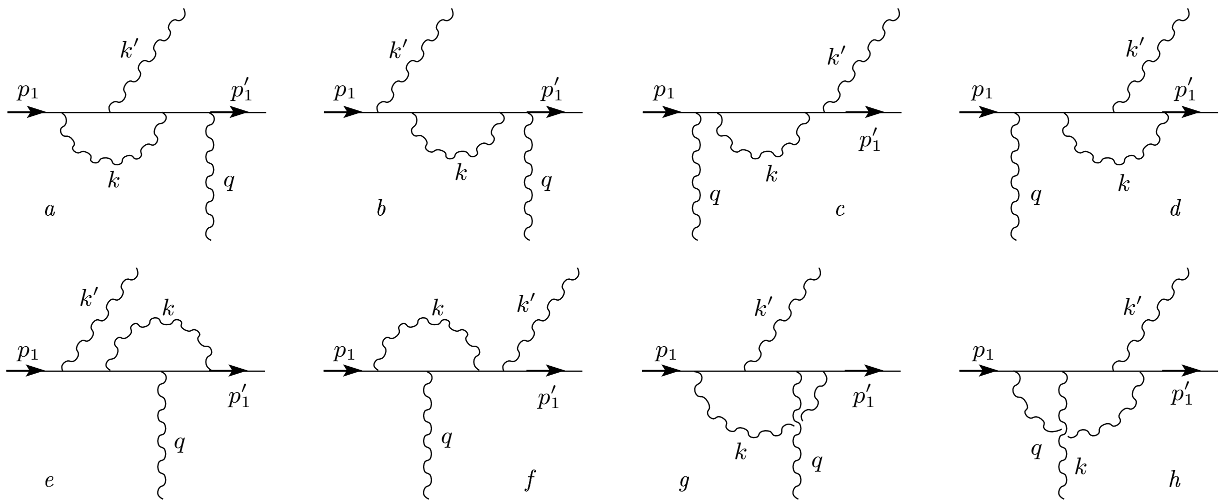

Fig. 4. Electron impact factor diagrams

terms of the structure-function approach for the electron impact factor as

$$d\tau^{e,coll} = \int\limits_0^1 dz_1 \left[P_\theta(z_1)\frac{\alpha}{2\pi}(l_u-1)+\frac{\alpha}{\pi}K^{i,e}_{coll}\right]\times$$
$$\times\, d\tau^{i,e}(z_1p_1,p'_1)+d\tau^{i,e}_{comp}+$$
$$+\int\limits_0^1\frac{dz_2}{z_2}\left[P_\theta\left(\frac{x'}{z_2}\right)\frac{\alpha}{2\pi}(l_u-1)+\frac{\alpha}{\pi}K^{f,e}_{coll}\right]\times$$
$$\times\, d\tau^{f,e}(p_1,\frac{p'_1}{z_2})+d\tau^{f,e}_{comp}, \quad (48)$$

(chiral indices are suppressed) and for the photon impact factor as

$$d\tau^{\gamma,coll} = \int\limits_0^1\frac{dz_-}{z_-}\left[P_\theta\left(\frac{x_-}{z_-}\right)\frac{\alpha}{2\pi}(l_s-1)+\frac{\alpha}{\pi}K^{-,\gamma}_{coll}\right]\times$$
$$\times\, d\tau\left(\frac{q_-}{z_-},q_+\right)+d\tau^{-,\gamma}_{comp}+$$
$$+\int\limits_0^1\frac{dz_+}{z_+}\left[P_\theta\left(\frac{x_+}{z_+}\right)\frac{\alpha}{2\pi}(l_s-1)+\frac{\alpha}{\pi}K^{+,\gamma}_{coll}\right]\times$$
$$\times\, d\tau\left(q_-,\frac{q_+}{z_+}\right)+d\tau^{+,\gamma}_{comp}, \quad (49)$$

where

$$P_\theta(z)=\frac{1+z^2}{1-z}\theta(1-z-\Delta), \quad (50)$$

and the nonleading contributions are given by

$$d\tau^{-,\gamma}_{comp}=\frac{\alpha}{2\pi}\int\limits_0^1\frac{dz_-}{z_-}P_\theta\left(\frac{x_-}{z_-}\right)\times$$
$$\times\, d\tau^\gamma_B\left(\frac{q_-}{z_-},q_+\right)\ln\theta_0^2,$$
$$K^{-,\gamma}_{coll}=\frac{1}{2}\int\limits_{x_-}^1\frac{dz_-}{z_-}P_\theta\left(\frac{x_-}{z_-}\right)(r_-+1-\tilde{x}_-)\times$$
$$\times\, d\tau^\gamma_B\left(\frac{q_-}{z_-},q_+\right),$$
$$d\tau^{+,\gamma}_{comp}=\frac{\alpha}{2\pi}\int\limits_0^1\frac{dz_+}{z_+}P_\theta\left(\frac{x_+}{z_+}\right)\times \quad (51)$$
$$\times\, d\tau^\gamma_B\left(q_-,\frac{q_+}{z_+}\right)\ln\theta_0^2,$$
$$K^{+,\gamma}_{coll}=\frac{1}{2}\int\limits_{x_+}^1\frac{dz_+}{z_+}P_\theta\left(\frac{x_+}{z_+}\right)(r_+-1-\tilde{x}_+)\times$$
$$\times\, d\tau^\gamma_B\left(q_-,\frac{q_+}{z_+}\right)$$

for the photon impact factor and

$$d\tau^{i,e}_{comp} = \frac{\alpha}{2\pi}\int\limits_0^1 dz_1 P_\theta(z_1)\, d\tau^e_B(z_1 p_1, p_1')\ln\theta_0^2,$$
$$K^{i,e}_{coll} = \frac{1}{2}\int\limits_0^1 dz_1 P_\theta(z_1)(l_1+1-z_1)\times$$
$$\times\, d\tau^e_B(z_1 p_1, p_1'),$$
$$d\tau^{f,e}_{comp} = \frac{\alpha}{2\pi}\int\limits_{x'}^1 \frac{dz_2}{z_2}P_\theta\left(\frac{x'}{z_2}\right)\times \quad (52)$$
$$\times\, d\tau^e_B\left(p_1, \frac{p_1'}{z_2}\right)\ln\theta_0^2,$$
$$K^{f,e}_{coll} = \frac{1}{2}\int\limits_{x'}^1 \frac{dz_2}{z_2}P_\theta\left(\frac{x'}{z_2}\right)\left(l_2+1-\frac{x'}{z_2}\right)\times$$
$$\times\, d\tau^e_B\left(p_1, \frac{p_1'}{z_2}\right)$$

for the electron impact factor. The terms with $\ln\theta_0^2$ are to be compensated by additional noncollinear photon emission terms.

5. NONCOLLINEAR HARD PHOTON EMISSION CONTRIBUTION

The contribution to the electron impact factor from the channel of the double Compton scattering process

$$e(p_1,\lambda_1)+\gamma^*(q)\to\gamma(k_1,\lambda_1)+$$
$$+\gamma(k_2,\lambda_2)+e(p_1',\lambda_e), \quad (53)$$
$$u=2p_1p_1', \quad \chi_i=2k_ip_1, \quad \chi_i'=2k_ip_1',$$

with the emission of both final electrons outside the narrow cone $\theta>\theta_0$, can be calculated using the chiral amplitude technique [4]. The result is

$$d\tau^{e\gamma\gamma}_{\lambda_e\lambda_1\lambda_2} =$$
$$= \frac{\alpha^3}{2\pi^2}|m^{\lambda_e}_{\lambda_1\lambda_2}|^2\,\frac{d^2k_1 d^2k_2 dx_1 dx_2}{x_1x_2x'}\bigg|_{\theta_{1,2}>\theta_0}, \quad (54)$$
$$x'=1-x_1-x_2, \quad \mathbf{q}=\mathbf{k}_1+\mathbf{k}_2+\mathbf{p}_1',$$

where

$$|m^+_{++}|^2 = \frac{4\mathbf{q}^2u}{\chi_1\chi_2\chi_1'\chi_2'},$$
$$|m^+_{--}|^2 = \frac{4(x')^2\mathbf{q}^2u}{\chi_1\chi_2\chi_1'\chi_2'}, \quad |m^+_{-+}|^2=|m^+_{+-}|^2, \quad (55)$$
$$|m^+_{+-}|^2 = \frac{4}{u^2\chi_1\chi_2\chi_1'\chi_2'}\mathrm{Tr}\,\hat p_1' B^+_{+-}\omega_+\hat p_1\tilde B^+_{+-},$$

with

$$B^+_{+-} = -\frac{1}{(p_1+q)^2}\hat p_1\hat k_1\hat p_1'\hat p_1\hat k_2(\hat p_1+\hat q)\frac{\hat p_2}{s} -$$
$$-\frac{1}{(p_1'-q)^2}\frac{\hat p_2}{s}(\hat p_1'-\hat q)\hat k_1\hat p_1'\hat p_1\hat k_2\hat p_1' +$$
$$+\hat p_1(\hat p_1'+\hat k_1)\frac{\hat p_2}{s}(\hat p_1-\hat k_2)\hat p_1'. \quad (56)$$

It was explicitly shown [9] that the quantity B^+_{+-} tends to zero as $|\mathbf{q}|\to 0$; this property is a consequence of the gauge invariance requirement for the virtual photon with momentum q.

Below, to check the θ_0-dependence cancellation for the sum of the collinear and noncollinear kinematics contributions, we evaluate the limit expressions for $|m^+_{ij}|^2$ for emission in the real photon kinematics

$$\theta_1>\theta_0, \quad \theta_1\to\theta_0, \quad \theta_2\gg\theta_0, \quad (57)$$

with θ_1 being the angle of the emission of a photon with momentum k_1 to the initial or final electron momentum. These limit values are

$$(|m^+_{+-}|^2+|m^+_{++}|^2)_{\chi_1\to 0} =$$
$$= \frac{4\mathbf{q}^2}{\chi_1}\,\frac{((x')^2+(1-x_1)^2)}{x_1(1-x_1)^2\chi_2\chi_2'},$$
$$(|m^+_{+-}|^2+|m^+_{++}|^2)_{\chi_1'\to 0} =$$
$$= \frac{4\mathbf{q}^2}{\chi_1'}\,\frac{x'}{x_1\chi_2\chi_2'}[1+(1-x_2)^2], \quad (58)$$
$$(|m^+_{-+}|^2+|m^+_{--}|^2)_{\chi_1\to 0} =$$
$$= \frac{4\mathbf{q}^2}{\chi_1}\,\frac{1}{x_1\chi_2\chi_2'}[(x')^2+(1-x_1)^2],$$
$$(|m^+_{-+}|^2+|m^+_{--}|^2)_{\chi_1'\to 0} =$$
$$= \frac{4\mathbf{q}^2}{\chi_1'}\,\frac{(x')^3}{x_1(1-x_2)^2\chi_2\chi_2'}[1+(1-x_2)^2].$$

At small emission angles, we can express all the invariants in terms of angular two-dimensional vectors in the plane transverse to the beams axis:

$$\mathbf{k}_1 = Ex_1\boldsymbol{\theta}_1, \quad \mathbf{p}_1' = Ex'\boldsymbol{\theta}',$$
$$\int\limits_{\theta_1>\theta_0}\frac{d^2k_1}{\chi_1} = \pi x_1\ln\frac{1}{\theta_0^2}+\ldots,$$
$$\int\frac{d^2k_1}{\chi_1'} = \frac{x_1}{x'}\int\limits_{|\boldsymbol{\theta}_1-\boldsymbol{\theta}'|>\theta_0}\frac{d^2\theta_1}{(\boldsymbol{\theta}_1-\boldsymbol{\theta}')^2} = \quad (59)$$
$$= \frac{\pi x_1}{x'}\ln\frac{1}{\theta_0^2}+\ldots$$

It can be explicitly verified that the θ_0-dependence is absent in the sum of collinear kinematics and the noncollinear contributions to the electron impact factor summed over the final-state hard photon chiral states:

$$d\tau^e_{hard,nc} = \sum_{\lambda_1,\lambda_2} (d\tau^{e\gamma\gamma}_{+\lambda_1\lambda_2} + d\tau^{i,e}_{comp} + d\tau^{f,e}_{comp}). \quad (60)$$

But this value depends essentially on the experimental photon detection set-up. Similar calculations of the photon impact factor in the noncollinear kinematics of a hard photon emission

$$\begin{gathered}\gamma(k,\lambda_\gamma) + \gamma^*(q) \to e^-(q_-,\lambda_-) + \\ + e^+(q_+,-\lambda_+) + \gamma(k_1,\lambda_1), \\ s_1 = 2q_-q_+, \quad \chi_\pm = 2kq_\pm, \quad \chi_{1\pm} = 2k_1q_\pm,\end{gathered} \quad (61)$$

with chiral amplitudes defined as $m^{\lambda_\gamma}_{\lambda_1\lambda_-}$, give

$$\begin{gathered}d\tau^{e^+e^-\gamma}_{\lambda_\gamma\lambda_1\lambda_-} = \frac{\alpha^3}{2\pi^2}|m^{\lambda_\gamma}_{\lambda_1\lambda_-}|^2 \frac{d^2q_- d^2q_+ dx_+ dx_-}{x_1x_+x_-}, \\ x_1 = 1 - x_+ - x_-, \quad \mathbf{q} = \mathbf{q}_- + \mathbf{q}_+ + \mathbf{k}_1,\end{gathered} \quad (62)$$

where

$$\begin{gathered}|m^+_{++}|^2 = \frac{4\mathbf{q}^2 s_1 x_+^2}{\chi_-\chi_{1-}\chi_+\chi_{1+}}, \\ |m^+_{+-}|^2 = \frac{4\mathbf{q}^2 s_1 x_-^2}{\chi_-\chi_{1-}\chi_+\chi_{1+}}, \\ |m^+_{--}(k,k_1)|^2 = |m^+_{-+}(-k_1,-k)|^2, \\ |m^+_{-+}|^2 = \frac{4}{s_1^2\chi_-\chi_{1-}\chi_+\chi_{1+}} \mathrm{Tr}\, \hat{q}_- A^+_{-+}\omega_+\hat{q}_+\tilde{A}^+_{-+}\end{gathered} \quad (63)$$

and

$$\begin{gathered}A^+_{-+} = \frac{s_1}{(q_+-q)^2}\hat{k}\hat{q}_+\hat{k}_1(-\hat{q}_+ + \hat{q})\frac{\hat{p}_2}{s} - \\ - \frac{s_1}{(q_--q)^2}\frac{\hat{p}_2}{s}(\hat{q}_- - \hat{q})\hat{k}\hat{q}_-\hat{k}_1 - \\ - \hat{q}_+(\hat{q}_- - \hat{k})\frac{\hat{p}_2}{s}(\hat{q}_+ + \hat{k}_1)\hat{q}_-.\end{gathered} \quad (64)$$

Again, the proportionality $A^+_{-+} \propto |\mathbf{q}|$ at small $|\mathbf{q}|$ was demonstrated in [9].

To check the cancellation of the θ_0-dependence, we evaluate the limit values of $|m^+_{\lambda_-\lambda_1}|^2$ in the limit of the emission angles close to the momentum directions of one of the charged particles,

$$\begin{gathered}(|m^+_{++}|^2 + |m^+_{+-}|^2)_{\chi_{1-}\to 0} = \\ = \frac{4\mathbf{q}^2}{\chi_{1-}} \frac{(x_+)^2 x_-}{x_1(1-x_+)^2\chi_+\chi_-}[x_-^2 + (1-x_+)^2], \\ (|m^+_{-+}|^2 + |m^+_{--}|^2)_{\chi_{1-}\to 0} = \\ = \frac{4\mathbf{q}^2}{\chi_{1-}} \frac{x_-}{x_1\chi_+\chi_-}[x_-^2 + (1-x_+)^2], \\ (|m^+_{-+}|^2 + |m^+_{--}|^2)_{\chi_{1+}\to 0} = \\ = \frac{4\mathbf{q}^2}{\chi_{1-}} \frac{x_+x_-}{x_1(1-x_-)^2\chi_+\chi_-}[x_+^2 + (1-x_-)^2], \\ (|m^+_{++}|^2 + |m^+_{+-}|^2)_{\chi_{1+}\to 0} = \\ = \frac{4\mathbf{q}^2}{\chi_{1-}} \frac{x_-}{x_1\chi_+\chi_-}[x_+^2 + (1-x_-)^2].\end{gathered} \quad (65)$$

It can be verified that the θ_0-dependence cancels in the sum of collinear kinematics and the noncollinear contributions to the photon impact factor summed over the hard photon chiral states:

$$d\tau^\gamma_{hard,nc} = d\tau^{e^+e^-\gamma}_{+\lambda,\lambda} + d\tau^{-,\gamma}_{comp} + d\tau^{+,\gamma}_{comp}. \quad (66)$$

The numerical value of $d\tau^\gamma_{hard,nc}$ also depends on the experimental set-up and is not considered here.

6. DISCUSSION AND CONCLUSIONS

We have obtained that the impact factors of both electron and photon in the leading logarithmic approximation can be written in the partonic form of the Drell – Yan process in terms of the structure functions for any chiral states of the initial and final particles

$$\begin{gathered}\left(d\tau_B + d\tau_{SV} + \sum d\tau_{hard}\right)^{\gamma,\,\lambda\sigma}(q_-,q_+) = \\ = \int\limits_{x_-}^{1}\frac{dz_-}{z_-}\int\limits_{x_+}^{1}\frac{dz_+}{z_+} D\left(\frac{x_-}{z_-},l_s\right) D\left(\frac{x_+}{z_+},l_s\right) \times \\ \times d\tau_B^{\gamma,\,\lambda\sigma}\left(\frac{q_-}{z_-},\frac{q_+}{z_+}\right) \times \\ \times \left(1 + \frac{\alpha}{\pi}[K^{\gamma,\,\lambda\sigma}_{SV} + K^{-,\gamma}_{coll} + K^{+,\gamma}_{coll} + K^\gamma_{ncoll}]\right),\end{gathered}$$

$$\left(d\tau_B + d\tau_{SV} + \sum d\tau_{hard}\right)^{e,\sigma\lambda}(p_1, p_1') =$$
$$= \int\limits_0^1 dz_1 \int\limits_{x'}^1 \frac{dz_2}{z_2} D(z_1, l_u) D\left(\frac{x'}{z_2}, l_u\right) \times$$
$$\times\, d\tau_B^{e,\sigma\lambda}\left(z_1 p_1, \frac{p_1'}{z_2}\right) \times$$
$$\times \left(1 + \frac{\alpha}{\pi}[K_{SV}^{e,\sigma\lambda} + K_{coll}^{i,e} + K_{coll}^{f,e} + K_{ncoll}^{e}]\right).$$

Here, the chirality indices are suppressed and D is the nonsinglet structure function of a fermion [6]:

$$\begin{aligned} D(z,l) &= \delta(z-1) + \frac{\alpha}{2\pi}(l-1)P^{(1)}(z) + \ldots, \\ P^{(1)}(z) &= \left(\delta(1-z)\left(2\ln\Delta + \frac{3}{2}\right) + \right. \\ &\quad \left. +\theta(1-z-\Delta)\frac{1+z^2}{1-z}\right)_{\Delta\to 0}. \end{aligned} \tag{67}$$

The explicit form of K_{SV} is given in Appendix B for definite chiral states. The explicit form of K_{coll} is given above (see (51) and (52)). The form of K_{ncoll} (after a proper regularization compensating the divergent terms in the limit as $\theta_0 \to 0$) strongly depends on the details of experiment tagging the additional hard photon. The nonleading terms are free from infrared and collinear divergences (are independent of λ, Δ, and θ_0).

The terms containing the vector product arise due to a nonzero imaginary part of the LP amplitudes.

We can be convinced in the validity of the gauge-invariance check: the squares of chiral amplitudes in the Born approximation and the one-loop-corrected ones tend to zero as $|\mathbf{q}|^2$ at small $|\mathbf{q}|$.

The electron impact factor also has contributions from pair production channels [9], which are not considered here.

The accuracy of the formulas given above is determined by the omitted terms (2):

$$1 + \mathcal{O}\left(\frac{m^2}{s_1^2}, \frac{m^2}{u^2}, \left(\frac{\alpha}{\pi}\right)^2\right). \tag{68}$$

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APPENDIX A

We evaluate asymptotic expressions for a part of the scalar, vector, and tensor integrals corresponding to the absorption of a virtual photon by the electron from the pair created in $\gamma(p_1)\gamma^*(q)$ collisions.

We first give the scalar integrals with two, three, and four (different) denominators

$$\begin{aligned} (0) &= k^2 - \lambda^2, \\ (2) &= (q_- - k)^2 - m^2 + i0 = k^2 - 2q_- k + i0, \\ (\bar{2}) &= (-q_+ - k)^2 - m^2 + i0, \\ (q) &= (p_1 - q_+ - k)^2 - m^2 + i0. \end{aligned} \tag{69}$$

The loop momentum integrals with the denominator

$$(\bar{q}) = (q_- - p_1 - k)^2 - m^2$$

instead of

$$(q) = (p_1 - q_+ - k)^2 - m^2,$$

including scalar, vector, and tensor ones can be obtained from those listed below by replacement (26):

$$\begin{aligned} &q_- \to -q_+, \quad q_+ \to -q_-, \quad p_1 \to -p_1, \\ &\chi_\pm \to \chi_\mp, \quad (2) \to (\bar{2}), \quad (q) \to (\bar{q}). \end{aligned} \tag{70}$$

We can therefore restrict ourselves to considering only the integrals with denominators (0), (2), $(\bar{2})$, and (q).

In this Appendix, we use the same conservation law, on-shell conditions, and kinematic invariants as for the photon impact factor in (12) and (13),

$$s_1 + \mathbf{q}^2 = \chi_+ + \chi_-. \tag{71}$$

Two denominator scalar integrals are defined as

$$I_{ij} = \int \frac{d^4k}{i\pi^2}\frac{1}{(i)(j)}.$$

The explicit expressions for them are

$$\begin{aligned} &I_{02} = L+1, \quad I_{2q} = L - l_q + 1, \quad I_{0q} = L - l_+ + 1, \\ &I_{0\bar{2}} = L+1, \quad I_{2\bar{2}} = L - L_s + 1, \quad I_{\bar{2}q} = L - 1. \end{aligned} \tag{72}$$

Here and below, we use the notation

$$\begin{aligned} &L = \ln\frac{\Lambda^2}{m^2}, \quad l_\pm = \ln\frac{\chi_\pm}{m^2}, \quad l_q = \ln\frac{\mathbf{q}^2}{m^2}, \\ &L_s = \ln\frac{s_1}{m^2} - i\pi = l_s - i\pi, \quad l_l = \ln\frac{m^2}{\lambda^2}, \\ &\mathrm{Li}_2(z) = -\int\limits_0^z \frac{dx}{x}\ln(1-x). \end{aligned} \tag{73}$$

We recall once more that we assume all the kinematic invariants to be greater than the electron mass squared,

$$s_1 \sim \mathbf{q}^2 \sim \chi_\pm \gg m^2$$

and systematically omit the terms of the order of m^2/s_1 and similar ones in presenting the asymptotic expressions.

The scalar integrals with three denominators

$$I_{ijk} = \int \frac{d^4k}{i\pi^2(i)(j)(k)}$$

are given by

$$\begin{aligned}
I_{0\bar{2}q} &= -\frac{1}{2\chi_+}\left[l_+^2 + \frac{2\pi^2}{3}\right],\\
I_{02\bar{2}} &= \frac{1}{2s_1}\left[l_s^2 + 2l_s l_l - \frac{4\pi^2}{3} - i\pi(2l_s + 2l_l)\right],\\
I_{2\bar{2}q} &= -\frac{1}{2(s_1+\mathbf{q}^2)}\left[l_q^2 - l_s^2 + \pi^2 + 2i\pi l_s\right], \qquad (74)\\
I_{02q} &= \frac{1}{\chi_+ - \mathbf{q}^2}\times\\
&\times\left[l_q(l_q - l_+) + \frac{1}{2}(l_q - l_+)^2 + 2\mathrm{Li}_2\left(1 - \frac{\chi_+}{\mathbf{q}^2}\right)\right].
\end{aligned}$$

The integral

$$I_{02\bar{2}q} = \int \frac{d^4k}{i\pi^2(0)(2)(\bar{2})(q)}$$

with four denominators is given by

$$\begin{aligned}
I_{02\bar{2}q} = \frac{1}{s_1\chi_+}\Bigg[l_q^2 - 2l_+l_s - l_s l_l + 2\mathrm{Li}_2\left(1 + \frac{\mathbf{q}^2}{s_1}\right) +\\
+ \frac{\pi^2}{6} + i\pi\left(2l_+ + l_l - 2\ln(1 + \frac{\mathbf{q}^2}{s_1})\right)\Bigg]. \qquad (75)
\end{aligned}$$

We now describe the vector integrals

$$I_r^\mu = \int \frac{d^4k k^\mu}{r} = a_r^+ q_+^\mu + a_r^- q_-^\mu + a_r^1 p_1^\mu \qquad (76)$$

with

$$r = (ij), (ijk), (ijkl), \quad i,j,k,l = (0), (2), (\bar{2}), (q).$$

For the vector integrals with two denominators, we have (indicating only nonzero coefficients)

$$\begin{aligned}
a_{2q}^- &= a_{2q}^1 = -a_{2q}^+ = \frac{1}{2}\left(L - l_q + \frac{1}{2}\right),\\
a_{0q}^1 &= -a_{0q}^+ = \frac{1}{2}\left(L - l_+ + \frac{1}{2}\right),\\
a_{2\bar{2}}^- &= -a_{2\bar{2}}^+ = \frac{1}{2}\left(L - L_s + \frac{1}{2}\right), \qquad (77)\\
a_{\bar{2}q}^1 &= -\frac{1}{2}a_{\bar{2}q}^+ = \frac{1}{2}\left(L - \frac{3}{2}\right),\\
a_{02}^- &= \frac{1}{2}L - \frac{1}{4}, \quad a_{0\bar{2}}^+ = -\frac{1}{2}L + \frac{1}{4}
\end{aligned}$$

and the coefficients for the vector integrals with three denominators are

$$\begin{aligned}
a_{02q}^- &= \frac{1}{a}\left(\chi_+ I_{02q} + \frac{2\chi_+}{a}l_+ - \frac{\mathbf{q}^2 + \chi_+}{a}l_q\right),\\
a_{02q}^+ &= -a_{02q}^1 = \frac{1}{a}(l_+ - l_q),\\
a_{0\bar{2}q}^1 &= \frac{1}{\chi_+}(-l_+ + 2), \quad a = \chi_+ - \mathbf{q}^2,\\
a_{0\bar{2}q}^+ &= -I_{0\bar{2}q} - \frac{1}{\chi_+}l_+, \quad a_{02\bar{2}}^- = -a_{02\bar{2}}^+ = \frac{1}{s_1}L_s, \qquad (78)\\
a_{2\bar{2}q}^- &= \frac{1}{c}(L_s - l_q), \quad a_{2\bar{2}q}^+ = -I_{2\bar{2}q} + \frac{1}{c}(L_s - l_q),\\
a_{2\bar{2}q}^1 &= \frac{s_1}{c}I_{2\bar{2}q} + \frac{1}{c}(-l_q + 2) - \frac{2s_1}{c^2}(L_s - l_q),\\
c &= s_1 + \mathbf{q}^2 = \chi_+ + \chi_-.
\end{aligned}$$

Finally, the coefficients of the vector integrals with four denominators are given by

$$\begin{aligned}
a^1 &= \frac{s_1}{d}(\chi_+ A + \chi_- B - s_1 C),\\
a^+ &= \frac{\chi_-}{d}(\chi_+ A - \chi_- B + s_1 C),\\
a^- &= \frac{\chi_+}{d}(-\chi_+ A + \chi_- B + s_1 C), \qquad (79)\\
d &= 2s_1\chi_+\chi_-,\\
A &= I_{2\bar{2}q} - I_{0\bar{2}q}, \quad B = I_{02q} - I_{2\bar{2}q},\\
C &= I_{02q} - I_{02\bar{2}} - \chi_+ I_{02\bar{2}q}.
\end{aligned}$$

We parameterize the second-rank tensor integrals as

$$\begin{aligned}
I_r^{\mu\nu} = \int \frac{d^4k}{i\pi^2}\frac{k_\mu k_\nu}{r} = \Big[a_r^g g + a_r^{11} p_1 p_1 + a_r^{++} q_+ q_+ +\\
+ a_r^{--} q_- q_- + a_r^{1+}(p_1 q_+ + q_+ p_1) +\\
+ a_r^{1-}(p_1 q_- + q_- p_1) + a_r^{+-}(q_+ q_- + q_- q_+)\Big]_{\mu\nu}. \qquad (80)
\end{aligned}$$

The coefficients for the tensor integral with four denominators are (we suppressed the index $02\bar{2}q$)

$$\begin{aligned}
a^{1+} &= \frac{1}{\chi_+}(A_6 + A_7 - A_{10}),\\
a^{+-} &= \frac{1}{s_1}(A_2 + A_6 - A_{10}),\\
a^{1-} &= \frac{1}{\chi_-}(A_2 + A_7 - A_{10}),\\
a^{11} &= \frac{1}{\chi_-}\left(A_1 - s_1 a^{1+}\right), \qquad (81)\\
a^{--} &= \frac{1}{s_1}\left(A_5 - \chi_+ a^{1-}\right),\\
a^{++} &= \frac{1}{s_1}\left(A_3 - \chi_- a^{1+}\right),\\
a^g &= \frac{1}{2}\left(A_{10} - A_2 - \chi_+ a^{1+}\right),
\end{aligned}$$

with

$$A_1 = a^1_{2\bar{2}q} - a^1_{0\bar{2}q},\quad A_6 = a^+_{02q} - a^+_{2\bar{2}q},$$
$$A_2 = a^-_{2\bar{2}q},\quad A_7 = a^1_{02q} - a^1_{02\bar{2}} - \chi_+ a^1,$$
$$A_3 = a^+_{2\bar{2}q} - a^+_{0\bar{2}q},\quad A_8 = a^-_{02q} - a^-_{02\bar{2}} - \chi_+ a^-, \tag{82}$$
$$A_4 = a^1_{02q} - a^1_{2\bar{2}q},\quad A_9 = a^+_{02q} - a^+_{02\bar{2}} - \chi_+ a^+,$$
$$A_5 = a^-_{02q} - a^-_{2\bar{2}q},\quad A_{10} = I_{2\bar{2}q}.$$

It can be verified that these coefficients satisfy the relations

$$A_4 = \chi_+ a^{11} + s_1 a^{1-},\quad A_8 = \chi_- a^{--} + \chi_+ a^{+-},\quad A_9 = \chi_+ a^{++} + \chi_- a^{+-} \tag{83}$$

that we use to check the calculation.

The coefficients entering the tensor integral $I^{\mu\nu}_{02q}$ have the form

$$a^g_{02q} = \frac{1}{4}L + \frac{3}{8} + \frac{\mathbf{q}^2}{4a} l_q - \frac{\chi_+}{a} l_+,$$
$$a^{+-}_{02q} = -a^{1-}_{02q} = \frac{1}{2a}\left[\frac{\chi_+}{a}(l_+ - l_q) - 1\right],$$
$$a^{++}_{02q} = a^{11}_{02q} = -a^{1+}_{02q} = \frac{1}{2a}(l_q - l_+), \tag{84}$$
$$a^{--}_{02q} = \frac{1}{a^2}\left[\chi_+^2 I_{02q} + \frac{3\chi_+^2}{a} l_+ + \frac{(\mathbf{q}^2)^2 - 4\mathbf{q}^2\chi_+ - 3\chi_+^2}{2a} l_q + \frac{\mathbf{q}^2 - 3\chi_+}{2}\right].$$

The coefficients entering the tensor integral $I^{\mu\nu}_{02\bar{2}}$ are given by

$$a^g_{02\bar{2}} = \frac{1}{4}(L - L_s) + \frac{3}{8},$$
$$a^{++}_{02\bar{2}} = a^{--}_{02\bar{2}} = \frac{1}{2s_1}(L_s - 1), \tag{85}$$
$$a^{+-}_{02\bar{2}} = -\frac{1}{2s_1}.$$

The coefficients for the tensor integral $I^{\mu\nu}_{0\bar{2}q}$ are given by

$$a^g_{0\bar{2}q} = \frac{1}{4}(L - l_+) + \frac{3}{8},\quad a^{1+}_{0\bar{2}q} = \frac{1}{\chi_+}\left(l_+ - \frac{5}{2}\right),$$
$$a^{11}_{0\bar{2}q} = \frac{1}{2\chi_+}(-l_+ + 2), \tag{86}$$
$$a^{++}_{0\bar{2}q} = I_{0\bar{2}q} + \frac{1}{2\chi_+}(3l_+ - 1).$$

For the tensor integral $I^{\mu\nu}_{2\bar{2}q}$, these coefficients are

$$a^g_{2\bar{2}q} = \frac{1}{2}\left[\frac{1}{2}L + \frac{3}{4} - \frac{s_1}{2c}L_s - \frac{\mathbf{q}^2}{2c} l_q\right],$$
$$a^{--}_{2\bar{2}q} = -\frac{1}{2c}(l_q - L_s),$$
$$a^{++}_{2\bar{2}q} = I_{2\bar{2}q} + \frac{3}{2c}(l_q - L_s),\quad a^{+-}_{2\bar{2}q} = \frac{1}{2c}(l_q - L_s),$$
$$a^{1-}_{2\bar{2}q} = \frac{1}{c}\left[-\frac{1}{2} + \frac{s_1}{2c}L_s - \frac{s_1}{2c} l_q\right], \tag{87}$$
$$a^{1+}_{2\bar{2}q} = \frac{1}{c}\left[-\frac{5}{2} - s_1 I_{2\bar{2}q} + \frac{5s_1}{2c}L_s + \frac{2\mathbf{q}^2 - 3s_1}{2c} l_q\right],$$
$$a^{11}_{2\bar{2}q} = \frac{1}{c^2}\left[4s_1 + \mathbf{q}^2 + s_1^2 I_{2\bar{2}q} - \frac{3s_1^2}{c}L_s + \frac{3s_1^2 - (\mathbf{q}^2)^2 - 4s_1\mathbf{q}^2}{2c} l_q\right].$$

The check-up equations for coefficients (87) can be obtained after multiplying $I^{\mu\nu}_{2\bar{2}q}$ by $2(q_+ + q_-)^\nu$ or $2p_1^\nu$, using the relations

$$2k(q_+ + q_-) = (\bar{2}) - (2),\quad 2p_1 k = (\bar{2}) - (q) - \chi_+$$

and using vector integrals (76). They are given by

$$2a^g_{2\bar{2}q} + s_1 a^{--}_{2\bar{2}q} + c a^{1-}_{2\bar{2}q} + s_1 a^{+-}_{2\bar{2}q} = a^-_{2q} - a^-_{\bar{2}q},$$
$$2a^g_{2\bar{2}q} + s_1 a^{++}_{2\bar{2}q} + c a^{1+}_{2\bar{2}q} + s_1 a^{+-} = a^+_{2q} - a^+_{\bar{2}q}, \tag{88}$$
$$c a^{11}_{2\bar{2}q} + s_1 a^{1+}_{2\bar{2}q} + s_1 a^{1-}_{2\bar{2}q} = a^1_{2q} - a^1_{\bar{2}q}.$$

The integrals for calculation of the electron impact factor with the denominators

$$(0)_e = k^2 - \lambda^2,$$
$$(1)_e = (p_1 - k)^2 - m^2 + i0,$$
$$(2)_e = (p_1' - k)^2 - m^2 + i0, \tag{89}$$
$$(q)_e = (p_1 - k_1 - k)^2 - m^2 + i0$$

can be obtained from those given above by the substitution

$$\int \frac{d^4k}{i\pi^2}\frac{1,\,k,\,kk}{(0\,1\,2\,q)_e} = \mathcal{P}(q_- \to p_1', q_+ \to -p_1, p_1 \to -k_1, q \to q) \times \int \frac{d^4k}{i\pi^2}\frac{1,\,k,\,kk}{(0\,2\,\bar{2}\,q)}. \tag{90}$$

An additional set of relevant integrals for the electron impact factor can be obtained by substitution (37).

APPENDIX B

The explicit expressions for the photon K-factor are (in the case of two different polarizations)

$$\frac{1}{2}K_{SV}^{\gamma+-} = -\frac{1}{2}\ln^2\frac{x_+}{x_-} - \frac{3}{4}l_{qs}^2 + l_{qs}l_{ps} + \frac{1}{2}l_{qs}l_{ms} + \frac{1}{4}l_{ms} + \frac{3}{4}l_{qs} + \frac{1}{2}l_{qs} - \frac{3}{4} +$$
$$+ \frac{3}{2}\mathrm{Li}_2\left(1+\frac{\mathbf{q}^2}{s_1}\right) - \mathrm{Li}_2\left(1-\frac{\chi_+}{\mathbf{q}^2}\right) - \frac{1}{2}\mathrm{Li}_2\left(1-\frac{\chi_-}{\mathbf{q}^2}\right) + x_+\frac{\chi_-x_+ - 2\chi_+ - 2s_1}{4x_-^2 s_1} +$$
$$+ \left(\frac{\chi_-x_+}{\chi_+x_-} - \frac{(x_+\chi_- - s_1)^2}{2x_-^2\chi_+^2}\right)\left(\frac{1}{2}l_{qs}^2 - l_{qs}l_{ms} + \frac{\pi^2}{3} - \mathrm{Li}_2\left(1+\frac{\mathbf{q}^2}{s_1}\right) + \mathrm{Li}_2\left(1-\frac{\chi_-}{\mathbf{q}^2}\right)\right) -$$
$$- x_+\frac{l_{ms}-l_{qs}}{x_-^2}\left(\frac{a^2x_+ - 2s_1x_-\chi_-}{4\tilde{a}^2} + \frac{4\chi_- - s_1 - 2x_+\chi_-}{2\tilde{a}} - \frac{x_+\chi_-^2}{2\tilde{a}\chi_+} + \frac{2x_+\chi_- - s_1}{2\chi_+x_+}\right) +$$
$$+ \frac{\pi}{2x_-^2\chi_+}[\mathbf{q}_+\mathbf{q}_-]_z\left[\frac{l_{ms} - \ln\left(1+\dfrac{\mathbf{q}^2}{s_1}\right)}{\chi_+}(s_1 - x_+\chi_- + x_-\chi_+) + \frac{s_1 + 3x_-\chi_+ - x_+\chi_-}{c}\right] +$$
$$+ \frac{l_{qs}}{2x_-^2}\left(\frac{\chi_+^2 + s_1^2 + 2x_+s_1\chi_+}{c^2} - \frac{2x_-s_1 + \chi_+}{c} + \frac{s_1^2}{\chi_+c} + x_+\frac{-2s_1 + x_+\chi_-}{\chi_+}\right) +$$
$$+ \frac{\chi_+^2 + s_1^2 + 2x_+s_1\chi_+}{2x_-^2s_1c} + \frac{\chi_+^2}{2s_1\mathbf{q}^2x_-^2}\left(\frac{1}{2}x_+^2 - 1\right)\frac{\chi_+x_+}{4\tilde{a}s_1\mathbf{q}^2x_-^2}\left(x_+\chi_+^2 + \mathbf{q}^2(x_+\chi_- - 2\chi_+)\right) +$$
$$+ \frac{1}{\mathbf{q}^2x_-}\left(\frac{1}{4}\chi_- - \frac{\mathbf{q}^2}{2} + \chi_+x_- - \frac{1}{2}s_1x_- - \frac{1}{4}x_+^2s_1 + \frac{1}{4}x_+^2\chi_+\right), \quad (91)$$

$$\frac{1}{2}K_{SV}^{\gamma++} = -\frac{1}{2}\ln^2\frac{x_+}{x_-} - \frac{3}{4}l_{qs}^2 + l_{qs}l_{ms} + \frac{1}{2}l_{qs}l_{ps} + \frac{1}{4}(l_{ps}+l_{qs}) - \mathrm{Li}_2\left(1-\frac{\chi_-}{\mathbf{q}^2}\right) +$$
$$+ \frac{3}{2}\mathrm{Li}_2\left(1+\frac{\mathbf{q}^2}{s_1}\right) - \frac{1}{2}\mathrm{Li}_2\left(1-\frac{\chi_+}{\mathbf{q}^2}\right) - 1 +$$
$$+ \frac{2x_-x_+\chi_-\chi_+ - (x_-\chi_+ - s_1)^2}{2\chi_-^2x_+^2}\left(\frac{1}{2}l_{qs}^2 - l_{qs}l_{ps} - \mathrm{Li}_2\left(1+\frac{\mathbf{q}^2}{s_1}\right) + \mathrm{Li}_2\left(1-\frac{\chi_+}{\mathbf{q}^2}\right) + \frac{\pi^2}{3}\right) -$$
$$- \frac{l_{ps}-l_{qs}}{x_+^2}\left[\frac{x_-(-2s_1\chi_+x_+ + x_-\tilde{a}^2)}{4a^2} + \frac{x_-(4x_+s_1\chi_+ - x_+s_1^2 - x_-\tilde{a}^2)}{2as_1} - \frac{(s_1 - x_-\chi_+)^2}{2\chi_-s_1}\right] +$$
$$+ \frac{l_{qs}}{x_+^2}\left[\frac{2x_+s_1\chi_+ + \tilde{a}^2}{2c^2} + \frac{s_1^2}{2c\chi_-} - \frac{x_+\mathbf{q}^2 + s_1x_- + \chi_-/2}{c} - \frac{x_-}{\chi_-}\left(s_1 - \frac{1}{2}x_-\chi_+\right) - \frac{1}{2} - x_+x_-\right] +$$
$$+ \pi\frac{[\mathbf{q}_+\mathbf{q}_-]_z}{2x_+^2}\left[\frac{x_+\chi_- + s_1 - x_-\chi_+}{\chi_-^2}\left(l_{ps} - \ln\left(1+\frac{\mathbf{q}^2}{s_1}\right)\right) + \frac{2x_+ + 1}{c} - \frac{x_-}{\chi_-} + \frac{s_1}{\chi_-c}\right] -$$
$$- \frac{1}{x_+^2}\left(x_-\frac{2s_1 - x_-\mathbf{q}^2}{\tilde{a}} + \frac{x_+x_-}{2} - \frac{2x_+s_1\chi_+ + (\chi_+ - s_1)^2}{2cs_1}\right) +$$
$$+ \frac{1}{\mathbf{q}^2x_+^2}\left(\frac{x_-^2s_1^2}{4a} + \frac{\mathbf{q}^2}{4} + \frac{5}{4}\chi_+ + \frac{3}{2}s_1x_+ - s_1 - x_+\chi_+ - \frac{1}{2}x_+^2s_1\right) - \frac{\chi_+^2}{2s_1x_+^2\mathbf{q}^2}, \quad (92)$$

$$a=\chi_+-\mathbf{q}^2,\quad \tilde a=\chi_--\mathbf{q}^2,\quad c=\chi_++\chi_-,$$
$$l_{qs}=\ln\frac{\mathbf{q}^2}{s_1},\quad l_{ps}=\ln\frac{\chi_+}{s_1},\quad l_{ms}=\ln\frac{\chi_-}{s_1}. \tag{93}$$

The explicit expression for the electron K-factor is

$$\begin{aligned}K_{SV}^{e,+-}&=-\ln^2 x'-\frac{3}{2}\ln^2\frac{\mathbf{q}^2}{u}+2\ln\frac{\chi}{u}\ln\frac{\mathbf{q}^2}{u}+\ln\frac{\chi'}{u}\ln\frac{\mathbf{q}^2}{u}+\frac{1}{2}\ln\frac{\chi'}{\mathbf{q}^2}+3\ln\frac{\mathbf{q}^2}{u}-\\
&-2\mathrm{Li}_2\left(1-\frac{\chi}{\mathbf{q}^2}\right)+3\mathrm{Li}_2\left(1-\frac{\mathbf{q}^2}{u}\right)-\mathrm{Li}_2\left(1+\frac{\chi'}{\mathbf{q}^2}\right)-\frac{\pi^2}{4}-2-\\
&-\frac{1}{(x')^2(\chi')^2}\left(\frac{1}{2}\ln^2\frac{\mathbf{q}^2}{u}-\ln\frac{\mathbf{q}^2}{u}\ln\frac{\chi'}{u}-\mathrm{Li}_2\left(1-\frac{\mathbf{q}^2}{u}\right)+\mathrm{Li}_2\left(1+\frac{\chi'}{\mathbf{q}^2}\right)+\frac{\pi^2}{12}\right)\times\\
&\times\left(d^2-2x'(-u\chi'+u^2+\chi\chi')+(x')^2u^2\right)-\\
&-\frac{1}{2(x')^2\mathbf{q}^2}\left(-u+\chi+2x'\chi-(x')^2u-3(x')^2\chi+\frac{u^2}{\tilde d}\right)+\frac{(\chi')^2x^2}{(x')^2\mathbf{q}^2u}\left(1+\frac{\mathbf{q}^2}{c}\right)-\\
&-\frac{1}{(x')^2}\left(\frac{x(-u+2\chi+ux')}{c}+\frac{1}{2}-x'-\frac{\chi'-2u+2x'u}{2\tilde d}\right)-\\
&-\frac{1}{(x')^2}\ln\frac{\chi'}{\mathbf{q}^2}\left(\frac{x'(2d+x'u)}{\chi'}-\frac{x'(4\chi'-u)}{\tilde d}-\frac{d^2(-2u-3\chi)}{2\tilde d^2\chi}+\frac{x'u\chi'}{\tilde d^2}\right)-\\
&-\frac{1}{(x')^2}\ln\frac{\mathbf{q}^2}{u}\left(\frac{u\,x}{\chi(\chi')^2}-\frac{x((\chi-u)^2x-2\chi u\,x')}{c^2}-\frac{2u\,x\,x'}{c}+\frac{(u^2-\chi^2)x^2}{c\chi}\right)+\\
&+4\pi\frac{[\mathbf{p}_1',\mathbf{k}_1]_z}{(x')^2}\left(\ln\frac{\mathbf{q}^2}{u}-\ln\frac{\chi'}{\mathbf{q}^2}\right)\frac{\chi'-u-x'(\chi-u)}{\chi^2}+2\pi\frac{[\mathbf{p}_1',\mathbf{k}_1]_z}{(x')^2}\left(\frac{d(-2u-3\chi)}{\tilde d^2\chi}+x'\frac{-2u+\chi}{\chi\tilde d}\right),\end{aligned} \tag{94}$$

where

$$d=\chi-\mathbf{q}^2,\quad \tilde d=u+\chi,\quad c=u-\mathbf{q}^2. \tag{95}$$

For $K_{SV}^{e,++}$, we have

$$\begin{aligned}K_{SV}^{e,++}&=-\ln^2 x'-\frac{3}{2}\ln^2\frac{\mathbf{q}^2}{u}+2\ln\frac{\mathbf{q}^2}{u}\ln\frac{\chi'}{u}\ln\frac{\mathbf{q}^2}{u}+\ln\frac{\chi}{u}+\frac{1}{2}\ln\frac{\chi}{u}+\frac{5}{2}\ln\frac{\mathbf{q}^2}{u}-\\
&-2\mathrm{Li}_2\left(1+\frac{\chi'}{\mathbf{q}^2}\right)+3\mathrm{Li}_2\left(1-\frac{\mathbf{q}^2}{u}\right)-\mathrm{Li}_2\left(1-\frac{\chi}{\mathbf{q}^2}\right)-\frac{\pi^2}{4}-\frac{3}{2}-\\
&-\frac{1}{(\chi')^2}\left(-\ln^2\frac{\mathbf{q}^2}{u}+\ln\frac{\chi}{u}\ln\frac{\mathbf{q}^2}{u}-\frac{\pi^2}{12}+\frac{(\chi')^2}{c^2}\ln\frac{\mathbf{q}^2}{u}+\left(\ln\frac{\chi}{u}-\ln\frac{\mathbf{q}^2}{u}+\frac{\chi'}{c}\right)\frac{\chi'}{u}+\right.\\
&\left.+\mathrm{Li}_2\left(1-\frac{\mathbf{q}^2}{u}\right)-\mathrm{Li}_2\left(1-\frac{\chi}{\mathbf{q}^2}\right)\right)\left(-u^2x^2+2x'\chi\tilde d+(x')^2\chi(-2u-\chi)\right)+\\
&+\frac{\chi^2(1-2x')}{u\,c}\left(1+\frac{u}{c}\ln\frac{\mathbf{q}^2}{u}\right)-\frac{2x'\chi}{u}\left(-\ln\frac{\chi}{u}-\frac{\chi'}{c}+\ln\frac{\mathbf{q}^2}{u}\left(1-\frac{u\chi'}{c^2}\right)\right)-\\
&-\frac{x'}{d}\left(u\,x-\frac{1}{2}x'\chi\right)+\frac{x'}{u\,d}\ln\frac{\chi}{\mathbf{q}^2}\left(u^2+4\chi u-x'\tilde d^2-\frac{u(-2\chi u+x'\tilde d^2)}{2d}\right)-\\
&-\ln\frac{\mathbf{q}^2}{u}\left(x'\frac{-2ux+x'\chi}{\chi'}+\frac{2u(1-(x')^2)-\chi x^2}{c}-\frac{u^2x^2}{c\chi'}\right)-\\
&-\frac{1}{\mathbf{q}^2}\left(-\frac{1}{2}\chi+x'u+3x'\chi-2(x')^2u-\frac{5}{2}(x')^2\chi-\frac{d\,x^2u^2}{2}\right)+\frac{x^2\chi^2}{u\mathbf{q}^2}.\end{aligned} \tag{96}$$

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