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RADIATIVE CORRECTIONS TO POLARIZATION OBSERVABLES IN ELASTIC ELECTRON–DEUTERON SCATTERING IN LEPTONIC VARIABLES

G. I. Gakh, M. I. Konchatnij, N. P. Merenkov***

Kharkov Institute of Physics and Technology
61108, Kharkov, Ukraine

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The model-independent QED radiative corrections to polarization observables in elastic scattering of unpolarized and longitudinally polarized electron beams by a deuteron target are calculated in leptonic variables. The experimental setup when the deuteron target is arbitrarily polarized is considered and the procedure for applying the derived results to the vector or tensor polarization of the recoil deuteron is discussed. The calculation is based on taking all essential Feynman diagrams into account, which results in the form of the Drell–Yan representation for the cross section, and the use of the covariant parameterization of the deuteron polarization state. Numerical estimates of the radiative corrections are given in the case where event selection allows undetected particles (photons and electron–positron pairs) and the restriction on the lost invariant mass is used.

1. INTRODUCTION

The process of elastic electron–deuteron scattering has long been a reaction used for investigating the electromagnetic structure of the deuteron. These investigations, both theoretical and experimental, can help clarify a number of important problems: the properties of the nucleon–nucleon interaction, non-nucleonic degrees of freedom in nuclei (such as the meson exchange currents, the isobar configurations), and the importance of relativistic effects (see, e. g., the recent reviews on the deuteron [1–4]).

The electromagnetic structure of the deuteron as a bound two-nucleon system with spin one is completely determined by three functions of one variable, the four-momentum transfer squared Q^2. These are the so-called electromagnetic form factors of the deuteron: G_C (the charge monopole), G_M (the magnetic dipole) and G_Q (the charge quadrupole). They are real functions in the space-like region of the four-momentum transfer squared (the scattering channel, for example, in elastic electron–deuteron scattering) and complex functions in the time-like region (the annihilation channel, for example, $e^+ + e^- \to D + \bar{D}$). Hence, the main experimental problem is to determine the electromagnetic deuteron form factors with a high accuracy and in a wide range of Q^2. A recent review of past and future measurements of the elastic electromagnetic deuteron form factors is given in Ref. [5].

We also note that the deuteron is used as an effective neutron target in studies of the neutron electromagnetic form factors [6] and the spin structure functions of the neutron in deep-inelastic scattering [7].

When addressing the electromagnetic properties of the deuteron more specifically, the corresponding question concerns the ability to predict the three deuteron form factors starting from the calculated deuteron wave function and nucleon form factors known from electron–nucleon scattering. At low momentum transfers, predictions and data agree quite well when accounting for one-body terms only, and at the higher momentum transfers, two-body contributions are known to be important. Whether the quark degrees of freedom need to be allowed for is still a matter of debate. We note that each deuteron form factor may be sensitive to some specific contribution. For example, the deuteron charge form factor G_C is particularly interesting for the understanding of the role of the meson exchange currents. Therefore, it is necessary to separate the three deuteron form factors. Measurements of the unpolarized cross section yield the structure functions $A(Q^2)$ and $B(Q^2)$: they can be separately determined by vary-

*E-mail: konchatnij@kipt.kharkov.ua
**E-mail: merenkov@kipt.kharkov.ua

ing the scattered electron angle θ_e for a given squared momentum Q^2 transfered to the deuteron. Hence, all three form factors can be separated when either the tensor analyzing power T_{20} or the recoil deuteron polarization t_{20} is also measured (the electron beam is unpolarized in both cases). This has prompted the development of both polarized deuterium targets for use with internal or external beams and polarimeters for measuring the polarization of recoil hadrons [8]. Both types of experiment result in the same combinations of form factors.

Two techniques are basically available to measure such spin observables.

i) At storage rings, polarized internal deuteron gas targets from an atomic beam source can be used [9–13]. The high intensity of the circulating electron beam allows achieving acceptable luminosities despite the very low thickness of the gas target.

ii) At facilities with external beams, polarimeters can be used to measure the polarization of recoil deuterons [14–16]. High beam intensities are a prerequisite as the polarization measurement, which requires a second reaction of the deuteron, involves a loss of a few orders of magnitude in count rate.

Current experiments at modern accelerators reached a new level of precision, and this requires a new approach to data analysis and inclusion of all possible systematic uncertainties. An important source of such uncertainties is the electromagnetic radiative effects caused by physical processes that occur in higher orders of the perturbation theory with respect to the electromagnetic interaction.

While radiative corrections have been taken into account for the unpolarized cross section, the radiative corrections for polarization observables in the elastic electron–deuteron scattering at large momentum transfer are not known at present [17]. For example, in the experiment on precise measurement of the deuteron elastic structure function $A(Q^2)$ (at $Q = 0.66–1.8$ GeV), the radiative corrections (about 20 %) due to losses in the radiative tail were calculated as in [18]. On the other hand, the authors of recent experiments [12, 13, 16] on measuring the polarization observables did not present the evidence about taking radiative corrections into account.

The importance of taking radiative corrections into account can be seen in the example of the discrepancy between the Rosenbluth [19] and the polarization transfer methods [20] for determination of the ratio of the electric to magnetic proton form factors. For a given value of Q^2, it suffices to measure the unpolarized elastic electron–nucleon scattering cross section for two values of ε (the virtual photon polarization parameter) to determine the G_{M_p} and G_{E_p} form factors (the Rosenbluth method). But the measurement of polarization observables in this reaction (using the longitudinally polarized electron beam) allows determining the ratio G_{M_p} to G_{E_p} [20]. Two experimental set ups were used: measurement of the asymmetry on the polarized target and measurement of the recoil-proton polarization (the polarization transfer method). Recent experiments show that the ratios G_{Ep}/G_{Mp} extracted using the Rosenbluth and polarization transfer methods are incompatible at large Q^2 [21, 22]. This discrepancy is a serious problem as it generates confusion and doubt about the whole methodology of lepton scattering experiments [23]. One plausible explanation of this problem is given by two-photon exchange effects [24]. The data are consistent with simple estimates of the two-photon contributions to explain the discrepancy (see, e. g., [25] and the references therein).

The precise calculation of radiative corrections is also important for the study of the two-photon exchange effects in elastic electron–deuteron scattering. It was observed in [26–29] that the relative role of the two-photon exchange can increase significantly in the region of large Q^2 due to a steep decrease of the deuteron form factors as functions of Q^2. Because one- and two-photon amplitudes have very different spin structures, the polarization phenomena have to be more sensitive to the interference effects than the differential cross section (with unpolarized particles).

An attempt to evaluate the presence of two intermediate hard photons in box diagrams using the existing data on the elastic electron–deuteron scattering was done in Ref. [30]. The authors searched for a deviation from the linear dependence of the cross section on $\mathrm{ctg}^2(\theta_e/2)$ using a Rosenbluth fit, with the cross section parameterized in a model-independent way according to crossing symmetry considerations.

The two-photon contribution to the structure functions and polarization observables in the elastic scattering of longitudinally polarized electrons on polarized deuterons was recently calculated in Ref. [31] (the references to earlier papers can be found there).

The radiative corrections to deep-inelastic scattering of unpolarized and longitudinally polarized electron beams on a polarized deuteron target were considered in Ref. [32] in the particular case of deuteron polarization (which can be obtained from the general covariant spin-density matrix [33] when spin functions are eigenvectors of the spin projection operator). The leading-log model-independent radiative corrections in deep-inelastic scattering of an unpolarized electron beam on a tensor-polarized deuteron target have been considered in Ref. [34]. The calculation was based on the covariant parameterization of the deuteron quadrupole polarization tensor and the use of the Drell–Yan like representation in electrodynamics. The model-independent QED radiative corrections to the polarization observables in the elastic scattering of unpolarized and longitudinally polarized electron beams by a polarized deuteron target were calculated in the hadronic variables in Ref. [35].

In this paper, we calculate the model-independent QED radiative corrections in leptonic variables to the polarization observables in the elastic scattering of unpolarized and longitudinally polarized electron beams by a deuteron target

$$e^-(k_1) + D(p_1) \to e^-(k_2) + D(p_2), \qquad (1)$$

where the four-momenta of the corresponding particles are indicated in the brackets. The experimental setup with an arbitrarily polarized deuteron target is considered and the procedure for applying the derived results to the vector or tensor polarization of the recoil deuteron is discussed. The basis of the calculations consists of the account for all essential Feynman diagrams which results in the form of the Drell–Yan representation for the cross section and use of the covariant parameterization of the deuteron polarization state. The numerical estimates of the radiative corrections are given for the case when event selection allows the undetected particles (photons and electron-positron pairs) and the restriction on the lost invariant mass is used.

2. THE BORN APPROXIMATION

From the theoretical standpoint, different polarization observables in the process of elastic electron–deuteron scattering have been investigated in many papers (see, e. g., Refs. [36–41]). The polarization observables were expressed in terms of the deuteron electromagnetic form factors. An up-to-date status of the experimental and theoretical research into the deuteron structure can be found in reviews [2, 4]. Here, we reproduce most of these results using the method of covariant parameterization of the deuteron polarization state in terms of the particle four-momenta and demonstrate the advantage of this approach.

We consider the process of elastic scattering of a polarized electron beam by a polarized deuteron target. In the one-photon-exchange approximation, we define the cross section of process (1) in terms of the contraction of the leptonic $L_{\mu\nu}$ and hadronic $H_{\mu\nu}$ tensors (we neglect the electron mass wherever possible),

$$d\sigma = \frac{\alpha^2}{2Vq^4} L^B_{\mu\nu} H_{\mu\nu} \frac{d^3k_2}{\varepsilon_2}\frac{d^3p_2}{E_2}\delta(k_1+p_1-k_2-p_2), \qquad (2)$$

where $V = 2k_1p_1, \varepsilon_2$ and E_2 are the respective energies of the scattered electron and recoil deuteron and $q = k_1 - k_2 = p_2 - p_1$ is the four-momentum of the heavy virtual photon that probes the deuteron structure. In the case of a longitudinally polarized electron beam, the leptonic tensor, in the Born approximation, is given by

$$L^B_{\mu\nu} = q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu}) + 2iP_e(\mu\nu q k_1), \qquad (3)$$
$$(\mu\nu ab) = \varepsilon_{\mu\nu\lambda\rho}a_\lambda b_\rho, \quad \varepsilon_{1230} = 1,$$

where P_e is the degree of the electron beam polarization (in what follows, we assume that the electron beam is completely polarized and hence $P_e = 1$).

The hadronic tensor can be expressed in terms of the deuteron electromagnetic current J_μ describing the transition $\gamma^* D \to D$ as

$$H_{\mu\nu} = J_\mu J^*_\nu. \qquad (4)$$

Using the requirements of Lorentz invariance, current conservation, and parity and time-reversal invariances of the hadron electromagnetic interaction, the general form of the electromagnetic current for the spin-one deuteron can be completely described by three form factors and can be written as [20]

$$J_\mu = (p_1+p_2)_\mu \Big[-G_1(Q^2) U_1\cdot U_2^* + \frac{G_3(Q^2)}{M^2}\Big(U_1\cdot q U_2^*\cdot q - \frac{q^2}{2}U_1\cdot U_2^*\Big)\Big] + G_2(Q^2)(U_{1\mu}U_2^*\cdot q - U^*_{2\mu}U_1\cdot q), \qquad (5)$$

where $U_{1\mu}$ and $U_{2\mu}$ are the polarization four-vectors for the initial and final deuteron states, and M is the deuteron mass. The functions $G_i(Q^2)$ $(i = 1, 2, 3)$ are the deuteron electromagnetic form factors depending only on the virtual photon four-momentum squared. Because the current is Hermitian, the form factors $G_i(Q^2)$ are real functions in the region of space-like momentum transfer. We here use the convention $Q^2 = -q^2$.

These form factors can be related to the standard deuteron form factors G_C (the charge monopole), G_M (the magnetic dipole), and G_Q (the charge quadrupole) as

$$G_M = -G_2, \quad G_Q = G_1 + G_2 + 2G_3,$$
$$G_C = \frac{2}{3}\eta(G_2-G_3) + \Big(1+\frac{2}{3}\eta\Big)G_1, \quad \eta = \frac{Q^2}{4M^2}. \qquad (6)$$

The standard form factors have the normalizations

$$G_C(0)=1, \quad G_M(0)=(M/m_n)\mu_d,$$
$$G_Q(0)=M^2 Q_d,$$

where m_n is the nucleon mass and $\mu_d(Q_d)$ is deuteron magnetic (quadrupole) moment, and their values are $\mu_d = 0.857$ [42] and $Q_d = 0.2859 fm^2$ [43].

If we write the electromagnetic current in the form

$$J_\mu = J_{\mu\alpha\beta} U_{1\alpha} U^*_{2\beta},$$

then the $H_{\mu\nu}$ tensor can be written as

$$H_{\mu\nu} = J_{\mu\alpha\beta} J^*_{\nu\sigma\gamma} \rho^i_{\alpha\sigma} \rho^f_{\gamma\beta}, \tag{7}$$

where $\rho^i_{\alpha\sigma}$ ($\rho^f_{\gamma\beta}$) is the spin-density matrix of the initial (final) deuteron.

Because we consider the case of a polarized deuteron target and an unpolarized recoil deuteron, the hadronic tensor $H_{\mu\nu}$ can be expanded according to the polarization state of the initial deuteron as

$$H_{\mu\nu} = H_{\mu\nu}(0) + H_{\mu\nu}(V) + H_{\mu\nu}(T), \tag{8}$$

where the spin-independent tensor $H_{\mu\nu}(0)$ corresponds to an unpolarized initial deuteron and the spin-dependent tensor $H_{\mu\nu}(V)$ ($H_{\mu\nu}(T)$) describes the case where the deuteron target has a vector (tensor) polarization.

We consider the general case of the initial deuteron polarization state described by the spin-density matrix. We use the general expression for the deuteron spin-density matrix in the coordinate representation [44]

$$\rho^i_{\alpha\beta} = -\frac{1}{3}\left(g_{\alpha\beta} - \frac{p_{1\alpha}p_{1\beta}}{M^2}\right) + \frac{i}{2M}(\alpha\beta s p_1) + Q_{\alpha\beta}, \tag{9}$$

where s_μ is the polarization four-vector describing the vector polarization of the deuteron target,

$$p_1 \cdot s = 0, \quad s^2 = -1,$$

and $Q_{\mu\nu}$ is the tensor describing the tensor (quadrupole) polarization of the initial deuteron,

$$Q_{\mu\nu} = Q_{\nu\mu}, \quad Q_{\mu\mu} = 0, \quad p_{1\mu} Q_{\mu\nu} = 0.$$

In the laboratory system (the initial deuteron rest frame), all time components of $Q_{\mu\nu}$ are zero and the tensor polarization of the deuteron target is described by five independent spatial components,

$$Q_{ij} = Q_{ji}, \quad Q_{ii} = 0, \quad i,j = x,y,z.$$

In Appendix B, we give the relation between elements of the deuteron spin-density matrix in the helicity and spherical tensor representations and also in the coordinate representation. We also give the relation between the polarization parameters s_i and Q_{ij} and the population numbers n_+, n_-, and n_0 describing the polarized deuteron target, which is often used in spin experiments.

We assume that the polarization of the recoil deuteron is not measured. Therefore, its spin-density matrix can be written as

$$\rho^f_{\alpha\beta} = -\left(g_{\alpha\beta} - \frac{p_{2\alpha}p_{2\beta}}{M^2}\right).$$

The spin-independent tensor $H_{\mu\nu}(0)$ describes unpolarized initial and final deuterons and has the general form

$$H_{\mu\nu}(0) = -W_1(Q^2)\tilde{g}_{\mu\nu} + \frac{W_2(Q^2)}{M^2}\tilde{p}_{1\mu}\tilde{p}_{1\nu}, \tag{10}$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad \tilde{p}_{1\mu} = p_{1\mu} - \frac{p_1 \cdot q}{q^2} q_\mu.$$

Two real structure functions $W_{1,2}(Q^2)$ are expressed in terms of the deuteron electromagnetic form factors as

$$\begin{aligned} W_1(Q^2) &= \frac{2}{3}Q^2(1+\eta)G_M^2, \\ W_2(Q^2) &= 4M^2\left(G_C^2 + \frac{2}{3}\eta G_M^2 + \frac{8}{9}\eta^2 G_Q^2\right). \end{aligned} \tag{11}$$

In the considered case, the spin-dependent tensor $H_{\mu\nu}(V)$, that describes the vector-polarized initial deuteron and the unpolarized final deuteron can be written as

$$H_{\mu\nu}(V) = \frac{i}{M}S_1(\mu\nu s q) + \frac{i}{M^3}S_2[\tilde{p}_{1\mu}(\nu s q p_1) - $$
$$-\tilde{p}_{1\nu}(\mu s q p_1)] + \frac{1}{M^3}S_3[\tilde{p}_{1\mu}(\nu s q p_1) + \tilde{p}_{1\nu}(\mu s q p_1)], \tag{12}$$

where three real structure functions $S_i(Q^2)$, $i=1,2,3$, can be expressed in terms of the deuteron electromagnetic form factors. They are

$$\begin{aligned} S_1(Q^2) &= M^2(1+\eta)G_M^2, \quad S_3(Q^2) = 0, \\ S_2(Q^2) &= M^2\left[G_M^2 - 2\left(G_C + \frac{\eta}{3}G_Q\right)G_M\right]. \end{aligned} \tag{13}$$

The third structure function $S_3(Q^2)$ is zero since deuteron form factors are real functions in elastic scattering (space-like momentum transfers). In the time-like region of momentum transfers (for annihilation processes, for example, $e^- + e^+ \to D + \bar{D}$), where the form factors are complex functions, the structure function $S_3(Q^2)$ is not zero and is determined by the imaginary part of the form factors:

$$S_3(Q^2) = 2M^2\,\mathrm{Im}\left(G_C - \frac{\eta}{3}G_Q\right)G_M^*$$

(in this case, Q^2 is the square of the virtual photon four-momentum).

In the case of a tensor-polarized deuteron target, the general structure of the spin-dependent tensor $H_{\mu\nu}(T)$ can be written in terms of the five structure functions as

$$H_{\mu\nu}(T) = V_1(Q^2)\bar{Q}\tilde{g}_{\mu\nu} + V_2(Q^2)\frac{\bar{Q}}{M^2}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + \\ + V_3(Q^2)(\tilde{p}_{1\mu}\widetilde{Q}_\nu + \tilde{p}_{1\nu}\widetilde{Q}_\mu) + V_4(Q^2)\widetilde{Q}_{\mu\nu} + \\ + iV_5(Q^2)(\tilde{p}_{1\mu}\widetilde{Q}_\nu - \tilde{p}_{1\nu}\widetilde{Q}_\mu), \quad (14)$$

where we introduce the notation

$$\widetilde{Q}_\mu = Q_{\mu\nu}q_\nu - \frac{q_\mu}{q^2}\bar{Q}, \quad \widetilde{Q}_\mu q_\mu = 0, \\ \widetilde{Q}_{\mu\nu} = Q_{\mu\nu} + \frac{q_\mu q_\nu}{q^4}\bar{Q} - \frac{q_\nu q_\alpha}{q^2}Q_{\mu\alpha} - \frac{q_\mu q_\alpha}{q^2}Q_{\nu\alpha}, \quad (15) \\ \widetilde{Q}_{\mu\nu}q_\nu = 0, \quad \bar{Q} = Q_{\alpha\beta}q_\alpha q_\beta.$$

The structure functions $V_i(Q^2)$, $i = 1, \ldots, 5$, which describe the part of the hadronic tensor due to the tensor polarization of the deuteron target, have the following form in terms of the deuteron form factors:

$$V_1(Q^2) = -G_M^2, \quad V_5(Q^2) = 0, \\ V_2(Q^2) = G_M^2 + \frac{4}{1+\eta}\left(G_C + \frac{\eta}{3}G_Q + \eta G_M\right)G_Q, \\ V_3(Q^2) = -2\eta[G_M^2 + 2G_QG_M], \\ V_4(Q^2) = 4M^2\eta(1+\eta)G_M^2. \quad (16)$$

The fifth structure function $V_5(Q^2)$ is zero since the deuteron form factors are real functions in the considered kinematical region. In the time-like region of momentum transfers, this structure function is not zero and is given by

$$V_5(Q^2) = -4\eta \operatorname{Im} G_Q G_M^*$$

(in this case, Q^2 is the square of the virtual photon four-momentum).

Using the definitions of cross-section (2) and leptonic (3) and hadronic (8) tensors, we can easily derive an expression for the unpolarized differential cross section (in the Born (one-photon-exchange) approximation) in terms of the invariant variables suitable for the calculation of the radiative corrections:

$$\frac{d\sigma_B^{un}}{dQ^2} = \frac{\pi\alpha^2}{VQ^4}\left\{2\rho W_1 + \frac{W_2}{\tau}[1-\rho(1+\tau)]\right\}, \quad (17)$$

$$\rho = \frac{Q^2}{V}, \quad \tau = \frac{M^2}{V}.$$

In the laboratory system, this expression can be written in a more familiar form using the standard structure functions $A(Q^2)$ and $B(Q^2)$. Then the unpolarized differential cross section for elastic electron–deuteron scattering takes the form

$$\frac{d\sigma_B^{un}}{d\Omega} = \sigma_M\left\{A(Q^2) + B(Q^2)\operatorname{tg}^2\left(\frac{\theta_e}{2}\right)\right\}, \quad (18)$$

$$\sigma_M = \frac{\alpha^2 E' \cos^2\left(\frac{\theta_e}{2}\right)}{4E^3\sin^4\left(\frac{\theta_e}{2}\right)},$$

where σ_M is the Mott cross section, E and E' are the incident and scattered electron energies, and θ_e is the electron scattering angle:

$$E' = E[1 + 2(E/M)\sin^2(\theta_e/2)]^{-1}, \\ Q^2 = 4EE'\sin^2(\theta_e/2).$$

The scattering angle in laboratory system can be written in terms of invariants as

$$\cos\theta_e = \frac{1-\rho-2\rho\tau}{1-\rho}, \quad \sin\theta_e = \frac{2\sqrt{\rho\tau(1-\rho-\rho\tau)}}{1-\rho}.$$

Two structure functions $A(Q^2)$ and $B(Q^2)$ are quadratic combinations of three electromagnetic form factors describing the deuteron structure:

$$A(Q^2) = G_C^2(Q^2) + \frac{8}{9}\eta^2G_Q^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2), \quad (19)$$

$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_M^2(Q^2).$$

It follows from Eq. (18) that the measurement of the unpolarized cross section at various values of the electron scattering angle and the same value of Q^2 allows determining the structure functions $A(Q^2)$ and $B(Q^2)$. Therefore, it is possible to determine the magnetic form factor $G_M(Q^2)$ and the combination

$$G_C^2(Q^2) + \frac{8}{9}\eta^2G_Q^2(Q^2)$$

of form factors. Hence, the separation of the charge G_C and quadrupole G_Q form factors requires polarization measurements.

Before writing similar distributions for the scattering of polarized particles, we note that in the general case, an azimuthal correlation between the reaction (electron scattering) plane and the plane $(\mathbf{k_1}, \mathbf{s})$ can exist for such experimental conditions if the initial deuteron is polarized. But in the Born approximation, with the P- and T-invariance of the hadron electromagnetic interaction taken into account, such correlation is absent. In what follows in this section, we consider the situation where the polarization three-vector $\mathbf{s}$ belongs to the reaction plane and the corresponding azimuthal angle is equal to zero. Therefore, there exist only two independent components of the polarization vector $\mathbf{s}$, which we call longitudinal and transverse ones.

To calculate radiative corrections to the polarization observables, it is convenient to parameterize the polarization state of the target (in our case, the deuteron polarization four-vector s_μ describing the deuteron vector polarization and the quadrupole polarization tensor $Q_{\mu\nu}$ describing the deuteron tensor polarization) in terms of the four-momenta of the particles in the reaction. This parameterization is not unique and depends on the directions chosen to define the longitudinal and transverse components of the deuteron polarization in its rest frame.

As mentioned above, we have to define the longitudinal $s^{(L)}$ and transverse $s^{(T)}$ polarization four-vectors. (The longitudinal and transverse components of the deuteron polarization are often defined along the z and x axes.) In our case, it is natural to choose the longitudinal direction, in the laboratory system, along the three-momentum transferred $\mathbf{q}$ (the virtual photon momentum) and the transverse direction perpendicular to the longitudinal one in the reaction plane. The corresponding polarization four-vectors can be written as [34]

$$s_\mu^{(T)} = \frac{(4\tau+\rho)k_{1\mu} - (1+2\tau)q_\mu - (2-\rho)p_{1\mu}}{\sqrt{V\,c(4\tau+\rho)}},$$
$$s_\mu^{(L)} = \frac{2\tau q_\mu - \rho p_{1\mu}}{M\sqrt{\rho(4\tau+\rho)}}, \quad c = 1-\rho-\rho\tau. \tag{20}$$

These four-vectors satisfy the conditions

$$s^{(L,T)}\cdot p_1 = 0, \quad s^{(L)}\cdot s^{(T)} = 0, \quad s^{(L,T)2} = -1.$$

It follows that they have the necessary properties of polarization four-vectors.

It can be verified that the set of four-vectors $s_\mu^{(L,T)}$ in the rest frame of the deuteron (the laboratory system) has the form

$$s_\mu^{(L)} = (0,\mathbf{L}), \quad s_\mu^{(T)} = (0,\mathbf{T}), \tag{21}$$

$$\mathbf{L} = \frac{\mathbf{k}_1-\mathbf{k}_2}{|\mathbf{k}_1-\mathbf{k}_2|}, \quad \mathbf{T} = \frac{\mathbf{n}_1 - (\mathbf{n}_1\cdot\mathbf{L})\mathbf{L}}{\sqrt{1-(\mathbf{n}_1\cdot\mathbf{L})^2}}, \quad \mathbf{n}_1 = \frac{\mathbf{k}_1}{|\mathbf{k}_1|}.$$

This leads to simple expressions for the spin-dependent hadronic tensors (due to the vector polarization of the deuteron target) corresponding to the longitudinal and transverse direction of the spin four-vector s_μ:

$$H_{\mu\nu}^T(V) = -\frac{iG_MG}{4}\sqrt{\frac{(4\tau+\rho)}{\tau c}}\times$$
$$\times\left[(4\tau+\rho)(\mu\nu qk_1) - (2-\rho)(\mu\nu qp_1)\right], \tag{22}$$
$$H_{\mu\nu}^L(V) = \frac{iG_M^2}{4\tau}(\mu\nu qp_1)\sqrt{\rho(4\tau+\rho)},$$

where

$$G = 2G_C + \frac{2}{3}\eta G_Q.$$

The spin-dependent parts of the cross section, due to the vector polarization of the initial deuteron and longitudinal polarization of the electron beam, can be written as

$$\frac{d\sigma_B^L}{dQ^2} = -\frac{\pi\alpha^2}{4\tau V^2}\frac{2-\rho}{\rho}\sqrt{\rho(4\tau+\rho)}G_M^2, \tag{23}$$

$$\frac{d\sigma_B^T}{dQ^2} = -\frac{\pi\alpha^2}{VQ^2}\sqrt{\frac{(4\tau+\rho)c}{\tau}}G_MG, \tag{24}$$

where we assume that P_e in Eq. (3) is equal to unity and the degree of the vector polarization (longitudinal or transverse) of the deuteron target is 100 percent.

In the laboratory system, these expressions lead to asymmetries (or the spin correlation coefficients) in the elastic electron–deuteron scattering in the Born approximation. These asymmetries are due to the vector polarization of the deuteron target, corresponding to the longitudinal and transverse direction of the spin four-vectors $s_\mu^{(L)}$ and $s_\mu^{(T)}$, and the longitudinal polarization of the electron beam:

$$I_0A_B^L = -\eta\sqrt{(1+\eta)\left(1+\eta\sin^2\left(\frac{\theta_e}{2}\right)\right)}\times$$
$$\times\operatorname{tg}\left(\frac{\theta_e}{2}\right)\sec\left(\frac{\theta_e}{2}\right)G_M^2, \tag{25}$$

$$I_0A_B^T = -2\operatorname{tg}\left(\frac{\theta_e}{2}\right)\sqrt{\eta(1+\eta)}\times$$
$$\times G_M\left(G_C+\frac{\eta}{3}G_Q\right), \tag{26}$$

where

$$I_0 = A(Q^2) + B(Q^2)\operatorname{tg}^2\left(\frac{\theta_e}{2}\right).$$

It is worth noting that the ratio of the longitudinal polarization asymmetry to the transverse one is

$$\frac{A_B^L}{A_B^T} = \sqrt{\eta\left(1+\eta\sin^2\left(\frac{\theta_e}{2}\right)\right)}\sec\left(\frac{\theta_e}{2}\right)\frac{G_M}{G}. \tag{27}$$

This ratio is expressed in terms of the deuteron form factors G_M and G in the same way as the corresponding ratio in the case of elastic electron–proton scattering is expressed in terms of proton electromagnetic form factors G_{M_p} and G_{E_p} [20, 45]. This is a direct consequence of the relation between the proton $H^p_{\mu\nu}(V)$ and deuteron $H_{\mu\nu}(V)$ spin-dependent hadronic tensors, which respectively depend on the proton polarization and deuteron vector polarization:

$$H_{\mu\nu}(V)(G_M, G) = -\frac{4\tau+\rho}{8\tau} H^p_{\mu\nu}(V)(G_{M_p}, G_{E_p}). \quad (28)$$

We now consider the tensor polarized deuteron target. If, for completeness, we introduce the four-vector $s^{(N)}_\mu$ orthogonal to the reaction plane,

$$s^{(N)}_\mu = \frac{2\varepsilon_{\mu\lambda\rho\sigma} p_{1\lambda} k_{1\rho} k_{2\sigma}}{V\sqrt{V\,c\rho}}, \quad (29)$$

then we can verify that the set of the four-vectors $s^{(I)}_\mu$, $I = L, T, N$, satisfies the conditions

$$s^{(\alpha)}_\mu s^{(\beta)}_\mu = -\delta_{\alpha\beta}, \quad s^{(\alpha)}_\mu p_{1\mu} = 0, \quad \alpha, \beta = L, T, N.$$

In the rest frame of the deuteron (the laboratory system), the four-vector $s^{(N)}_\mu$ has the form

$$s^{(N)}_\mu = (0, \mathbf{N}), \quad \mathbf{N} = \frac{\mathbf{n}_1 \times \mathbf{n}_2}{\sqrt{1-(\mathbf{n}_1\cdot\mathbf{n}_2)^2}}, \quad \mathbf{n_2} = \frac{\mathbf{k}_2}{|\mathbf{k}_2|},$$

with the vector $\mathbf{N}$ directed along the y axis. If we add one more four-vector

$$s^{(0)}_\mu = \frac{p_{1\mu}}{M}$$

to the set of the four-vectors defined in Eqs. (20) and (29), then we obtain a complete set of orthogonal four-vectors with the properties

$$s^{(m)}_\mu s^{(m)}_\nu = g_{\mu\nu}, \quad s^{(m)}_\mu s^{(n)}_\mu = g_{mn}, \quad m, n = 0, L, T, N.$$

In the general case, this set of four-vectors allows expressing the deuteron quadrupole polarization tensor as

$$Q_{\mu\nu} = s^{(m)}_\mu s^{(n)}_\nu R_{mn} \equiv s^{(\alpha)}_\mu s^{(\beta)}_\nu R_{\alpha\beta}, \quad R_{\alpha\beta} = R_{\beta\alpha}, \quad R_{\alpha\alpha} = 0, \quad (30)$$

because the time components R_{00}, $R_{0\alpha}$, and $R_{\alpha 0}$ are identically zero due to the condition $Q_{\mu\nu}p_{1\nu} = 0$. The $R_{\alpha\beta}$ are in fact the tensor polarization degrees of the deuteron target in its rest system (laboratory system). In the Born approximation, the components R_{NL} and R_{NT} do not contribute to the observables and this expansion can be rewritten in the standard form

$$Q_{\mu\nu} = \left[s^{(L)}_\mu s^{(L)}_\nu - \frac{1}{2}s^{(T)}_\mu s^{(T)}_\nu\right] R_{LL} + \frac{1}{2}s^{(T)}_\mu s^{(T)}_\nu \times (R_{TT} - R_{NN}) + \left(s^{(L)}_\mu s^{(T)}_\nu + s^{(T)}_\mu s^{(L)}_\nu\right) R_{LT}, \quad (31)$$

where we took into account that

$$R_{LL} + R_{TT} + R_{NN} = 0.$$

The part of the cross section in the Born approximation that depends on the tensor polarization of the deuteron target can be written as

$$\frac{d\sigma^Q_B}{dQ^2} = \frac{d\sigma^{LL}_B}{dQ^2} R_{LL} + \frac{d\sigma^{TT}_B}{dQ^2}(R_{TT} - R_{NN}) + \frac{d\sigma^{LT}_B}{dQ^2} R_{LT},$$

where

$$\frac{d\sigma^{LL}_B}{dQ^2} = \frac{\pi\alpha^2}{Q^4} 2c\eta \times \left\{8G_C G_Q + \frac{8}{3}\eta G_Q^2 + \frac{2c + 4\tau\rho + \rho^2}{2c} G_M^2\right\},$$
$$\frac{d\sigma^{TT}_B}{dQ^2} = \frac{\pi\alpha^2}{Q^4} 2c\eta G_M^2,$$
$$\frac{d\sigma^{LT}_B}{dQ^2} = -\frac{\pi\alpha^2}{Q^4} 4\eta(2-\rho)\sqrt{\frac{c\rho}{\tau}} G_Q G_M. \quad (32)$$

In the laboratory system, these expressions lead to the following asymmetries (or analyzing powers) in the elastic electron–deuteron scattering caused by tensor polarization of the deuteron target and an unpolarized electron beam (in the Born approximation)

$$I_0 A^Q_B = A^{LL}_B R_{LL} + A^{TT}_B (R_{TT} - R_{NN}) + A^{LT}_B R_{LT}, \quad (33)$$

where

$$I_0 A^{LL}_B = \frac{1}{2}\left\{8\eta G_C G_Q + \frac{8}{3}\eta^2 G_Q^2 + \eta\left[1 + 2(1+\eta)\,\mathrm{tg}^2\left(\frac{\theta_e}{2}\right)\right] G_M^2\right\},$$
$$I_0 A^{TT}_B = \frac{1}{2}\eta G_M^2, \quad I_0 A^{LT}_B = -4\eta \times \sqrt{\eta + \eta^2 \sin^2\left(\frac{\theta_e}{2}\right)} \sec\left(\frac{\theta_e}{2}\right) G_Q G_M. \quad (34)$$

Using the P-invariance of the hadron electromagnetic interaction, we can parameterize the differential cross section for elastic scattering of a longitudinally polarized electron beam on the polarized deuteron target (in the coordinate representation of the deuteron and electron spin-density matrices) as

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^{un}}{dQ^2}[1 + A^N s_y + A^{LL} R_{LL} + \\ + A^{LT} R_{LT} + A^{TT}(R_{TT} - R_{NN}) + \\ + P_e(A^L s_z + A^T s_x + A^{LN} R_{LN} + A^{TN} R_{TN})], \quad (35)$$

where $d\sigma^{un}/dQ^2$ is the differential cross section for unpolarized particles, A^N is the asymmetry (analyzing power) due to the normal component of the deuteron vector polarization (s_y), A^{LL}, A^{LT}, and A^{TT} are the asymmetries (analyzing powers) due to the deuteron tensor polarization corresponding to the R_{LL}, R_{LT}, and $(R_{TT} - R_{NN})$ components of the quadrupole tensor, A^L and A^T are the correlation parameters due to the longitudinal polarization of the electron beam, s_z and s_x are the components of the deuteron vector polarization, A^{TN} and A^{LN} are the correlation parameters due to the longitudinal polarization of the electron beam, and R_{TN} are R_{LN} the quadrupole tensor components. We note that the elastic electron–deuteron scattering amplitude is real in the Born (one-photon-exchange) approximation. This leads to zero values of the polarization observables A^N, A^{TN}, and A^{LN} in this approximation.

The formalism of spherical tensors is also used for parameterizing the deuteron spin-density matrix (see Appendix B for the details). In this case, Eq. (35) can be written as

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^{un}}{dQ^2}[1 + 2\,\mathrm{Im}\,t_{11}T_{11} + \\ + t_{20}T_{20} + 2\,\mathrm{Re}\,t_{21}T_{21} + 2\,\mathrm{Re}\,t_{22}T_{22} + \\ + P_e(t_{10}C_{10} + 2\,\mathrm{Re}\,t_{11}C_{11} + 2\,\mathrm{Im}\,t_{21}C_{21} + \\ + 2\,\mathrm{Im}\,t_{22}C_{22})], \quad (36)$$

where t_{kq} are the polarization tensor describing the polarization state of the deuteron target and T_{kq} and C_{kq} are the analyzing powers and correlation parameters of the reaction.

The relations between the polarization observables in the coordinate representation and within the approach of spherical tensors are

$$T_{11} = -\frac{1}{\sqrt{3}}A_y, \quad T_{20} = -\frac{\sqrt{2}}{3}A_{zz}, \\ T_{21} = \frac{1}{2\sqrt{3}}A_{xz}, \quad T_{22} = -\frac{1}{\sqrt{3}}A_{xx}, \\ C_{10} = \sqrt{\frac{2}{3}}A_z, \quad C_{11} = -\frac{1}{\sqrt{3}}A_x, \\ C_{21} = \frac{1}{2\sqrt{3}}A_{yz}, \quad C_{22} = -\frac{1}{2\sqrt{3}}A_{xy}. \quad (37)$$

If the longitudinal direction is determined by the recoil deuteron three-momentum, then relations (21) are not affected by hard photon radiation in the lepton part of the interaction (this corresponds to the use of the so-called hadronic variables) because $\mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1$. But when this direction is reconstructed from the experiment using the three-momentum of the detected scattered electron (lepton variables), these relations break down because $\mathbf{q} \neq \mathbf{k}_1 - \mathbf{k}_2$ in this case. This means that in the leptonic variables, parameterization (20) is unstable and radiation of a hard photon by the electron leads to a mixture of the longitudinal and transverse polarizations.

We can eliminate such a mixture if, in the laboratory system of reaction (1), we choose the longitudinal direction $\mathbf{l}$ along the electron beam momentum and the transverse directions $\mathbf{t}$, in the plane $(\mathbf{k}_1, \mathbf{k}_2)$ and perpendicular to $\mathbf{l}$. Then the corresponding parameterization of the polarization four-vectors is [34]

$$s^{(l)}_\mu = \frac{2\tau k_{1\mu} - p_{1\mu}}{M}, \quad s^{(n)}_\mu = s^{(N)}_\mu, \\ s^{(t)}_\mu = \frac{k_{2\mu} - (1 - \rho - 2\rho\tau)k_{1\mu} - \rho p_{1\mu}}{\sqrt{V\,c\rho}}. \quad (38)$$

It can be verified that the set of these polarization four-vectors $s^{(l,t,n)}_\mu$ in the rest frame of the deuteron (the laboratory system) has the form

$$s^{(l)}_\mu = (0, \mathbf{l}), \quad s^{(t)}_\mu = (0, \mathbf{t}), \quad s^{(n)}_\mu = (0, \mathbf{n}), \quad (39)$$

$$\mathbf{l} = \mathbf{n}_1, \quad \mathbf{t} = \frac{\mathbf{n}_2 - (\mathbf{n}_1 \cdot \mathbf{n}_2)\mathbf{n}_1}{\sqrt{1 - (\mathbf{n}_1 \cdot \mathbf{n}_2)^2}}, \quad \mathbf{n} = \frac{\mathbf{n}_1 \times \mathbf{n}_2}{\sqrt{1 - (\mathbf{n}_1 \cdot \mathbf{n}_2)^2}}.$$

This set of the polarization four-vectors (together with $s^{(0)}_\mu$) is also a complete set of orthogonal four-vectors with the properties

$$s^{(m)}_\mu s^{(m)}_\nu = g_{\mu\nu}, \quad s^{(m)}_\mu s^{(n)}_\mu = g_{mn}, \quad m, n = 0, l, t, n.$$

The hadronic tensors $H^{l,t}_{\mu\nu}$ corresponding to the longitudinal and transverse directions of the new spin four-vectors are given by

$$H^{l}_{\mu\nu} = i\frac{4\tau+\rho}{4\tau} \times \times \left\{ G\left[-2\tau(\mu\nu q k_1) + \frac{2\tau(2-\rho)}{4\tau+\rho}(\mu\nu q p_1)\right] + + G_M \frac{\rho(1+2\tau)}{4\tau+\rho}(\mu\nu q p_1)\right\} G_M, \quad (40)$$

$$H^{t}_{\mu\nu} = i\sqrt{\frac{\rho\tau}{c}} \times \times \left\{ G(1+2\tau)\left[\frac{2-\rho}{4\tau}(\mu\nu q p_1) - \frac{4\tau+\rho}{4\tau}(\mu\nu q k_1)\right] - - G_M \frac{c}{2\tau}(\mu\nu q p_1)\right\} G_M. \quad (41)$$

In the case of scattering on a vector-polarized deuteron target, the tensors $H^{L,T}_{\mu\nu}$ and $H^{l,t}_{\mu\nu}$ corresponding to the two choices of the spin four-vectors are connected by trivial relations

$$H^{L}_{\mu\nu} = \cos\theta\, H^{l}_{\mu\nu} + \sin\theta\, H^{t}_{\mu\nu},$$

$$H^{T}_{\mu\nu} = -\sin\theta\, H^{l}_{\mu\nu} + \cos\theta\, H^{t}_{\mu\nu},$$

where

$$\cos\theta = -(s^{(L)}s^{(l)}), \quad \sin\theta = -(s^{(L)}s^{(t)}).$$

Simple calculation leads to

$$\cos\theta = \frac{\rho(1+2\tau)}{\sqrt{\rho(4\tau+\rho)}}, \quad \sin\theta = -2\sqrt{\frac{c\tau}{4\tau+\rho}}. \quad (42)$$

These relations are a consequence of the fact that two sets of spin four-vectors are connected by means of an orthogonal matrix that describes a rotation in the plane perpendicular to the direction $\mathbf{n} = \mathbf{N}$:

$$s^{(A)}_{\mu} = V_{A\beta}(\theta)s^{(\beta)}_{\mu}, \quad V(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix},$$

where $A = L, T$ and $\beta = l, t$.

Using this rotation matrix, we can write the spin-dependent parts (due to the vector polarization of the target) of the Born cross section, which correspond to parameterization (38), in the simple way

$$\frac{d\sigma^{\beta}_B}{dQ^2} = V_{\beta A}(-\theta)\frac{d\sigma^{A}_B}{dQ^2}, \quad (43)$$

where $d\sigma^{L}_B/dQ^2$ and $d\sigma^{T}_B/dQ^2$ are defined in Eqs. (23) and (24). Therefore, we can write

$$\frac{d\sigma^{l}_B}{dQ^2} = -\frac{\pi\alpha^2}{V^2}\left[\frac{1+2\tau}{4\tau}(2-\rho)G_M + \frac{2c}{\rho}G\right]G_M, \quad (44)$$

$$\frac{d\sigma^{t}_B}{dQ^2} = \frac{\pi\alpha^2}{VQ^2}\sqrt{\frac{c\rho}{\tau}}\left[\frac{1}{2}(2-\rho)G_M - (1+2\tau)G\right]G_M. \quad (45)$$

In the case of the tensor polarization of the deuteron target, the relations analogous to Eq. (43) are

$$\frac{d\sigma^{\beta}_B}{dQ^2} = T_{\beta A}(-\theta)\frac{d\sigma^{A}_B}{dQ^2}, \quad (46)$$

where now $A = LL, TT, LT$ and $\beta = ll,\ tt,\ lt$. The rotation matrix can then be written as

$$T(\theta) = \begin{pmatrix} \frac{1}{4}(1+3\cos 2\theta) & \frac{3}{4}(1-\cos 2\theta) & \frac{3}{4}\sin 2\theta \\ \frac{1}{4}(1-\cos 2\theta) & \frac{1}{4}(3+\cos 2\theta) & -\frac{1}{4}\sin 2\theta \\ -\sin 2\theta & \sin 2\theta & \cos 2\theta \end{pmatrix},$$

where the partial cross sections $d\sigma^{IJ}_B/dQ^2$, $I, J = L, T$, are defined in Eq. (32) as the coefficients in front of the respective quantities R_{LL}, $R_{TT} - R_{NN}$, and R_{LN}, and the partial cross sections in the left-hand side of Eq. (46) are defined as

$$\frac{d\sigma^{Q}_B}{dQ^2} = \frac{d\sigma^{ll}_B}{dQ^2}R_{ll} + \frac{d\sigma^{tt}_B}{dQ^2}(R_{tt} - R_{nn}) + \frac{d\sigma^{lt}_B}{dQ^2}R_{lt}. \quad (47)$$

The partial spin-dependent cross sections in this case are

$$\begin{aligned} \frac{d\sigma^{tt}_B}{dQ^2} &= \frac{2\pi\alpha^2}{Q^4}c\eta\left[(1+\tau\rho)G_M^2 + \right. \\ &\left. + \frac{2\rho}{\rho+4\tau}(2-\rho)(1+2\tau)G_M G_Q + \frac{2c}{1+\eta}G_Q G\right], \\ \frac{d\sigma^{lt}_B}{dQ^2} &= -\frac{8\pi\alpha^2}{Q^4}\eta\sqrt{c\eta} \times \\ &\times \left\{\frac{1+2\tau}{1+\eta}\left[\rho\tau(1+\eta)G_M^2 + 2cG_Q G\right] + \right. \\ &\left. + 2\left[2\rho\frac{1+2\tau}{\rho+4\tau}(c+\tau+\tau\eta) - c\right]G_M G_Q\right\}, \\ \frac{d\sigma^{ll}_B}{dQ^2} &= -\frac{2\pi\alpha^2}{Q^4}\eta\left\{\left[\frac{\rho}{2}(2+4\tau-\rho) - 1 - \right.\right. \\ &\left. - 3\rho^2\tau(1+\tau)\right]G_M^2 + 6c\rho(2-\rho)\frac{1+2\tau}{\rho+4\tau} \times \\ &\left. \times G_M G_Q + \frac{c}{\tau(1+\eta)}\left[2\tau - \rho - 6\rho\tau(1+\tau)\right]G_Q G\right\}. \end{aligned} \quad (48)$$

It now follows that the spin-dependent part of the cross section, due to the tensor polarization of the deuteron target, is expressed in terms of new polarization parameters R_{ll}, $R_{tt} - R_{nn}$, and R_{lt}, which are defined in accordance with the new longitudinal and transverse directions given by Eq. (38), and the coefficients in front of these quantities, in the right-hand side of Eq. (47), define the corresponding partial cross sections $d\sigma_B^{ij}/dQ^2$. The new polarization parameters are related to R_{LL}, $R_{TT}-R_{NN}$, and R_{LT} as

$$R_{ll} = \frac{1}{4}(1+3\cos 2\theta)R_{LL} + \frac{1}{4}(1-\cos 2\theta)(R_{TT}-R_{NN}) - \sin 2\theta R_{LT},$$

$$R_{ll}-R_{nn} = \frac{3}{4}(1-\cos 2\theta)R_{LL} + \frac{1}{4}(3+\cos 2\theta)(R_{TT}-R_{NN}) + \sin 2\theta R_{LT},$$

$$R_{lt} = \frac{3}{4}\sin 2\theta R_{LL} - \frac{1}{4}\sin 2\theta(R_{TT}-R_{NN}) + \cos 2\theta R_{LT}.$$

We now consider the scattering of a longitudinally polarized electron beam by the unpolarized deuteron target in the case where the recoil deuteron is polarized. We can then calculate both vector and tensor polarizations of the recoil deuteron using the results given above. For this, we note that the polarization state of the recoil deuteron can be described by the longitudinal and transverse polarization four-vectors $S_\mu^{(L)}$ and $S_\mu^{(T)}$ that satisfy the relations

$$S^2 = -1, \quad S\cdot p_2 = 0$$

and are given by

$$S_\mu^{(L)} = \frac{2\tau q_\mu + \rho p_{2\mu}}{M\sqrt{\rho(4\tau+\rho)}}, \quad S_\mu^{(T)} = s_\mu^{(T)}. \tag{49}$$

We note that the spin-dependent part of the hadronic tensor describing the vector polarization of the deuteron target, Eq. (12), can be written in the equivalent form

$$H_{\mu\nu}(V) = \frac{iG_M}{2M}[(G_M-G)s\cdot p_2(\mu\nu q p_1) + 2M^2(1+\eta)G(\mu\nu q s)]. \tag{50}$$

The spin-dependent part of the hadronic tensor $H_{\mu\nu}^R(V)$, which corresponds to the case of the vector polarized recoil deuteron, can be derived from this equation by the substitution

$$s_\mu \to S_\mu, \quad p_1 \leftrightarrow -p_2.$$

In fact, this means that we have to replace the term $s\cdot p_2$ in the right-hand side of Eq. (50) with the term $S\cdot p_1$. The vector polarization of the recoil deuteron (longitudinal P^L or transverse P^T) is defined as the ratio of the spin-dependent part of the cross section to the unpolarized one. Taking into account that

$$S^{(L)}\cdot p_1 = -s^{(L)}\cdot p_2$$

and

$$s^{(T)}\cdot q = 0,$$

we conclude that

$$P^L = -A^L, \quad P^T = A^T, \tag{51}$$

where A^L and A^T are the respective vector asymmetries for the scattering of the longitudinally polarized electron beam by the vector polarized deuteron target (we assume that the beam and the target have 100 percent polarization).

Here, we emphasize that determining the ratio G_M/G by measuring the ratio of the asymmetries A^L/A^T, in the scattering of a longitudinally polarized electron beam by a vector-polarized deuteron target, may be more attractive than by measuring the ratio of polarizations P^L/P^T in the process of polarization transfer (from the longitudinally polarized electron beam to the recoil deuteron) because a second scattering is necessary in the latter case. This decreases the corresponding event number by about two orders [46] and essentially increases the statistical error. The problem with the depolarization effect that appears in the scattering of high-intensity electron beams on polarized solid targets can be avoided using the polarized gas deuteron target [13].

The tensor polarization components of the recoil deuteron are defined similarly by the ratios of the corresponding partial spin-dependent cross sections to the unpolarized one,

$$\widetilde{R}_{LL} = \frac{d\sigma_B^{LL}}{d\sigma_B^{un}}, \quad \widetilde{R}_{LT} = \frac{d\sigma_B^{LT}}{d\sigma_B^{un}}, \quad \widetilde{R}_{TT}-\widetilde{R}_{NN} = \frac{d\sigma_B^{TT}}{d\sigma_B^{un}}.$$

The spin-dependent part of the hadronic tensor $H_{\mu\nu}^R(T)$, which corresponds to the case of a tensor-polarized recoil deuteron, can be derived from Eq. (14) by changing the sign of the structure function $V_3(Q^2)$. Straightforward calculations using this updated tensor and parameterization (49) lead to the following results: i) both diagonal partial cross sections in the right-hand side of the last equations are the same as those defined by the first three lines in Eqs. (32) for the scattering on a polarized target, and ii) the partial cross section $d\sigma_B^{LT}/dQ^2$ changes sign compared with the one in the fourth line of Eqs. (32).

3. RADIATIVE CORRECTIONS

There exist two sources of radiative corrections when corrections of the order of α are taken into account. The first is caused by virtual and soft photon emission that cannot affect the kinematics of process (1). The second arises due to the radiation of a hard photon,

$$e^-(k_1) + D(p_1) \to e^-(k_2) + \gamma(k) + D(p_2), \qquad (52)$$

because cuts on the event selection used in the current experiments allow photons to be radiated with the energy about 100 MeV and even more [16, 46]. Such photons cannot be interpreted as "soft" ones. The form of the radiative corrections caused by the contribution due to the hard photon emission depends strongly on the choice of the variables that are used to describe process (52) [47].

We calculate the radiative corrections in the leptonic variables. This corresponds to the experimental setup where the energy and momentum of the virtual photon are determined using the measured energy and momentum of the scattered electron.

The hadronic variables (with the virtual photon kinematics reconstructed using the recoil deuteron energy and momentum) were previously used to compute the radiative corrections in the elastic and deep-inelastic polarized electron–proton scattering [47, 48], and elastic polarized electron–deuteron scattering [35].

We here calculate the model-independent radiative corrections that include all QED corrections to the lepton part of the interaction and an insertion of the vacuum polarization into the exchanged virtual photon propagator.

The general analysis of the two-photon exchange contribution (box diagrams) to the polarization observables in the elastic electron–deuteron scattering was done in Refs. [49, 50]. The effect of a two-photon exchange on the deuteron electromagnetic form factors was estimated numerically in Ref. [31, 51].

3.1. Unpolarized cross section

The model-independent radiative corrections to the unpolarized and polarized cross sections (due to the vector polarization of the deuteron target) of the elastic electron–deuteron scattering can be obtained using the results in [45], where the QED corrections for the polarized elastic electron–proton scattering were calculated in the framework of electron structure functions.

The spin-independent part of the cross section for elastic electron–deuteron scattering can be derived from the respective part of elastic electron–proton scattering by a simple rule using following relation between spin-independent hadronic tensors describing electron–deuteron and electron–proton scattering:

$$H^{d(un)}_{\mu\nu} = \frac{4\tau + xyr}{4\tau} H^{p(un)}_{\mu\nu} \left(G^2_{Mp} \to \frac{2}{3} G^2_M, \right.$$
$$\left. G^2_{Ep} \to G^2_C + \frac{x^2y^2r^2}{18\tau^2} G^2_Q \right), \qquad (53)$$

where

$$x = \frac{Q^2}{2p_1(k_1-k_2)}, \quad y = \frac{2p_1(k_1-k_2)}{V}, \quad r = \frac{-(k_1-k_2-k)^2}{Q^2}. \qquad (54)$$

We recall that k is the four-momentum of the hard photon in reaction (52).

The radiatively corrected cross section can be written in terms of the electron structure functions in the form (master formula) [52]

$$\frac{d\sigma(k_1,k_2)}{d\,Q^2} = \int\limits_{y_{min}}^{y_{max}} dy \int\limits_{z_{1m}}^{1} dz_1 \int\limits_{z_{2m}}^{1} dz_2 D(z_1,L) \times$$
$$\times \frac{1}{z_2^2} D(z_2,L) \frac{d\sigma_{hard}(\tilde{k}_1,\tilde{k}_2)}{d\tilde{Q}^2\, d\tilde{y}}, \quad L = \ln\frac{Q^2}{m^2}, \qquad (55)$$

where m is the electron mass, the integration limits for y, z_1, and z_2 are defined below, and the quantity $D(z,L)$ is the electron structure function. Numerical estimate of the radiative corrections (see below) have been done using the exponentiated form of the electron structure function given in Ref. [45, 53]. For the different representations of the photon contribution to the electron structure function, see, e. g., Ref. [54].

The reduced variables that define the cross section with emission of a hard photon in the integrand are

$$\tilde{k}_1 = z_1 k_1, \quad \tilde{k}_2 = \frac{k_2}{z_2}, \quad \tilde{Q}^2 = \frac{z_1}{z_2} Q^2, \quad \tilde{y} = 1 - \frac{1-y}{z_1 z_2}.$$

The hard part of the unpolarized (spin-independent) cross section can be written as

$$\frac{d\sigma_{hard}}{dQ^2 dy} = \frac{d\sigma^{un}_B}{dQ^2 dy}\left(1 + \frac{\alpha}{2\pi}\delta(x,\rho)\right) + H_x + H_{xr}, \quad (56)$$

where

$$\delta(x,\rho) = -1 - \frac{\pi^2}{3} - 2f\left(\frac{x-\rho(1+x\tau)}{x(1-\rho)(1-z_+)}\right) - \ln^2\frac{(1-\rho)}{1-z_+},$$

$$f(x) = \int_0^x \frac{dt}{t} \ln(1-t),$$

$$H_x = \frac{\alpha}{V^2}\left[\frac{1-r_1}{1-\rho}\hat{P}_1 - \frac{1-r_2}{1-z_+}\hat{P}_2\right] N(x,r)\frac{\alpha^2(rQ^2)}{r},$$

$$H_{xr} = \frac{\alpha}{V^2}\left\{\int_{r_-}^{r_+}\frac{2xW(x,r)\,dr}{\sqrt{\rho^2+4\rho x^2\tau}} + P\int_{r_-}^{r_+}\frac{dr}{1-r}\left[\frac{1-\hat{P}_1}{|r-r_1|}\times\right.\right.$$
$$\times\left(\frac{1+r^2}{1-\rho}N(x,r) + (r_1-r)T_1(x,r)\right) - \frac{1-\hat{P}_2}{|r-r_2|}\times$$
$$\left.\left.\times\left(\frac{1+r^2}{1-z_+}N(x,r) + (r_2-r)T_2(x,r)\right)\right]\right\}\frac{\alpha^2(rQ^2)}{r},$$

$$N(x,r) = \frac{2}{3}(1+\lambda_r)G_M^2(rQ^2) + \\ + \frac{2}{\rho^2 r}\left(1-\frac{\rho}{x}-\rho\tau\right)G^2(\lambda_r, rQ^2),$$

$$W(x,r) = \frac{2}{3}(1+\lambda_r)G_M^2(rQ^2) - \frac{2\tau}{r\rho}G^2(\lambda_r, rQ^2),$$

$$T_1(x,r) = -\frac{2}{\rho^2 r}\left[1-\frac{r(x-\rho)}{x}\right]G^2(\lambda_r, rQ^2),$$

$$T_2(x,r) = -\frac{2}{\rho^2 r}\left[1-r-\frac{\rho}{x}\right]G^2(\lambda_r, rQ^2), \quad \lambda_r = \frac{\rho r}{4\tau},$$

$$G^2(\lambda_r, rQ^2) = G_C^2(rQ^2) + \frac{2}{3}\lambda_r G_M^2(rQ^2) + \\ + \frac{8}{9}\lambda_r^2 G_Q^2(rQ^2), \quad z_+ = \frac{\rho}{x}(1-x).$$

The integration limits for r in the expression for H_{xr} can be written as

$$r_\pm = \frac{1}{2x^2(\tau+z_+)}\left[2x^2\tau + (1-x)\left(\rho \pm \sqrt{\rho^2+4x^2\rho\tau}\right)\right].$$

The integration limits in master formula (55) at fixed values of ρ can be derived from the restriction on the lost invariant mass for the hard subprocess:

$$M^2 < (\tilde{k}_1 + p_1 - \tilde{k}_2)^2 < (M+\Delta_M)^2, \qquad (57)$$

where Δ_M is usually smaller than the pion mass to exclude inelastic hadronic events. This means that

$$z_{2m} = \rho + \frac{1-y}{z_1}, \quad z_{1m} = \frac{1-y}{1-\rho}, \quad y_{min} = \rho,$$

$$y_{max} = \rho + \Delta_{th}, \quad \Delta_{th} = \frac{\Delta_M^2 + 2M\Delta_M}{V}. \qquad (58)$$

The action of the projection operators $\hat{P}_1$ and $\hat{P}_2$ is defined as

$$\hat{P}_1 f(r,x) = f(r_1, x), \quad \hat{P}_2 f(r,x) = f(r_2, x), \qquad (59)$$

where

$$r_1 = \frac{x-\rho}{x(1-\rho)}, \quad r_2 = \frac{x}{x-\rho(1-x)} = \frac{1}{1-z_+}.$$

The principal value symbol P in the expression for H_{xr} means that the nonphysical singularity at $r=1$ must be ignored; in other words,

$$P\int_{r_-}^{r_+}\frac{f(r)\,dr}{(1-r)|r-r_1|} = \int_{r_-}^{r_+}\frac{dr}{(1-r)}\left[\frac{f(r)}{|r-r_1|} - \right. \\ \left. - \frac{f(1)}{|1-r_1|}\right] + \frac{f(1)}{|1-r_1|}\ln\frac{1-r_-}{r_+-1}. \qquad (60)$$

The Born unpolarized cross section that enters hard cross section (56) is defined by expression (17) multiplied by the delta function $\delta(y-\rho)$.

To compare our calculations with others it is very important to extract the first-order correction from the master formula. For this, it suffices to use the well-known iterative form of the electron structure function entering Eq. (55) taking terms of the order of α into account, namely,

$$D(z_i, L) = \lim_{\Delta_i\to 0}\left[\delta(1-z_i) + \frac{\alpha(L-1)}{2\pi}P_1(z_i)\right],$$

$$P_1(z) = \frac{1+z^2}{1-z}\Theta(1-z-\Delta_i) + \delta(1-z)\left(\frac{3}{2}+2\ln\Delta_i\right).$$

The exact form of the infrared parameters Δ_1 and Δ_2 is given in Ref. [45], but it is unessential because they cancel in the final result, which can be written as

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma_B}{dQ^2}\left[1 + \frac{\alpha}{2\pi}\left(\delta(1,\rho) + (L-1)G_0\right)\right] + \\ + \frac{\alpha}{2\pi}(L-1)G_1 + \int_{y_{min}}^{y_{max}}(H_x + H_{xr})\,dy, \qquad (61)$$

where

$$G_0 = g(z) + g(\tilde{z}), \quad G_1 = I(z) + \tilde{I}(\tilde{z}), \quad z = \frac{\Delta_{th}}{1-\rho},$$

$$g(x) = \frac{3}{2} - 2x + \frac{x^2}{2} + 2\ln x, \quad \tilde{z} = \Delta_{th},$$

$$I(z) = \int_{1-z}^{1}\frac{1+z_1^2}{1-z_1}\left[\frac{d\sigma_B(z_1k_1, k_2)}{dQ^2} - \frac{d\sigma_B(k_1,k_2)}{dQ^2}\right]dz_1$$

and $\tilde{I}$ can be derived from I by substitution $d\sigma_B(k_1, z_2^{-1}k_2)$ instead of $d\sigma_B(z_1k_1, k_2)$ and $z_1 \to z_2$.

3.2. Correction to the cross section part caused by the vector polarization of the deuteron target

The correction to the spin-dependent part of the cross section, in the case where the deuteron target has vector polarization, can be obtained from the corresponding formulas of electron–proton scattering totally similarly to the unpolarized case. The only difference consists in the relation between spin-dependent hadronic tensors, which becomes

$$H^{d(l,t)}_{\mu\nu} = -\frac{4\tau + xyr}{8\tau} \times$$
$$\times H^{p(l,t)}_{\mu\nu}\left(G_{Mp}\to G_M, G_{Ep}\to 2G_C+\frac{xyr}{6\tau}G_Q\right). \quad (62)$$

We can again start from the Drell–Yan representation, but now for the spin-dependent part of the cross section. We recall that this representation is valid in this case where the radiation of collinear photons by the initial and final electrons does not change the longitudinal (l) and transverse (t) polarizations. Such stabilized polarization four-vectors of the deuteron polarization can be written in the form

$$\widetilde{S}^{(l)}_\mu = \frac{2\tau k_{1\mu} - p_{1\mu}}{M},$$
$$\widetilde{S}^{(t)}_\mu = \frac{-xyp_{1\mu} + k_{2\mu} - [-2xy\tau + (1-y)]k_{1\mu}}{\sqrt{Vxy(1-y-xy\tau)}}. \quad (63)$$

It can be verified that the polarization four-vector $\widetilde{S}^{(l)}$ in the laboratory system has components $(0,\mathbf{n})$, where the three-vector $\mathbf{n}$ has the orientation of the initial electron three-momentum $\mathbf{k}_1$. It can also be verified that $\tilde{S}^{(t)}\widetilde{S}^{(l)} = 0$ and

$$\widetilde{S}^{(t)} = (0, \mathbf{n}_\perp), \quad \mathbf{n}^2_\perp = 1, \quad \mathbf{n}\cdot\mathbf{n}_\perp = 0$$

in the laboratory system, where the three-vector $\mathbf{n}_\perp$ lies in the plane $(\mathbf{k}_1, \mathbf{k}_2)$.

It is convenient to write the master formula for the spin-dependent differential cross sections $d\sigma^l$ and $d\sigma^t$ in the form

$$\frac{d\sigma^{l,t}(k_1,k_2,\widetilde{S})}{dQ^2} = \int\limits_{y_{min}}^{y_{max}} dy \frac{d\sigma^{l,t}}{dQ^2dy},$$
$$\frac{d\sigma^{l,t}}{dQ^2dy} = \int\limits_{z_{1m}}^{1} dz_1 \int\limits_{z_{2m}}^{1}\frac{dz_2}{z_2^2}D^{(p)}(z_1,L)\times$$
$$\times D(z_2,L)\frac{d\sigma^{l,t}_{hard}(\tilde{k}_1,\tilde{k}_2,\widetilde{S})}{d\tilde{Q}^2\,d\tilde{y}}. \quad (64)$$

The function $D^{(p)}(z,L)$ is the electron structure function for a longitudinally polarized electron. It differs from $D(z,L)$ in the second order due to collinear electron–positron pair production in the so-called nonsinglet channel (see Ref. [55] for the details).

If the longitudinal direction $\mathbf{L}$ is chosen along the three-momentum $\mathbf{k}_1 - \mathbf{k}_2$ in the laboratory system, which for nonradiative process coincides with the direction of the three-vector $\mathbf{q}$, and the transverse direction $\mathbf{T}$ lies in the plane $(\mathbf{k}_1, \mathbf{k}_2)$, then we have to use

$$S^{(L)}_\mu = \frac{2\tau(k_1-k_2)_\mu - y\,p_{1\mu}}{M\sqrt{y(y+4x\tau)}},$$

$$S^{(T)}_\mu = \frac{(1+2x\tau)k_{2\mu}-(1-y-2x\tau)k_{1\mu}-x(2-y)p_{1\mu}}{\sqrt{Vx(1-y-xy\tau)(y+4x\tau)}}.$$

In this case, we use the relations

$$\frac{d\sigma^{(A)}(k_1,k_2,S)}{dQ^2} =$$
$$= \int\limits_{y_{min}}^{y_{max}} dy\, V_{A\beta}(\psi)\frac{d\sigma^\beta(k_1,k_2,\widetilde{S})}{dQ^2dy}, \quad (65)$$

where, totally similarly to Eq. (42),

$$\cos\psi = -(S^{(L)}\tilde{S}^{(l)}), \quad \sin\psi = -(S^{(L)}\tilde{S}^{(t)}),$$

$$\cos\psi = \frac{y+2xy\tau}{\sqrt{y^2+4xy\tau}},$$

$$\sin\psi = -2\sqrt{\frac{xy\tau(1-y-xy\tau)}{y^2+4xy\tau}}.$$

The vector asymmetries in the considered process with the radiative corrections taken into account are defined as the ratios

$$A^{l,t}_C = \frac{d\sigma^{l,t}(k_1,k_2,\widetilde{S})}{d\sigma(k_1,k_2)},$$
$$A^{L,T}_C = \frac{d\sigma^{L,T}(k_1,k_2,S)}{d\sigma(k_1,k_2)}, \quad (66)$$

where the unpolarized cross section is described by Eq. (55), and the partial cross sections caused by correlation between the deuteron vector polarizations and the electron longitudinal polarization are respectively defined by Eqs. (64) and (65) (we use $P_e = 1$). Therefore, calculating the asymmetry, including the radiative corrections, requires knowing the radiative corrections for both spin-independent and spin-dependent parts of the cross section.

The hard part of the polarized cross section in the integrand in the right-hand side of Eq. (64) can be written in a form very similar to, although slightly different from, the unpolarized cross section:

$$\frac{d\sigma^{l,t}_{hard}}{dQ^2dy} = \frac{d\sigma^{l,t}_{B}}{dQ^2dy}\left(1+\frac{\alpha}{2\pi}\delta(x,\rho)\right)+H^{l,t}_x+H^{l,t}_{xr}, \quad (67)$$

$$H^{l,t}_x = -\frac{\alpha U^{l,t}}{8\tau V^2}\left\{-\frac{1-r_1}{1-\rho}\hat{P}_1N^{l,t}_1 - \right.\\ \left. -\frac{1-r_2}{1-z_+}\hat{P}_2N^{l,t}_2\right\}\frac{\alpha^2(rQ^2)}{r},$$

$$H^{l,t}_{xr} = -\frac{\alpha U^{l,t}}{8\tau V^2}\left\{\int\limits_{r_-}^{r_+}\frac{2xW^{l,t}dr}{\sqrt{\rho^2+4x^2\rho\tau}}+P\int\limits_{r_-}^{r_+}\frac{dr}{1-r}\times\right.\\ \times\left[\frac{1-\hat{P}_1}{|r-r_1|}\left(\frac{1+r^2}{1-\rho}N^{l,t}_1+(r_1-r)T^{l,t}_1\right)-\right.\\ \left.\left.-\frac{1-\hat{P}_2}{|r-r_2|}\left(\frac{1+r^2}{1-z_+}N^{l,t}_2+(r_2-r)T^{l,t}_2\right)\right]\right\}\frac{\alpha^2(rQ^2)}{r}.$$

The rest of our short-hand notation is

$$U^l=1,\quad U^t=\sqrt{\frac{\tau}{x\rho}\frac{1}{(x-\rho-x\rho\tau)}},$$

$$W^l=-2\frac{\rho\tau}{x}W,\quad W^t=-\frac{\rho^2}{x^2}(1+2x\tau)W,$$

$$W=[x(1+r)-1]G^2_M+\left[1+\frac{4x\tau}{\rho r}(1+r)\right]G_M\widetilde{G},$$

$$N^l_1=(2\tau+r)(2-\rho)G^2_M+8\tau\left(\frac{1-\rho}{\rho}-\frac{\tau}{r}\right)G_M\widetilde{G},$$

$$N^l_2=(2\tau+1)(2-\rho r)G^2_M+8\tau\left(\frac{1}{\rho r}-1-\tau\right)G_M\widetilde{G},$$

$$N^t_1=\left[1-\frac{\rho}{x}+r-\rho(r+2\tau)\right]\times\\ \times\left[-(2-\rho)G^2_M+2\left(1+\frac{2\tau}{r}\right)G_M\widetilde{G}\right],$$

$$N^t_2=\left[1-\frac{\rho}{x}+\frac{1}{r}-\rho(1+2\tau)\right]\times\\ \times\left[-(2-\rho r)G^2_M+2(1+2\tau)G_M\widetilde{G}\right],$$

$$T^l_1=2\left[(r+2\tau)G^2_M+2\tau\left(\frac{2}{\rho}-1\right)G_M\widetilde{G}\right],$$

$$T^l_2=-2\left[(1+2\tau)G^2_M+2\tau\left(\frac{2}{\rho r}-1\right)G_M\widetilde{G}\right],$$

$$T^t_1=2\left\{-\left[r(1-\rho)+1-\frac{\rho}{x}-2\rho\tau\right]G^2_M+\right.\\ \left.+\left[1-\frac{\rho}{x}-2\rho\tau+r+4\tau\right]G_M\widetilde{G}\right\},$$

$$T^t_2=2\left[\frac{1}{r}-\rho(1+2\tau)+1-\frac{\rho}{x}\right]G^2_M-\\ -2\left[\rho+2\frac{\tau-1}{r}+\frac{1}{r}+1-\frac{\rho}{x}\right]G_M\widetilde{G}.$$

We note that the argument of the electromagnetic form factors in Eq. (67) is $-Q^2r$ and

$$\widetilde{G}(rQ^2)=2G_C(rQ^2)+\frac{\rho r}{6\tau}G_Q(rQ^2).$$

The spin-dependent Born cross sections in the right-hand side of Eq. (67) are defined by expressions (44) and (45) multiplied by $\delta(y-\rho)$.

We can now write the first order-correction to the spin-dependent part of the cross section totally similarly to the unpolarized case,

$$\frac{d\sigma^{l,t}}{dQ^2}=\frac{d\sigma^{l,t}_B}{dQ^2}\left[1+\frac{\alpha}{2\pi}(\delta(1,\rho)+(L-1)G_0)\right]+\\ +\frac{\alpha}{2\pi}(L-1)G^{l,t}_1+\int\limits_{y_{min}}^{y_{max}}(H^{l,t}_x+H^{l,t}_{xr})\,dy, \quad (68)$$

where $G^{l,t}_1$ can be derived from G_1 (see Eq. (61)) by the simple substitution $d\sigma\to d\sigma^{l,t}$.

3.3. Correction to the cross section part caused by tensor polarization of the deuteron target

The radiative corrections to polarization observables in elastic electron–deuteron scattering caused by a tensor-polarized deuteron target can be obtained using the results in [56], where model-independent radiative corrections to the deep-inelastic scattering of an unpolarized electron beam on a tensor-polarized deuteron target have been calculated.

To obtain the required corrections, it is necessary first to derive the contribution of the elastic radiative tail (radiative corrections to elastic electron–deuteron scattering). It can be obtained using the results in Ref. [56], where radiative corrections to the deep-inelastic scattering of an unpolarized electron beam on a tensor-polarized deuteron target have been calculated.

We can obtain these radiative corrections from formula (46) in [56] by the substitution

$$B_i(q^2, x') \to -\frac{1}{q^2}\delta(1-x')B_i^{(el)}, \quad i = 1, \ldots, 4, \quad (69)$$

in the hadronic tensor, where $B_i(q^2, x')$ are the spin-dependent structure functions caused by the tensor polarization of the deuteron, describing the transition $\gamma^* d \to X$, and the functions $B_i^{(el)}$ are their elastic limit (when the final state X is the deuteron). Here

$$q^2 = (k_1 - k_2 - k)^2.$$

Hence, the elastic structure functions can be expressed in terms of the deuteron electromagnetic form factors as

$$\begin{aligned} B_1^{(el)} &= \bar\eta q^2 G_M^2, \\ B_3^{(el)} &= 2\bar\eta^2 q^2 G_M(G_M + 2G_Q), \\ B_2^{(el)} &= -2\bar\eta^2 q^2 \times \\ &\times \left[G_M^2 + \frac{4G_Q}{1+\bar\eta}\left(G_C + \frac{\bar\eta}{3}G_Q + \bar\eta G_M\right)\right], \\ B_4^{(el)} &= -2\bar\eta q^2(1+\bar\eta)G_M^2, \quad \bar\eta = -q^2/4M^2. \end{aligned} \quad (70)$$

After the substitution of the elastic functions $B_i^{(el)}$ in formula (46) in Ref. [56], we have to integrate over the z variable (which shows the degree of deviation of the deep-inelastic scattering from the elastic process) using the δ-function

$$\delta(1-x') = xyr\delta(z).$$

Hence, the value $z = 0$ corresponds to the elastic contribution (elastic electron–deuteron scattering) to the deep-inelastic process. We note that the z variable used in Ref. [56] has a different meaning compared to the z variable used here. And finally, to obtain the radiative corrections to the process of elastic electron–deuteron scattering, it is necessary to integrate the elastic radiative tail contribution over the x variable.

As a result, we obtain the radiatively corrected spin-dependent part of the cross section caused by the tensor polarization of the deuteron target in elastic electron–deuteron scattering in the form

$$\frac{d\sigma^Q}{dQ^2} = \frac{d\sigma^{ll}}{dQ^2}R_{ll} + \frac{d\sigma^{tt}}{dQ^2}(R_{tt} - R_{nn}) + \frac{d\sigma^{lt}}{dQ^2}R_{lt}, \quad (71)$$

where the spin-dependent parts of the cross section $d\sigma^{mn}$, $mn = ll, lt, tt$, can be written in terms of the electron structure function as

$$\begin{aligned} \frac{d\sigma^{mn}(k_1, k_2, \widetilde{S})}{dQ^2} &= \int\limits_{y_{min}}^{y_{max}} dy \frac{d\sigma^{mn}}{dQ^2 dy}, \\ \frac{d\sigma^{mn}}{dQ^2 dy} &= \int\limits_{z_{1m}}^{1} dz_1 \int\limits_{z_{2m}}^{1} \frac{dz_2}{z_2^2} D(z_1, L) \times \\ &\times D(z_2, L) \frac{d\sigma^{mn}_{hard}(\tilde k_1, \tilde k_2, \widetilde{S})}{d\tilde Q^2\, d\tilde y}. \end{aligned} \quad (72)$$

Totally similarly to Eq. (65), we can write the tensor partial cross sections defined relative to the **L** and **T** directions as

$$\frac{d\sigma^{(A)}(k_1, k_2, S)}{dQ^2} = = \int\limits_{y_{min}}^{y_{max}} dy\, T_{A\beta}(\psi) \frac{d\sigma^{\beta}(k_1, k_2, \widetilde{S})}{dQ^2 dy}. \quad (73)$$

In this paper, we define the partial tensor asymmetries in the same way as for the vector ones (see Eq. (66)),

$$A_C^{\beta} = \frac{d\sigma^{\beta}(k_1, k_2, \widetilde{S})}{d\sigma(k_1, k_2)}, \quad A_C^{A} = \frac{d\sigma^{A}(k_1, k_2, S)}{d\sigma(k_1, k_2)}, \quad (74)$$

where the indices A and β take the values

$$A = LL,\ TT,\ LT, \quad \beta = ll,\ tt,\ lt,$$

and the spin-dependent cross section parts due to the deuteron tensor polarization are determined by Eqs. (72) and (73).

The hard part of the spin-dependent cross sections in the integrand in the right-hand side of Eq. (72) can be written as

$$\frac{d\sigma^{mn}_{hard}}{dQ^2 dy} = \left(1 + \frac{\alpha}{2\pi}\delta(1, \rho)\right)\frac{d\sigma^{mn}_{B}}{dQ^2 dy} + H^{mn}_{x} + H^{mn}_{xr}, \quad (75)$$

where we introduce the notation

$$H^{mn}_{x} = \frac{\alpha}{2\pi}\left[\frac{(1-r_1)\hat P_1}{1-\rho} - \frac{(1-r_2)\hat P_2}{r_2}\right]\frac{d\sigma^{mn}_{B}}{dQ^2},$$

$$H_{xr}^{mn} = \frac{\alpha\eta}{Q^4}\left\{\frac{P}{1-xy}\int_{r_-}^{r_+}\frac{dr(1-\hat{P}_1)G^{mn}(r)}{(1-r)|r-r_1|} + \right.$$
$$\left. + \frac{P}{1-y(1-x)}\int_{r_-}^{r_+}\frac{dr(1-\hat{P}_2)\tilde{G}^{mn}(r)}{(1-r)|r-r_2|}\right\} - \frac{\alpha\rho}{4Q^4}\times$$
$$\times\frac{1}{\sqrt{y^2+4xy\tau}}\int_{r_-}^{r_+}dr\left[F_0^{mn}(r) + \xi_1\frac{Q^2}{V}F_1^{mn}(r) + \right.$$
$$\left. + \tau\xi_2\frac{Q^4}{V^2}F_2^{mn}(r)\right]\frac{\alpha^2(Q^2r)}{r^2}, \quad (76)$$

with the coefficients

$$\xi_1 = \frac{1}{y+4x\tau}\left[1-y-2x\tau+r(1+xy-2x+2x\tau)\right],$$
$$\xi_2 = 3\xi_1^2 - \frac{1}{y(y+4x\tau)}\left[r(1-xy)+y-1\right]^2.$$

The functions $G^{mn}(r)$, $\tilde{G}^{mn}(r)$, and $F_i^{mn}(r)$, $i=0$, 1, 2, are defined as

$$G^{mn}(r) = \frac{\alpha^2(Q^2r)}{r^2}\sum_{j=1}^{4}A_j^{mn}H_j,$$
$$F_i^{mn}(r) = \sum_{j=1}^{4}C_{ij}^{mn}H_j, \quad (77)$$
$$\tilde{G}^{mn}(r) = \frac{\alpha^2(Q^2r)}{r^2}\sum_{j=1}^{4}B_j^{mn}H_j,$$

where the expressions for the coefficients A_j^{mn}, B_j^{mn}, and C_{ij}^{mn}, with $mn = ll, lt, tt$, $i = 0, 1, 2$, and $j = 1, 2, 3, 4$, are given in Appendix A. The functions H_j, $j = 1, \ldots, 4$, in relations (76) depend on the shifted momentum transfer squared, i.e., $H_j = H_j(rQ^2)$, and

$$H_1(Q^2) = G_M^2, \quad H_4(Q^2) = -2(1+\eta)G_M^2,$$
$$H_2(Q^2) =$$
$$= -2\eta\left[G_M^2 + \frac{4}{1+\eta}G_Q\left(G_C + \frac{\eta}{3}G_Q + \eta G_M\right)\right], \quad (78)$$
$$H_3(Q^2) = 2\eta\, G_M(G_M + 2G_Q).$$

From the master formula (72), we can now extract the first-order correction to the spin-dependent parts of the cross section, caused by the tensor polarization of the deuteron target:

$$\frac{d\sigma^{mn}}{dQ^2} = \left\{1 + \frac{\alpha}{2\pi}\left[\delta(1,\rho) + (L-1)G_0\right]\right\}\frac{d\sigma_B^{mn}}{dQ^2} +$$
$$+ \frac{\alpha}{2\pi}(L-1)G_1^{mn} + \int_{y_{min}}^{y_{max}}(H_x^{mn} + H_{xr}^{mn})\,dy, \quad (79)$$

where $mn = ll, lt, tt$ and

$$G_1^{mn} = I_1^{mn}(z) + I_2^{mn}(\tilde{z}),$$
$$I_1^{mn}(z) = -\int_{1-z}^{1}dr_1\frac{(1+r_1^2)(1-\hat{P}_1)}{1-r_1}\frac{d\sigma_B^{mn}}{dQ^2},$$
$$I_2^{mn}(\tilde{z}) = -\int_{\frac{1}{1-\tilde{z}}}^{1}\frac{dr_2}{r_2^3}\frac{(1+r_2^2)(1-\hat{P}_2)}{1-r_2}\frac{d\sigma_B^{mn}}{dQ^2}.$$

4. NUMERICAL ESTIMATIONS

The recent measurements in polarized electron–deuteron scattering are as follows:

i) measurement of the analyzing power T_{20} in the region $0.126\ \mathrm{GeV}^2 < Q^2 < 0.397\ \mathrm{GeV}^2$ [12],

ii) measurement of recoil polarizations t_{20}, t_{21}, t_{22} at $0.66\ \mathrm{GeV}^2 < Q^2 < 1.7\ \mathrm{GeV}^2$ [16],

iii) measurement of analyzing powers T_{20}, T_{21}, T_{22} at $0.326\ \mathrm{GeV}^2 < Q^2 < 0.838\ \mathrm{GeV}^2$ [13].

For the deuteron form factors, we use the results in [57], where the world data for elastic electron–deuteron scattering was used to parameterize the three electromagnetic form factors of the deuteron in the four-momentum transfer range 0–7fm^{-1} in three different ways. The accuracy in the determination of these form factors is limited by the assumption of the one-photon exchange mechanism and the precise calculation of the radiative corrections. In the range of intermediate to high Q, other corrections such as the double scattering contribution to the two-photon exchange [30] should be considered, but they are at present by neither accurately calculated nor experimentally determined.

For numerical calculations, we use two different parameterizations labeled as I and II. In parameterization I, each form factor is given by a polynomial in Q^2. With 18 free parameters, a fit was obtained with $\chi^2/N_{d.f.} = 1.5$. Parameterization II has been proposed in Ref. [58]. Each form factor is proportional to the square of the dipole nucleon form factor and to a linear combination of reduced helicity transition amplitudes. In addition, the asymptotic behavior dictated by quark counting rules and helicity rules valid in perturbative QCD were incorporated in the fitting procedure. With 12 free parameters, a fit to the data set was obtained with $\chi^2/N_{d.f.} = 1.8$, whereas the original values of the parameters in Ref. [58] yield $\chi^2/N_{d.f.} = 7.5$. Parameterization II, unlike the other two presented in this paper, can be extrapolated well above 7fm^{-1}, albeit with some theoretical prejudice.

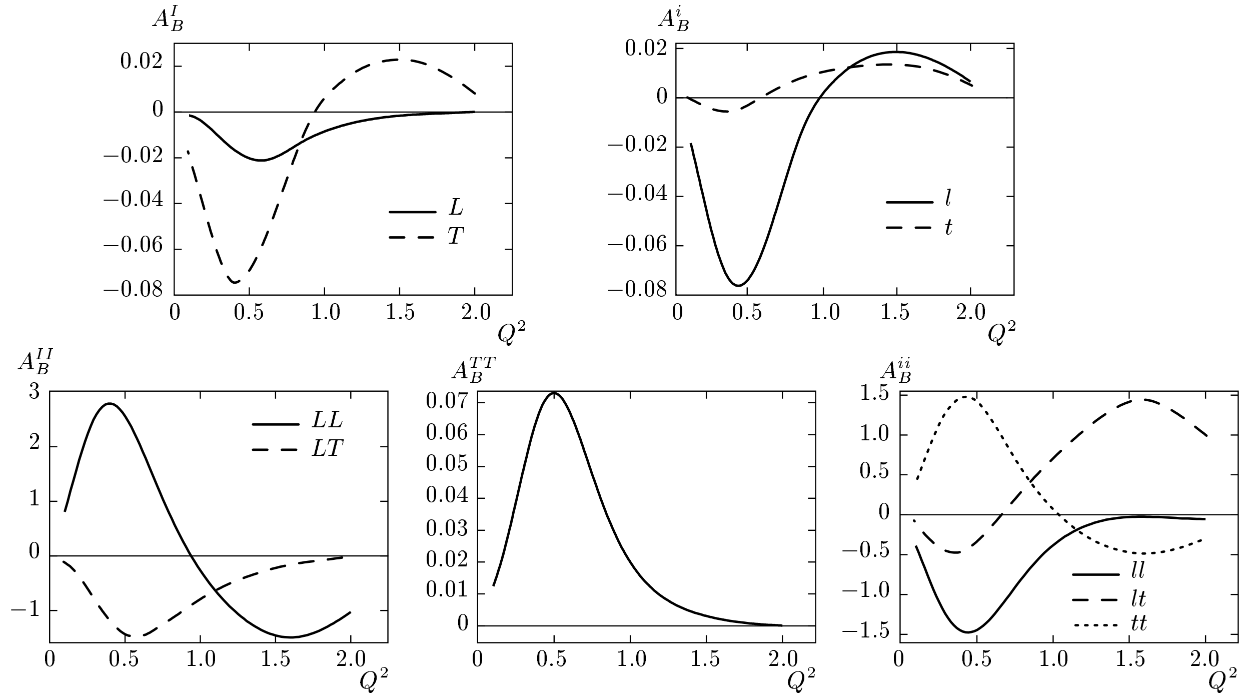

Fig. 1. The Born values of vector and tensor asymmetries calculated by means of Eqs. (25), (26), and (34). We note that the quantity A_B^{TT} is small compared with A_B^{tt}. Q^2 is given in GeV^2

To demonstrate the effect of radiative corrections in the considered polarized phenomena, we give the Q^2-dependence of the quantities δA^I and δA^{IJ} defined as

$$\delta A^I = A_C^I - A_B^I, \quad \delta A^{IJ} = A_C^{IJ} - A_B^{IJ}, \qquad (80)$$

where A_C^I and A_C^{IJ} are the values of the corresponding asymmetries with the radiative corrections taken into account (see Eqs. (66) and (74)) and A_B^I and A_B^{IJ} are their Born values.

In the calculation, we took $V = 2(k_1 p_1) = 10$ GeV^2 and 0.1 GeV$^2 \leq Q^2 \leq 2$ GeV^2 and used parameterizations I and II of the deuteron form factors given in Ref. [57]. It turns out that the difference between the asymmetries calculated with these two parameterizations are very small and we use parameterization I in what follows.

In Fig. 1, the Born values of vector and tensor asymmetries are shown. We can see that the absolute values of vector asymmetries are small compared with the tensor ones. Besides, the effect of the polarization direction choice is seen very clearly. The most pronounced feature is that A_B^{TT} is near zero at all values of Q^2 considered, whereas A_B^{tt} is large enough (of the order of unity).

In Fig. 2, we demonstrate the influence of radiative corrections on the single-spin tensor asymmetries. The corresponding effect depends strongly on the parameter Δ_{th} that defines the rules for event selection. If we set $\Delta_{th} = 0$, the radiation of hard photons is forbidden, and the effect vanishes, as can be seen from master formulas (55), (64) and (72) as well as from Eqs. (65) and (73).

In this case, the radiative corrections are determined by the soft photon emission and virtual loops and are factorized in both unpolarized and polarization-dependent cross sections. If the hard photon emission is allowed but the pion production threshold does not exceed $\Delta_{th} \leq 0.0526$ (see Eqs. (57) and (58)), the absolute value of the effect is smaller than 2.5 %. As the allowed photon energy increases, corrections to the tensor asymmetries can reach the values of the order of 15–20 %.

As regards the double-spin vector asymmetries, they are very small even at the Born level, and up to now there have been no attempts to measure them. We calculate them to give the complete picture of taking model-independent radiative corrections in to account in electron–deuteron scattering.

We emphasized in Sec. 2 that measurement of the polarization asymmetries in process (1), in principle, can be used to determine the deuteron magnetic form factor G_M. The reason is that the quantities A_B^L (see Eq. (26)) and A_B^{TT} (see Eq. (34)) are proportional to

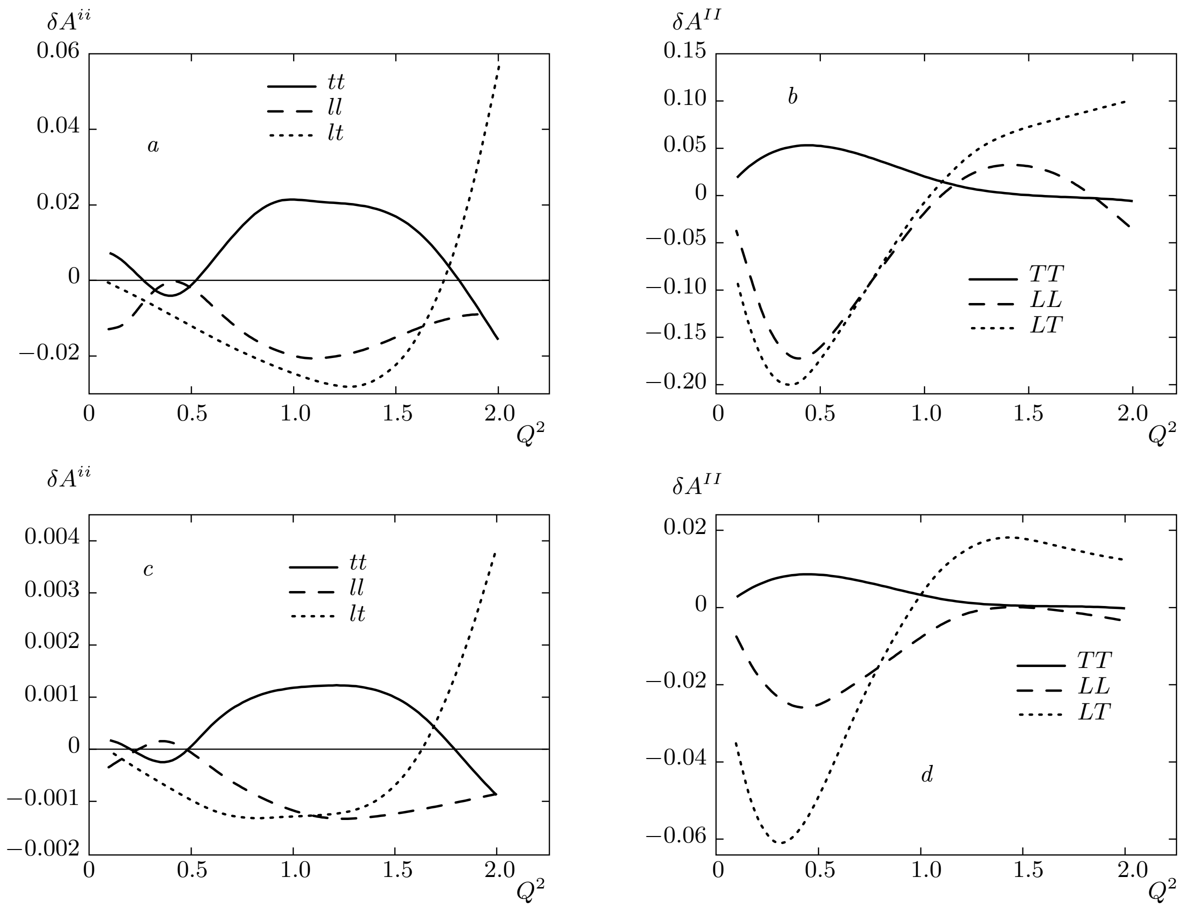

Fig. 2. Effect of radiative corrections for the single-spin tensor asymmetries defined with respect to the stable (a,c) and unstable (b,d) directions in radiating the collinear photons and e^+e^- pairs by the initial and final electrons. The curves in the upper row are calculated at $\Delta_{th} = 0.26$ and those in the lower row, at $\Delta_{th} = 0.0526$. In the Born approximation, all quantities δA^{ii} and δA^{II} are equal to zero. Q^2 is given in GeV^2

G_M^2 if the deuteron polarization states are determined with respect to directions defined by Eq. (21), with the longitudinal direction chosen along the transferred 3-momentum. If these states are defined by Eq. (39), with the longitudinal direction chosen along the initial electron 3-momentum, such a simple form of the relevant asymmetries is violated, and the analysis of polarization data becomes more complicated. This situation is preserved if the corrections due to a soft photon emission and a virtual loop are taken into account.

But the inclusion of radiative events with hard photon emission (process (52)) inevitably changes it because even the radiation of collinear photons alters the direction of the three-momentum transferred, and a rotation of polarization states necessarily occurs. In that case, to take the radiative corrections into account by the electron structure function method, we need, according to the spirit of this method, to use the set of polarization states that are stable under collinear photon radiation by both initial and scattered electrons. The corresponding spin-dependent parts of the hard cross sections in master formulas (64) and (72) include different combinations of all deuteron form factors. This means, for example, that the small partial cross section $d\sigma^{TT}/dQ^2$, which is expressed through the large ones $d\sigma^{tt}/dQ^2$, $d\sigma^{ll}/dQ^2$, and $d\sigma^{lt}/dQ^2$, can change significantly if undetected additional particles accompany process (1). At least for $\Delta_{th} = 0.26$, the radiative corrections almost double the tensor asymmetry A^{TT} compared with the Born one. With a decrease in Δ_{tr}, this effect is diminished. We note that a similar effect is absent if hadronic variables are used to describe radiative corrections (see Eqs. (64) and (69) in Ref. [35]). The reason is that in that case, the recoil deuteron momentum is measured independently of undetected particles in the leptonic part of interaction.

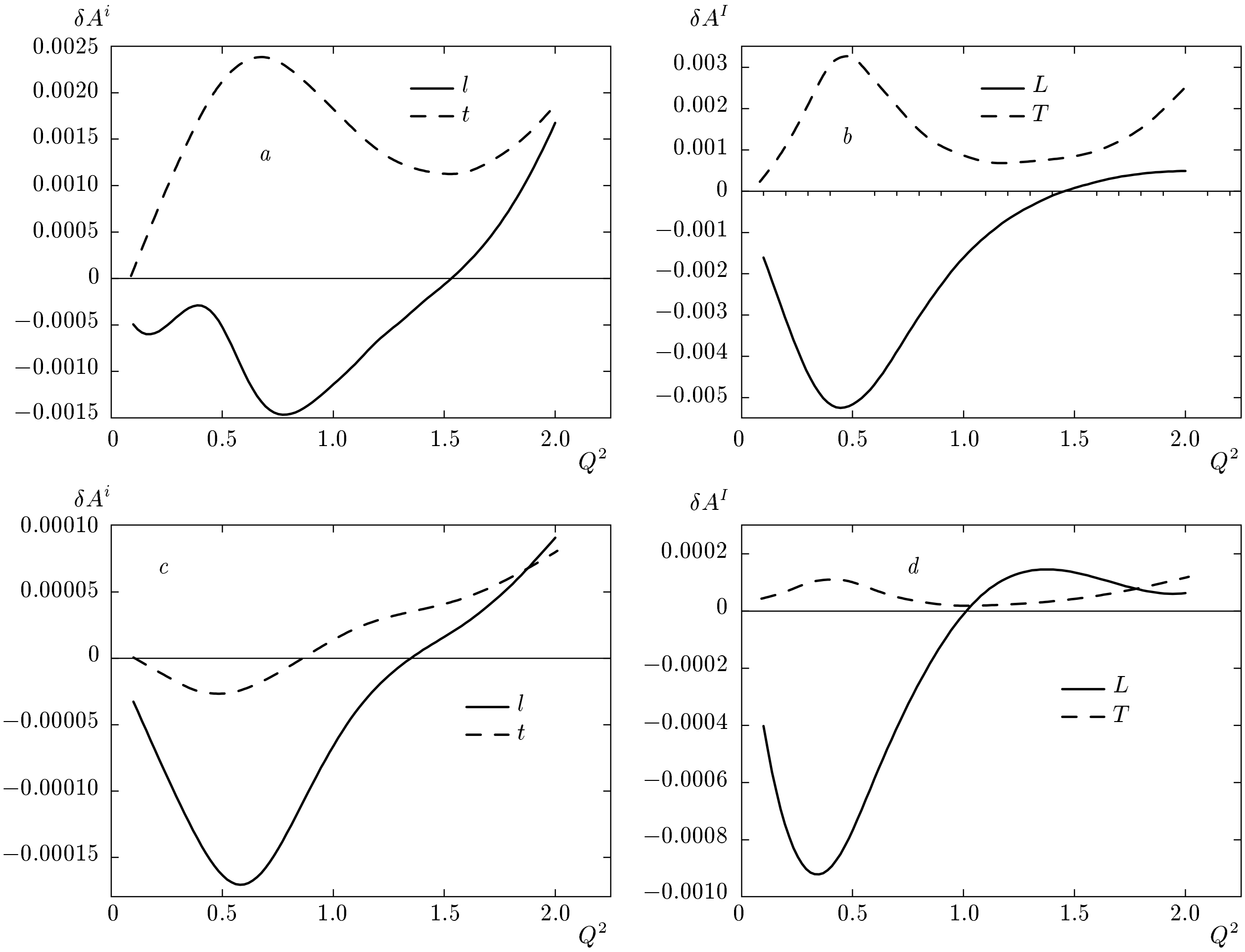

Fig. 3. The same as in Fig. 2 but for the double-spin vector asymmetries

We here give a consistent calculation of electromagnetic model-independent radiative corrections for polarization observables in the process of elastic electron–deuteron scattering. Our approach is based on the electron structure function method and covariant description of the polarization states, with the event selection done by a restriction on the lost invariant mass. The only additional parameter that must then be determined in measurements is Δ_{th}.

In real experiments, the rules for event selection typically include different cuts caused by the measurement method and the detector geometry. Each cut leaves a trace on the level of radiative corrections if undetected particles are allowed. Hence, radiative corrections are different in each independent experiment because the cut procedure distinguishes in general, and only a Monte Carlo event generator can take all the restrictions into account exactly. Our semi-analytic result can be incorporated in to such a generator to check its work for the appropriate event selection.

APPENDIX A

In this Appendix, we present formulas for the coefficients A_i^{mn}, B_i^{mn}, and C_{ij}^{mn} ($mn = ll, lt, tt$, $i = 0, 1, 2$, and $j = 1, 2, 3, 4$) that determine the partial cross sections in the case of a tensor-polarized target (see formula (75)).

A.1. Component *ll*

The coefficients determining the contribution proportional to the components R_{ll} of the tensor describing the tensor polarization of the deuteron target can be written as

$$A_1^{ll} = -xy(1+r^2)Z, \quad A_2^{ll} = \frac{ZZ_1}{xyr},$$

$$A_3^{ll} = \frac{1}{xy}\left\{(a+\bar{r})\left[2Z_1 + r\Delta_1(2a+r-\Delta_1)\right] - \right.$$
$$\left. - \Delta_1\left[r^2(r-\Delta_1)+2(b+\Delta_1)+r(a+b)(a+r-\Delta_1)\right]\right\},$$

$$A_4^{ll} = r\left\{(b-a)(1+r^2) + \Delta_1\left[1+r(2a-b)\right]\right\},$$

$$B_1^{ll} = xyr(1+r^2)\left[xyr(1+6\tau) - 2\tau(1-3ar)\right],$$

$$B_2^{ll} = -\frac{1}{xy}\left[xyr(1+6\tau) - 2\tau(1-3ar)\right] \times \times \left[b(1+r^2) - \Delta_2(\bar{r}-2a)\right],$$

$$B_3^{ll} = -\frac{1}{xy}\left\{2Z_2\left[1+(2a-b)r+\Delta_2\right] + + 3a\Delta_2\left[(b-a)r - 1 - \Delta_2\right]\right\},$$

$$B_4^{ll} = r\left[(a-b)(1+r^2) + \Delta_2(a+\bar{r})\right],$$

$$C_{01}^{ll} = \frac{2}{\tau}\left\{(\bar{r}-\Delta_1)^2 + a[3a(1+r^2) - 2(b+\Delta_1)]\right\},$$

$$C_{02}^{ll} = \frac{2}{xyr}\left\{(7-3y)(xyr)^2 + 3a(5-y+r)xyr + + 3a^2(3+r^2) - ar[5+3(a+b)^2]\right\},$$

$$C_{03}^{ll} = -xy\left[r(6a-16+9y) + 6\tau(y-3-r)\right],$$

$$C_{04}^{ll} = xyr\left[1+3(b-a)\right], \quad C_{11}^{ll} = 12xy(r+2\tau),$$

$$C_{12}^{ll} = 6\frac{\tau}{r}[4(r+\tau) - yr], \quad C_{13}^{ll} = 6\tau(2-y),$$

$$C_{21}^{ll} = 6, \quad C_{22}^{ll} = \frac{6\tau}{xyr}, \quad C_{14}^{ll} = C_{23}^{ll} = C_{24}^{ll} = 0.$$

A.2. Component *lt*

The coefficients determining the contribution proportional to the components R_{lt} of the tensor describing the tensor polarization of the deuteron target can be written as

$$A_1^{lt} = 2a(2\tau+r)(1+r^2)(2b+\Delta_1)\frac{Q^2}{Md},$$

$$A_2^{lt} = -2(2\tau+r)(2b+\Delta_1)Z_1\frac{\tau}{r}\frac{V}{Md},$$

$$A_3^{lt} = -\tau\frac{V}{Md}\left\{2Z_1(3b-a-r) + + \Delta_1[4r(1+b^2+3ab) - - 2a(1+r^2) + xyr(ar-3+5br)]\right\},$$

$$A_4^{lt} = ar\frac{V}{Md}\left\{2b(1+r^2) + \Delta_1\left[1 - r(3b-a)\right]\right\},$$

$$B_1^{lt} = 2ar(1+2\tau)(1+r^2)(\Delta_2 - 2br)\frac{Q^2}{Md},$$

$$B_2^{lt} = -2\tau(1+2\tau)(\Delta_2-2br)Z_2\frac{V}{Md},$$

$$B_3^{lt} = \tau\frac{V}{Md}\left\{2Z_2\left[(3b-a)r - 1\right] + + \Delta_2[(1+ar)(a-6b) - -(a+3b)(r^2+\Delta_2) + r(b^2-1) + b + \Delta_2(3r-2b)]\right\},$$

$$B_4^{lt} = ar\frac{V}{Md}\left[-2b(1+r^2) + \Delta_2(\bar{r}-2b)\right],$$

$$C_{01}^{lt} = \frac{4Q^2}{Md}\left[a(1+r^2)(y+2a) - 2b\bar{r} - - \Delta_1(xyr+2a-2b)\right],$$

$$C_{02}^{lt} = = -\frac{2V}{M\,dx\,yr}\left\{2axyr\left[y\bar{r} + (3b+a)(1+r) - y - 8a\right] - -(xyr)^2\left[2a+(2-y)(y+4a)\right] + 2a\left[2a(b-a+r) + + (y+2a)(r - a(1+r^2) + r(a+b)^2)\right]\right\},$$

$$C_{03}^{lt} = -\frac{Q^2}{Md}\left\{4\tau\left[2b\bar{r} - y^2 + 4(b^2-a)\right] - - r\left[3y(2-y) + 8a(1+a+2b)\right]\right\},$$

$$C_{04}^{lt} = xyr\frac{V}{Md}\left[1 + 4ab - (a-b)^2\right],$$

$$C_{11}^{lt} = \frac{4Q^2}{Md}\left[2\tau(2y+4a-1) + r(y+4a-2\tau)\right],$$

$$C_{12}^{lt} = \frac{4\tau V}{Mdr}\left[(a-b)(4\tau-yr) + 2\tau(1-r) + 2yr(1+4x\tau)\right],$$

$$C_{13}^{lt} = 4\tau(2-y)(y+2a)\frac{V}{Md}, \quad C_{14}^{lt} = C_{23}^{lt} = C_{24}^{lt} = 0,$$

$$C_{21}^{lt} = 4(y+2a)\frac{V}{Md}, \quad C_{22}^{lt} = 4\tau\frac{y+2a}{xyr}\frac{V}{Md}.$$

A.3. Component tt

The coefficients determining the contribution proportional to the components R_{tt} of the tensor describing the tensor polarization of the deuteron target can be written as

$$A_1^{tt} = -\frac{a}{b}(1+r^2)[b^2+(b+\Delta_1)^2],$$

$$A_2^{tt} = \frac{\tau}{b}\frac{Z_1}{xyr}[2b^2+\Delta_1(2b+\Delta_1)], \quad A_4^{tt} = -ar^2\Delta_1,$$

$$A_3^{tt} = \frac{\tau}{b}\{(\Delta_1-2b)[b(1+r^2)+(1-r+ry)\Delta_1] + \\ + b\Delta_1[1+r(b-a+\Delta_1)]\},$$

$$B_1^{tt} = \frac{a}{b}(1+r^2)[2b^2r^2-2br\Delta_2+\Delta_2^2],$$

$$B_2^{tt} = -\frac{\tau}{b}\frac{Z_2}{xyr}[2b^2r^2-2br\Delta_2+\Delta_2^2], \quad B_4^{tt} = -ar\Delta_2,$$

$$B_3^{tt} = \frac{\tau}{b}\{(2br-\Delta_2)Z_2+b\Delta_2[r(b-a+r)-\Delta_2]\},$$

$$C_{01}^{tt} = \frac{xy}{b}\left[(1+r^2)(y^2+4a-2ab)+(2b+\Delta_1)^2+\Delta_1^2\right],$$

$$C_{02}^{tt} = -\frac{1}{bxyr}\times \\ \times\{-2(xyr)^2[a+(1+a)(2-y)]+xyr[(3-2y+ \\ +a^2+b^2)(\bar{r}-2a)+4(ab+b-a^2)+4r(a-b^2)]- \\ -2a\left[(r-a)^2+b^2\right]+(1+2a-2b+ \\ +a^2+b^2)\left[r-a(1+r^2)+(a+b)^2r\right]\},$$

$$C_{03}^{tt} = -\frac{1}{b}\{3b-a-(a^2+b^2)(2+a+b)+ \\ +r[y^2+2y(2b-a)+2a(3-a)]+ \\ +xyr[y(1+y+3a)-4(1+a)-2ab]\},$$

$$C_{04}^{tt} = xyr(y+2a),$$

$$C_{11}^{tt} = -\frac{4}{b}[y(b-a+r)+2a(r-a)-xyr(1+a)],$$

$$C_{12}^{tt} = \frac{1}{bxyr}\{r[1+7a(1+a)-b(1+b)+(a+b)\times \\ \times(a^2+b^2)]+4\tau\left[(a-b)(1-r)+a^2+b^2-r\right]\},$$

$$C_{13}^{tt} = \frac{1}{bxy}(2-y)\left[y^2+2a(2-b)\right],$$

$$C_{14}^{tt} = C_{23}^{tt} = C_{24}^{tt}0,$$

$$C_{21}^{tt} = \frac{1}{ab}\left[y^2+2a(2-b)\right],$$

$$C_{22}^{tt} = \frac{1}{brx^2y^2}\left[y^2+2a(2-b)\right],$$

where we use the notation

$$\Delta_1 = (1-xy)r-a-b, \quad \Delta_2 = (1-y+xy)r-1,$$

$$Z_1 = b(1+r^2)+\Delta_1(1-r+yr),$$

$$Z_2 = b(1+r^2)+\Delta_2(1-y-r),$$

$$Z = xy(2\tau+r)^2-2\tau(b+\Delta_1),$$

$$a = xy\tau, \quad b = 1-y-a, \quad \bar{r} = a-b+r.$$

APPENDIX B

We here give some formulas describing the polarization state of a deuteron target in different cases. For an arbitrary polarization of the target, it is described by the general spin-density matrix (defined by 8 parameters in the general case), which in the coordinate representation has the form

$$\rho_{\mu\nu} = -\frac{1}{3}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{M^2}\right)+\frac{i}{2M}\varepsilon_{\mu\nu\lambda\rho}s_\lambda p_\rho+Q_{\mu\nu}, \quad \text{(B.1)}$$

$$Q_{\mu\nu} = Q_{\nu\mu}, \quad Q_{\mu\mu} = 0, \quad p_\mu Q_{\mu\nu} = 0,$$

where p_μ is the deuteron four-momentum, and s_μ and $Q_{\mu\nu}$ are the deuteron polarization four-vector and the deuteron quadrupole polarization tensor. In the deuteron rest frame, the above formula is written as

$$\rho_{ij} = \frac{1}{3}\delta_{ij}-\frac{i}{2}\varepsilon_{ijk}s_k+Q_{ij}, \quad i,j = x,y,z. \quad \text{(B.2)}$$

This spin-density matrix can be written in the helicity representation using the relation

$$\rho_{\lambda\lambda'} = \rho_{ij}e_i^{(\lambda)*}e_j^{(\lambda')}, \quad \rho_{\lambda\lambda'} = (\rho_{\lambda'\lambda})^*, \\ \lambda,\lambda' = +,-,0, \quad \text{(B.3)}$$

where $e_i^{(\lambda)}$ are the deuteron spin functions with the deuteron spin projection λ onto the quantization axis (z axis). They are

$$e^{(\pm)} = \mp\frac{1}{\sqrt{2}}(1,\pm i,0), \quad e^{(0)} = (0,0,1). \quad \text{(B.4)}$$

The elements of the spin-density matrix in the helicity representation are related to those in the coordinate representation as

$$
\begin{aligned}
\rho_{++} &= \frac{1}{3} + \frac{1}{2}s_z - \frac{1}{2}Q_{zz},\\
\rho_{--} &= \frac{1}{3} - \frac{1}{2}s_z - \frac{1}{2}Q_{zz},\\
\rho_{00} &= \frac{1}{3} + Q_{zz},\\
\rho_{+-} &= -\frac{1}{2}(Q_{xx} - Q_{yy}) + iQ_{xy},\\
\rho_{+0} &= \frac{1}{2\sqrt{2}}(s_x - is_y) - \frac{1}{\sqrt{2}}(Q_{xz} - iQ_{yz}),\\
\rho_{-0} &= \frac{1}{2\sqrt{2}}(s_x + is_y) - \frac{1}{\sqrt{2}}(Q_{xz} + iQ_{yz}).
\end{aligned}
\tag{B.5}
$$

To obtain this relations we used that

$$Q_{xx} + Q_{yy} + Q_{zz} = 0.$$

The polarized deuteron target, which is described by the population numbers n_+, n_-, and n_0, is often used in spin experiments (see, e.g., Ref. [59]). Here, n_+, n_-, and n_0 are the fractions of atoms with the nuclear spin projection onto the quantization axis $m = +1$, $m = -1$, and $m = 0$. If the spin-density matrix is normalized to 1, i.e., $\mathrm{Tr}\,\rho = 1$, then

$$n_+ + n_- + n_0 = 1.$$

Hence, the polarization state of the deuteron target is defined in this case by two parameters, the so-called V (vector) and T (tensor) polarizations

$$V = n_+ - n_-, \quad T = 1 - 3n_0. \tag{B.6}$$

Using the definitions

$$n_\pm = \rho_{ij} e_i^{(\pm)*} e_j^{(\pm)}, \quad n_0 = \rho_{ij} e_i^{(0)*} e_j^{(0)}, \tag{B.7}$$

we have the following relation between V and T parameters and the parameters of the spin-density matrix in the coordinate representation (in the case where the quantization axis is directed along the z axis):

$$n_0 = \frac{1}{3} + Q_{zz}, \quad n_\pm = \frac{1}{3} \pm \frac{1}{2}s_z - \frac{1}{2}Q_{zz}, \tag{B.8}$$

or

$$T = -3Q_{zz}, \quad V = s_z. \tag{B.9}$$

We now relate the parameters of the density matrix for a massive spin-one particle (deuteron) for two representations: the coordinate (see Eq. (B.1)) and spherical tensors.

According to the Madison Convention [60], the density matrix of a spin-one particle is given by

$$\rho = \frac{1}{3}\sum_{kq} t_{kq}^* \tau_{kq}, \tag{B.10}$$

where t_{kq} are the polarization parameters of the deuteron density matrix and τ_{kq} are the spherical tensors given by

$$
\begin{aligned}
\tau_{00} &= 1, \quad \tau_{10} = \sqrt{\frac{3}{2}}S_z,\\
\tau_{1\pm1} &= \mp\frac{\sqrt{3}}{2}(S_x \pm iS_y),\\
\tau_{20} &= \frac{3}{\sqrt{2}}\left(S_z^2 - \frac{2}{3}\right),\\
\tau_{2\pm2} &= \frac{\sqrt{3}}{2}(S_x \pm iS_y)^2,\\
\tau_{2\pm1} &= \mp\frac{\sqrt{3}}{2}\left[(S_x \pm iS_y)S_z + S_z(S_x \pm iS_y)\right],
\end{aligned}
\tag{B.11}
$$

$$
S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix},\quad
S_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix},\quad
S_z = \begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}.
\tag{B.12}
$$

From Eq. (B.11) and the Hermiticity of the spin operator, we immediately obtain

$$\tau_{kq}^+ = (-1)^q \tau_{k-q} \tag{B.13}$$

and the Hermiticity condition for the density matrix yields

$$t_{kq}^* = (-1)^q t_{k-q}. \tag{B.14}$$

It follows from this equation that

$$
\begin{aligned}
t_{10}^* = t_{10}, \quad t_{11}^* = -t_{1-1}, \quad t_{20}^* = t_{20},\\
t_{22}^* = t_{2-2}, \quad t_{21}^* = -t_{2-1},
\end{aligned}
\tag{B.15}
$$

i.e., the parameters t_{10} and t_{20} are real and the parameters t_{11}, t_{21}, and t_{22} are complex. Hence, in total, there are 8 independent real parameters, as must be the case for a spin-one massive particle.

We then have the explicit expression of the deuteron density matrix

$$\rho = \frac{1}{3}\begin{pmatrix} 1+\sqrt{\frac{3}{2}}t_{10}+\frac{1}{\sqrt{2}}t_{20} & \sqrt{\frac{3}{2}}(t_{1-1}+t_{2-1}) & \sqrt{3}t_{2-2} \\ -\sqrt{\frac{3}{2}}(t_{11}+t_{21}) & 1-\sqrt{2}t_{20} & \sqrt{\frac{3}{2}}(t_{1-1}-t_{2-1}) \\ \sqrt{3}t_{22} & -\sqrt{\frac{3}{2}}(t_{11}-t_{21}) & 1-\sqrt{\frac{3}{2}}t_{10}+\frac{1}{\sqrt{2}}t_{20} \end{pmatrix}. \quad (B.16)$$

The density matrix is normalized to 1, i.e., $\mathrm{Tr}\,\rho = 1$. Using the expression for the density matrix in the helicity representation, Eq. (B.5), we obtain the following relations between the parameters of the density matrix in the coordinate and spherical tensor representations:

$$\begin{gathered} t_{10} = \sqrt{\frac{3}{2}}s_z, \\ \mathrm{Re}\,t_{11} = -\mathrm{Re}\,t_{1-1} = -\frac{\sqrt{3}}{2}s_x, \\ \mathrm{Im}\,t_{11} = \mathrm{Im}\,t_{1-1} = -\frac{\sqrt{3}}{2}s_y, \\ t_{20} = -\frac{3}{\sqrt{2}}Q_{zz}, \\ \mathrm{Re}\,t_{21} = -\mathrm{Re}\,t_{2-1} = \sqrt{3}Q_{xz}, \\ \mathrm{Im}\,t_{21} = \mathrm{Im}\,t_{2-1} = \sqrt{3}Q_{yz}, \\ \mathrm{Re}\,t_{22} = \mathrm{Re}\,t_{2-2} = -\frac{\sqrt{3}}{2}(Q_{xx}-Q_{yy}), \\ \mathrm{Im}\,t_{22} = -\mathrm{Im}\,t_{2-2} = -\sqrt{3}Q_{xy}. \end{gathered} \quad (B.17)$$

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