ЖЭТФ, 2016, том 150, вып. 5 (11), стр. 917–928 

# DISTINCTIVE ASPECTS OF THE EVOLUTION OF GALACTIC MAGNETIC FIELDS

*D. Yar-Mukhamedov* [*]

*Cavendish Laboratory, University of Cambridge*
*CB3 0HE, Cambridge, United Kingdom*

Received December 4, 2015

We perform an in-depth analysis of the evolution of galactic magnetic fields within a semi-analytic galaxy formation and evolution framework, determine various distinctive aspects of the evolution process, and obtain analytic solutions for a wide range of possible evolution scenarios.

**DOI:** 10.7868/S0044451016110080

## 1. INTRODUCTION

The aim of this paper is to perform an in-depth analysis of the model we have developed earlier and introduced in [1]. In this section, we briefly outline some of the details, which are relevant to the analysis we perform later in the paper.

The model in [1] included the following energy components:

- $\mathcal{E}_o$, the energy of ordered magnetic fields;
- $\mathcal{E}_c$, the energy of chaotic (random or turbulent) magnetic fields;
- $\mathcal{E}_t$, the energy in turbulence of the gas in a galaxy.

In addition, the energy of the system as a whole was referred to as $\mathcal{E}_\Sigma$ and the total energy in magnetic fields as $\mathcal{E}_m$. All of these quantities are tied together by the energy balance equations

$$\dot{\mathcal{E}}_\Sigma = \dot{\mathcal{E}}_t + \dot{\mathcal{E}}_m, \qquad (1)$$

$$\dot{\mathcal{E}}_m = \dot{\mathcal{E}}_c + \dot{\mathcal{E}}_o, \qquad (2)$$

where all derivatives are taken with respect to time. Moreover, in [1] we also assumed equipartition of the total energy between the energy of turbulent motions of gas in a galaxy and galactic magnetic fields following the argument in [2]

$$\dot{\mathcal{E}}_t = \dot{\mathcal{E}}_m. \qquad (3)$$

[*] E-mail: danial.su@gmail.com

Three energy balance equations (1), (2), and (3) allow obtaining three of five unknowns. In [1], for the total energy of the system in terms of the rate of energy inputs and outputs $\dot{\mathcal{E}}_{io}$, the negative part of the gas mass rate $\dot{\mathcal{M}}_-$, and the mass of the cold gas $\mathcal{M}$, we had

$$\dot{\mathcal{E}}_\Sigma = \dot{\mathcal{E}}_{io} - \mathcal{E}_\Sigma \frac{\dot{\mathcal{M}}_-}{\mathcal{M}}. \qquad (4)$$

Finally, the energy of ordered magnetic fields was chosen in [1] as

$$\dot{\mathcal{E}}_o = \begin{cases} \frac{1}{2}\dot{\mathcal{E}}_\Sigma, & \mathcal{E}_o = \frac{1}{2}\mathcal{E}_\Sigma \text{ and } \dot{\mathcal{E}}_\Sigma < 0, \\ \frac{\mathcal{E}_c}{\tau} - \mathcal{E}_o \frac{\dot{\mathcal{M}}_-}{\mathcal{M}} & \text{otherwise,} \end{cases} \qquad (5)$$

where $\tau$ is a time scale of formation of ordered magnetic fields in a galaxy; this time scale is $k_\tau$ times greater than the corresponding period of rotation of the same galaxy.

In addition, in [1] we also provided formal solutions for the total energy and the energy of ordered magnetic fields. The solution for the total energy is

$$\mathcal{E}_\Sigma = \exp\left(-\int_{t_0}^{t} \frac{\dot{\mathcal{M}}_-}{\mathcal{M}} dt'\right) \times \\ \times \left[\mathcal{E}_{\Sigma,\,0} + \int_{t_0}^{t} \exp\left(\int_{t_0}^{t''} \frac{\dot{\mathcal{M}}_-}{\mathcal{M}}\, dt'\right) \dot{\mathcal{E}}_{io}\, dt''\right]. \qquad (6)$$

For the energy of ordered magnetic fields, we obtained in [1] a piecewise solution. When

$$\mathcal{E}_o = \frac{1}{2}\mathcal{E}_\Sigma \;\wedge\; \dot{\mathcal{E}}_\Sigma < 0$$

holds, we have

$$\mathcal{E}_o = \frac{1}{2}\mathcal{E}_\Sigma, \tag{7}$$

and in all other cases we obtain

$$\mathcal{E}_o = \exp\left[-\int\limits_{t_0}^{t}\left(\frac{1}{\tau}+\frac{\dot{\mathcal{M}}_-}{\mathcal{M}}\right)dt'\right]\times$$
$$\times\left\{\mathcal{E}_{o,\,0}+\int\limits_{t_0}^{t}\exp\left[\int\limits_{t_0}^{t''}\left(\frac{1}{\tau}+\frac{\dot{\mathcal{M}}_-}{\mathcal{M}}\right)dt'\right]\frac{\mathcal{E}_\Sigma}{2\tau}\,dt''\right\}. \tag{8}$$

In the most general case, the integrals in (6) and (8) should be calculated numerically. For this, we have developed a computer implementation of this model in [1]. As a source of input data, we have used outputs from Galacticus — a semi-analytic model of galaxy formation and evolution developed in [3]. Finally, we have fitted our model with observational data from [4,5]. For more details on our model, its implementation, and major results obtained by its means, we refer the reader to [1].

## 2. BEHAVIOR OF AVERAGED PHYSICAL PROPERTIES OF COLLECTIVES OF GALAXIES

To better understand the evolution of galactic magnetic fields in our model, it is very useful to analyze the behavior of various average physical quantites. They hold a key to revealing the prevaling processes and tendencies, which shape the behavior we have observed in [1].

### 2.1. Evolution of average energies

We proceed by analyzing the average total energy, the average energy of ordered magnetic fields, and the average energy of chaotic magnetic fields (Fig. 1).

#### 2.1.1. Rapid evolution during the early epoch

In [1], we defined the early epoch as the period of evolution between the formation of the first galaxies and 2.5 Gyr. This epoch is characterized by the rapid nonlinear evolution of magnetic fields. During this epoch, all average energies reach their maxima.

At the beginning, at $t \approx 0.5$ Gyr, a collective of galaxies starts accumulating energy and forming random magnetic fields. Initially, the energy in these fields follows the evolution of the total energy very closely. However, with the formation of ordered magnetic fields at $t \approx 0.75$ Gyr, the evolution of random magnetic

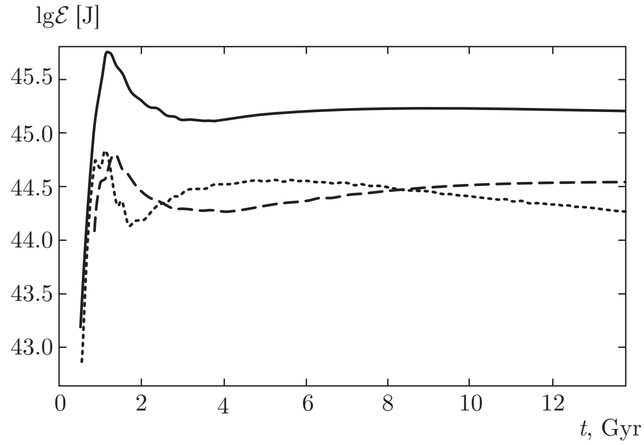


**Fig. 1.** The averaged total energy (solid line), the averaged energy of ordered magnetic fields (dashed line), and averaged energy of chaotic fields (dotted line) traced throughout the whole lifetime of the Universe

fields starts diverging from the path of the total energy and at $t \approx 1.1$ Gyr chaotic magnetic fields reach their global maximum, which is followed by a decline, almost as rapid as the initial increase. The total energy reaches its global maximum shortly after the chaotic fields do at $t \approx 1.25$ Gyr. Finally, the energy of ordered magnetic fields reaches its peak at $t \approx 1.4$ Gyr.

This order is quite natural for the following reasons. Random fields should be the first to reach their maximum values because their existence lies on the boundary of two competing processes. On one hand, they are fueled by turbulence, and therefore at first they follow the total energy of the system. On the other hand, they play the role as a source of energy for ordered fields, and therefore, as soon as the ordered component starts rapidly evolving, the growth of random fields should become slower or even give its place to a decline. The total energy should have its maximum earlier than the energy of ordered magnetic fields because of the delay that is preventing ordered fields from forming instantaneously.

The rapid growth at the beginning is associated with the fact that a majority of the galaxies are those belonging to group I, which we have defined in [1] as the group that represents rapidly evolving galaxies. They have undergone recent major mergers and accumulated mass very quickly, which has led to a significant increase in the total average energy of the whole collective and, consequently, to a swift formation of random magnetic fields. At the later stages of the evolution, there are many other galaxies in group I, but they do not represent the majority, and are not therefore able to affect the average behavior in the same way.

When galaxies are very young and relatively small, their ordered magnetic fields form and evolve relatively quickly. Shortly after their formation, the ordered fields cause rapid decline of the random magnetic fields, as a result of a very efficient transfer of energy between the random and the ordered energy containers, which disrupts monotonous evolution of the random component. At the stage of decline, which follows the aforementioned peaks, the behavior of the total energy is partially mirrored by the energy of ordered magnetic fields. It contrasts with the increase in average energies, where total and random energies are evolving closely. This is a very natural behavior for several reasons. First, at the decline, the energy is no longer transferred to random magnetic fields or the amount of the transferred per unit time is not very significant compared to the rate of the energy transfer from the chaotic to ordered fields. Moreover, the chaotic fields have already transferred most of their energy into the ordered fields. Thus, the ordered fields contain nearly a half of the total energy and evolve mainly under the influence of the gas mass losses in exactly the same way as the total energy does in the absence of energy inputs.

At $t \approx 1.65$ Gyr, the balance between ordered and random fields changes and the energy of random fields starts increasing again. At this point, group II (which we also defined in [1]) starts forming and initially is dominated by the galaxies, where most of the energy is concentrated in random magnetic fields. The group II may be considered fully formed at $t \approx 2.5$ Gyr, when the average energies of ordered and chaotic magnetic fields are approximately the same.

#### 2.1.2. Steady evolution in the course of the intermediate and the late epochs

After 2.5 Gyr (according to the definition in [1]), galaxies enter the intermediate epoch, which lasts until 6.75 Gyr. During this period, the evolution of magnetic fields is relatively steady in contrast with the early epoch. The late epoch takes place after the intermediate epoch and continues till the present day.

At the beginning of the intermediate epoch, the total energy continues to decrease due to the growth of group II. In [1], we discovered that galaxies in this group are capable of maintaining a power-law relation between their stellar masses and strengths of their ordered magnetic fields and that the energy density of ordered fields has a practicaly negligible dependence on the stellar mass. This maintainable value of the energy density is lower than the average value in rapidly evolving galaxies of group I. Thus, when group II grows, the average value slowly declines until galaxies of group II represent the majority and the average values are dominated by them. Only then is a slow increase in the average total energy possible. It should happen because of the increase in masses of galaxies in group II, which causes them to evolve along a power-law curve, thus causing an increase in their total energy. This can be seen in Fig. 1, where the total energy after reaching its local minimum value at approximately $t \approx 3.5$ Gyr begins very slowly growing again.

From the beginning of the intermediate epoch till $t \approx 4.5$ Gyr, the evolution of ordered magnetic fields undergoes a period of stagnation, while the energy of random fields, on the contrary, rapidly grows reaching a very flat maximum at $t \approx 5.5$ Gyr. The reason for this behavior is the same: growth of group II. Average values of energies of all galaxies slowly adjust themselves to reflect this.

Various processes associated with the constant mass losses dominate galaxies in the intermediate epoch, resulting in the accumulation of large amounts of energy in random magnetic fields. Chaotic fields are inevitably converted into ordered fields after they reach their local peak values, where galaxies of group II begin spreading in the energy dimension. At the end of this epoch, group II is significantly dispersed in the energy dimension, which in the following epoch leads to formation of subgroups IIA with strong ordered fields and IIB with strong random fields (Fig. 2).

The growth of the average energy of ordered magnetic fields at the end of the intermediate epoch is associated with a steady increase in the number of galaxies with a high percentage of their energy located in their ordered fields. The same tendencies are present at the late epoch, which is characterized by a slowly increasing gap between average values of the energy of ordered and random fields caused by the formation of distinct subgroups and consequent growth of subgroup IIA.

By the end of the intermediate epoch, the total energy enters a period of stagnation, which lasts till the present day, while random fields after reaching their local maximum decline and ordered fields, on the contrary, begin rising again. This behavior continues throughout the last epoch.

### 2.2. Average fraction of energy in ordered magnetic fields

So far, we have analyzed the evolution of average energies. It is possible to significantly simplify the overall picture by analyzing the behavior of the fraction of the total magnetic field energy or the total energy of

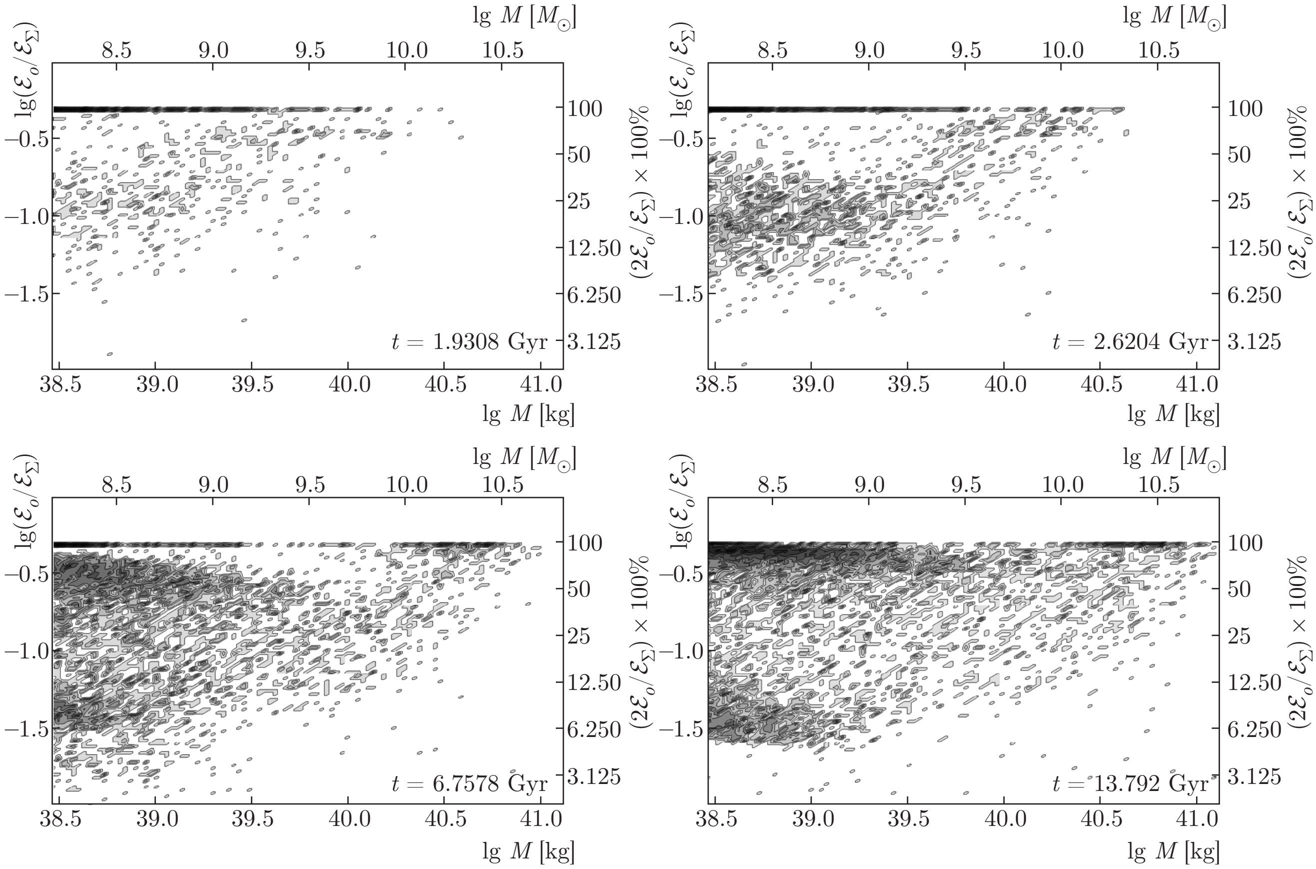


**Fig. 2.** Evolution of the energy fraction in ordered magnetic fields vs stellar mass. The left vertical axis on each plot is in the scale of the decimal logarithm of the total energy fraction in ordered magnetic fields, while the right vertical axis shows the percentage of the total magnetic field energy in ordered magnetic fields

a system, which goes into either its chaotic or its ordered component. Because the main goal of this work is to study the evolution of ordered magnetic fields, in Fig. 3, for consistency, we also show the evolution of the average fraction of the total magnetic field energy in the ordered magnetic fields.

It is very important to note that the local maximum at the early epoch does not represent a global maximum anymore, i. e., fractionwise, the ordered magnetic fields at the present day are much stronger compared to chaotic fields than they were during the earlier epochs, although they a certainly slightly weaker in terms of their absolute values. Another quality, which is immediately obvious, is a relative smoothness of the evolution path of the energy fraction, which is much smoother than the evolution path of the average energy we have observed earlier. This shows that despite the rapid changes in the total energy, resulting in an apparently rather chaotic behavior of the average energies at the early epochs, the underlying mechanisms that lead

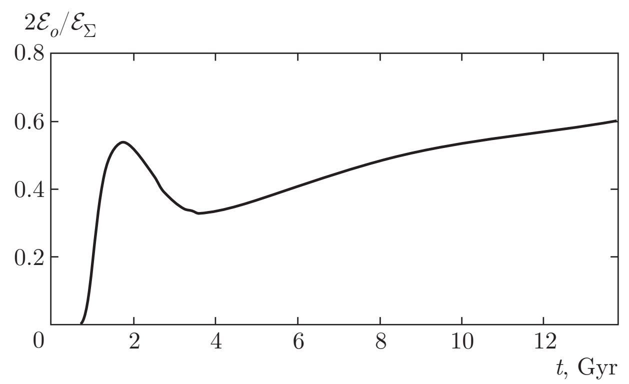


**Fig. 3.** Average of the fractions of energies in ordered magnetic fields vs cosmic time

to the redistribution of energy between various components of magnetic fields are more consistent and the evolution associated with them is much smoother.

The initial increase in the energy fraction of ordered magnetic fields is due to inefficient energy sinks in the

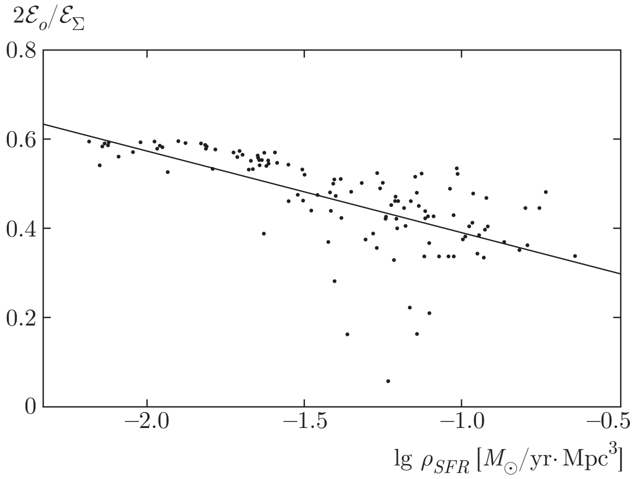


**Fig. 4.** The averaged energy fraction in ordered magnetic fields plotted against the density of comoving star formation rate. Points represent real galaxies with star formation rate values from [3] and approximations of energy fraction values from this paper. The line represents best linear fit to this data

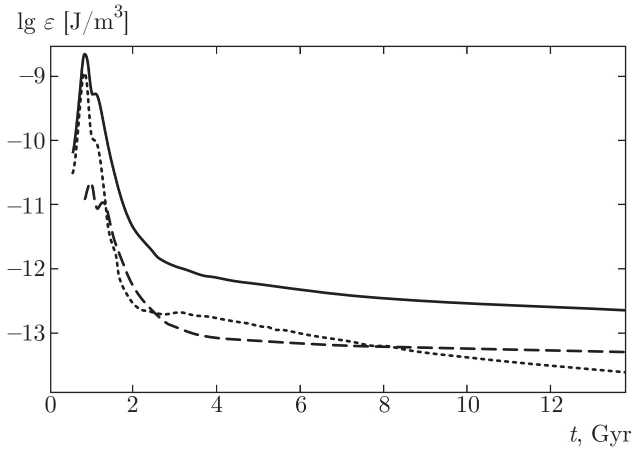


**Fig. 5.** The averaged total energy density (solid line), the averaged energy density of ordered magnetic fields (dashed line) and the averaged energy density of chaotic fields (dotted line) throughout the whole lifetime of the Universe

early galaxies. Then, at the local maximum at $t \approx$ $\approx 1.75$ Gyr, the efficiency of energy sinks becomes significant enough to turn the process around, and the energy fraction starts decreasing. Later at the next turning point at around $t \approx 3.7$ Gyr, the average energy of ordered magnetic fields starts increasing again. This local minimum corresponds to the maximum star formation rate throughout the history of the Universe (see, for example, the star formation data in [6]), showing that star formation plays a decisive role in establishing a balance between ordered and random magnetic fields. In addition, the increasing star formation rate in the early Universe explains the rapid decrease in the fraction of energy in ordered fields before the local minimum, and the slowly decreasing star formation rate in the late Universe explains the steady growth of the average fraction of energy in ordered magnetic fields after the minimum.

The connection between the star formation rate and the fraction of energy in ordered magnetic fields becomes even more obvious when each observed galaxy is associated with the average fraction of energy in ordered magnetic fields at its epoch. The result shows a considerable anticorrelation, −0.608005, between the star formation rate and the average fraction, which can be seen in Fig. 4, where comoving star formation rate densities are obtained from [3], which in turn represents a compilation of the data originally from [6] and [7].

### 2.3. Densities of various energy components

We next consider the evolution of average energy densities (Fig. 5). A first easily noticeable feature is a very dramatic change in the energy densities over time in the early epoch. Similar processes took place for energies, but they were around one or two orders of magnitude, whereas the drop here is four orders of magnitude. Such a drastic difference is due to a rapid increase in volumes of considered galaxies during the early stages of their evolution. Despite some similarities between the ordered energy and the ordered energy density plots, there are some significant differences.

The first significant difference is that at the later epochs, the energy density of ordered magnetic fields never increases. After a rapid drop in the early epoch, it always stays around the same value regardless of how the energy is distributed between random and ordered fields. It suggests that all galaxies tend to certain sustainable values of the energy densities associated with ordered fields. Due to the nature of the energy balance equations, which we discussed at the beginning, sustainable values should also exist for densities of random fields and consequently for densities of total energies in the considered systems.

Second, we note that shortly after entering the intermediate epoch, the energy density drops exponentially for all components. By fitting the simulation data with the function

$$\varepsilon = \frac{\mathcal{E}}{V} = a + be^{-ct}$$

for the total averaged energy density $\overline{\varepsilon}_\Sigma$, we obtain

$$\overline{\varepsilon}_\Sigma \left[\frac{\mathrm{J}}{\mathrm{m}^3}\right] \approx 2.21\cdot 10^{-13} + 1.87\cdot 10^{-12}\exp\left(-\frac{0.33t}{\mathrm{Gyr}}\right),$$

and for the energy density of ordered magnetic fields $\overline{\varepsilon}_o$ this gives

$$\overline{\varepsilon}_o \left[\frac{\mathrm{J}}{\mathrm{m}^3}\right] \approx 4.84\cdot 10^{-14} + 1.01\cdot 10^{-13}\exp\left(-\frac{0.25t}{\mathrm{Gyr}}\right).$$

Finally, for the energy density of chaotic magnetic fields $\overline{\varepsilon}_c$, we obtain

$$\overline{\varepsilon}_c \left[\frac{\mathrm{J}}{\mathrm{m}^3}\right] \approx 1.83\cdot 10^{-14} + 5.26\cdot 10^{-13}\exp\left(-\frac{0.31t}{\mathrm{Gyr}}\right),$$

where $t$ is the cosmic time, which should be specified in Gyr, and the resulting energy densities are in J/m$^3$. This result confirms that the energy density of the ordered magnetic fields is not the only container that has a limiting value, and that two other major components in the system also evolve similarly. Moreover, in the $t \to \infty$ limit, the ratio between ordered and random energy densities tends to

$$\lim_{t\to\infty} \frac{\overline{\varepsilon}_o}{\overline{\varepsilon}_c} \approx 2.64 \approx e.$$

This leads us to a very important conclusion that on average, the energy of random magnetic fields is never completely transferred into the ordered magnetic fields, but instead tends to become approximately $e$ times smaller.

In the general case, the limit of the ratio between the averaged ordered and random energy densities is a function of all the model parameters, i. e.,

$$\lim_{t\to\infty} \frac{\overline{\varepsilon}_o}{\overline{\varepsilon}_c} = \overline{\mathcal{R}}_\infty\left(k_{sn},\, k_a,\, k_m,\, \ldots,\, k_\tau\right).$$

Features of this function depend on the input data in our model as well as on a selection of galaxies over which the modeling results are averaged. It is important to note that our simulations for the aforediscussed collective of galaxies show only a negligible dependence of $\overline{\mathcal{R}}_\infty$ on $k_{sn}$, $k_a$, and $k_m$ for a wide range of the model parameters around the optimal parameter set. In contrast, the dependence of $\overline{\mathcal{R}}_\infty$ on $k_\tau$ is considerable and can be approximated with

$$\overline{\mathcal{R}}_\infty \approx 60.23 k_\tau^{-1.34}.$$

We note, however, that the behavior of averaged values patially reflects the overall tendencies of the evolution of the individual galaxies and is partially affected by the dispersion of galaxies in energy and energy density manifolds. As a result of the latter,

$$\frac{1}{2}\overline{\varepsilon}_\Sigma \neq \overline{\varepsilon}_o + \overline{\varepsilon}_c,$$

in contrast to naive expectations. In the following sections, we investigate the behavior of individual galaxies in greater detail using the analytic approach.

## 3. ANALYTIC SOLUTIONS IN VARIOUS SPECIAL CASES

We now consider analytic solutions in several extreme cases of the evolution of galactic magnetic fields in order to determine origins of the tendencies governing magnetic field evolution processes.

### 3.1. Stationary and quasistationary cases of magnetic fields evolution

We begin by considering a scenario where the energy inputs and outputs that contribute to the total energy and consequently to the energy of ordered magnetic fields of the considered system lead to a stationary system state with time-independent characteristics, namely, with $\dot{\mathcal{E}}_\Sigma = 0$. The same reasoning is applicable to quasistationary cases where changes in the total energy are negligible, i. e., $|\dot{\mathcal{E}}_\Sigma| \ll \dot{\mathcal{E}}_{io}$. In both cases, Eq. (4) takes the form

$$\mathcal{E}_\Sigma = \mathcal{M}\frac{\dot{\mathcal{E}}_{io}}{\dot{\mathcal{M}}_-}, \tag{9}$$

where the "=" sign should be replaced with "≈" in the quasistationary case. Equation (9) immediatly leads us to the conclusion that in this case $\dot{\mathcal{E}}_{io} \geq 0$ because all other values in this equation are not negative by definition. It also means that the total energy in the quasistationary system tends to some limit value, which can be easily determined when all energy inputs and outputs are explicitly defined.

The constraint on the behavior of the total energy is not sufficient to make the energy of ordered galactic magnetic field stationary. We now consider a stationary version of Eq. (5), which is

$$\mathcal{E}_o = \frac{\mathcal{E}_\Sigma \mathcal{M}}{2\left(\mathcal{M} + \tau\dot{\mathcal{M}}_-\right)} \tag{10}$$

and can be expressed directly through the inputs and outputs as

$$\mathcal{E}_o = \frac{\mathcal{M}^2}{2\left(\mathcal{M} + \tau\dot{\mathcal{M}}_-\right)}\frac{\dot{\mathcal{E}}_{io}}{\dot{\mathcal{M}}_-}, \tag{11}$$

which in the case where $\mathcal{M} \gg \tau\dot{\mathcal{M}}_-$ (the mass of gas in the disk being significanly greater than the mass extracted from the disk on a time scale of formation of ordered magnetic fields in a galaxy) gives the relation

$$\lim_{\mathcal{M}\gg\tau\dot{\mathcal{M}}_-} \mathcal{E}_o = \frac{1}{2}\mathcal{E}_\Sigma,$$

and we can assume that the energy is transfered instantaneously from the total energy reservoir into the energy of ordered magnetic fields.

Therefore, in stationary or quasistationary cases for galaxies with sufficiently large masses of gas and low gas mass loss rates, it may often be assumed that the energy of ordered magnetic fields follows one half of the total energy without any delays.

#### 3.1.1. Supernovae-dominated evolution and the ultimate limit of the evolution of galactic magnetic fields

We now consider an ultimate case where the contribution from supernovae dominates the total energy inputs and gas mass losses. This scenario, for example, may take place after a sufficiently long period of time when contributions of all other processes become negligible or vanish completely.

We first assume that in this limit, the accretion of intergalactic gas is halted or its rate is too low to be taken into account, and that mergers become very rare and therefore energy inputs of both these processes can be neglected. Furthermore, we assume that morphologies of considered galaxies become more or less defined and do not affect the evolution of ordered galactic magnetic fields as much as they did in the early stages of the evolution. Therefore, processes shaping morphological features of considered galaxies can also be presumed to contribute negligibly to the mass losses. Finally, galactic magnetic fields and other feedback processes prevent star formation or make its rate negligibly small, and therefore only the existing stars continue to burn and the mass of the gaseous component of the galaxy does not change significantly because of them.

This reasoning leads us to one remaining process that can both input energy into magnetic fields and take it away through mass losses. It is a supernovae feedback. Assuming that these two processes are proportional to the same power of the supernovae energy output (usually, the first power in both cases), Eq. (9) can be rewritten as

$$\mathcal{E}_\Sigma = \mathcal{M}\frac{k_{sn}}{f_{sn}}, \tag{12}$$

where $k_{sn}$ is the efficiency of energy inputs due to supernovae, and $f_{sn}$ is a function that determines the fraction of gas that should leave the galactic disk due to supernovae feedback processes. Depending on how this function is defined, there are several ultimate outcomes.

In the case where $f_{sn} \equiv$ const, the total energy does not depend on the stellar mass of the galactic disk or other parameters and depends only on the mass of the gas in the disk. This leads to a specific total energy and consequently ordered magnetic field energy per unit mass of the disk gas. However, in the previous paper, we indirectly assumed a different model of the supernovae feedback. As a galaxy formation and evolution base, we used Galacticus, which assumes a power-law dependence of $f_{sn}$ on the characteristic velocity $v$ of a considered galaxy [3], i. e., $f_{sn} = \mathcal{A}/v^\alpha$, where $\mathcal{A}$ and $\alpha$ are constant coefficients. This leads us to

$$\mathcal{E}_\Sigma = \mathcal{M}\frac{k_{sn}}{\mathcal{A}}v^\alpha, \tag{13}$$

which in its turn by means of the virial theorem can be described in terms of the total mass of a galaxy $M$ and its radius $R$ as

$$\mathcal{E}_\Sigma = \mathcal{M}\frac{k_{sn}}{\mathcal{A}}\left(G\frac{M}{R}\right)^{\alpha/2}, \tag{14}$$

where $G$ is the universal gravitational constant, and by introducing the average density of matter in the galaxy $\rho = 3M/4\pi R^3$, we obtain

$$\mathcal{E}_\Sigma = \mathcal{M}\frac{k_{sn}}{\mathcal{A}}G^{\alpha/2}\left(\frac{4\pi\rho}{3}\right)^{\alpha/6}M^{\alpha/3} \sim \mathcal{M}M^{\alpha/3}, \tag{15}$$

which yields a linear dependence if we assume the default value for this coefficient for galactic disks in Galacticus, $\alpha = 3$.

Therefore, in all cases we obtain a power-law dependence of the total energy and hence of the energy of ordered magnetic fields on the total mass of a galaxy. This means that in such an ultimate limit, we obtain a power-law dependence on stellar masses of galaxies, with the galaxies being scattered around depending on their average densities and so on their morphologies.

More importantly, this result means that for each and every galaxy, there is an ultimate energy of its ordered magnetic field. If, due to various processes, it becomes larger than this limit value, then it eventually drops back to a stationary value, which is determined by the mechanism of supernovae feedback and hence can be used to test various supernovae feedback models.

Finally, in our previous paper [1], we have seen galaxies in groups IIA and IIB to evolve along the lines of exactly this power-law dependence on the stellar mass. This suggests and confirms that the supernovae feedback plays a decisive role in the process of formation and evolution of galactic magnetic fields. However, there are some alternative scenarios, which may occur for some of the galaxies and at the different stages of the evolution. We consider them next.

#### 3.1.2. Gravitationally dominated evolution

At some stages of the evolution of galactic magnetic fields, galaxies may undergo a gravitationally dominated evolution mode. During this period, most of the energy inputs and outputs are due to accretion and mergers. A detailed description of how these mechanisms affect the total energy can be found in our previous paper [1].

Substituting energy inputs due to gravitation in (9), we obtain

$$\mathcal{E}_\Sigma = \mathcal{M}\frac{GM\left(k_a\dot{\mathcal{M}}_a + k_m\dot{\mathcal{M}}_m\right)}{\dot{\mathcal{M}}_- R},$$

where we can introduce the effective gravitational mass rate $k_+\dot{\mathcal{M}}_+ = k_a\dot{\mathcal{M}}_a + k_m\dot{\mathcal{M}}_m$; taking into account that mass losses and mass gains in stationary and quasistationary cases must be equal in order to maintain a constant gas mass, we obtain

$$\mathcal{E}_\Sigma = \mathcal{M}\frac{GMk_+}{R},$$

where the radius can be expressed through the mass and average matter density,

$$\mathcal{E}_\Sigma = \mathcal{M}\left(\frac{4\pi\rho}{3}\right)^{1/3} GM^{2/3}k_+ \sim \mathcal{M}M^{2/3}, \qquad (16)$$

which leads us to a conclusion that in gravitationally dominated stationary and quasistationary evolution modes, the energy of magnetic field is proportional to the stellar mass of galaxies to the power $2/3$. In this mode of evolution, the mass of the galaxy increases significantly along with its magnetic fields, leading to the condition where the previously accumulated energy is negligible, which makes the obtained power-law dependence so explicit and easily observable.

However, it should be noted that this mode of evolution is very unlikely to be the final mode, because, as soon as accretion of the gas becomes negligible, all other processes such as supernovae feedback and star formation become dominant, leading to other modes and thus slowly changing the established power-law dependence.

#### 3.1.3. Complete recycling evolution mode

We now consider a case characterized by the rapid disk gas recycling. We define this case as follows. In systems under consideration, all disk gas should be completely renewed during a characteristic time $\kappa\tau$, i. e., $\mathcal{M} = \kappa\tau\mathcal{M}'_-$. In the case of a stationary or quasistationary system, this leads to characteristic relations and dependences. Equation (9) changes to

$$\mathcal{E}_\Sigma = \kappa\tau\dot{\mathcal{E}}_{io} \qquad (17)$$

and Eq. (10) to

$$\mathcal{E}_o = \frac{\mathcal{E}_\Sigma\kappa}{2(1+\kappa)}, \qquad (18)$$

where it becomes immediately clear that in the case $\kappa \to 1$, the energy of the magnetic field tends to be equally divided between its ordered and chaotic components. If for any reason relation (18) is broken, then the system would evolve according to

$$\dot{\mathcal{E}}_o = \frac{\kappa\mathcal{E}_c - \mathcal{E}_o}{\kappa\tau}, \qquad (19)$$

which returns the system to the stationary condition described by (18). Furthermore, if $\kappa$ and $\tau$ are constants, then the system evolves according to an exponential decay law,

$$\mathcal{E}_o = \frac{\mathcal{E}_\Sigma\kappa}{2(1+\kappa)} + \exp\left[-\frac{(1+\kappa)\,(t-t_0)}{\kappa\tau}\right] \times \\ \times \left(\mathcal{E}_{o,\,0} - \frac{\mathcal{E}_\Sigma\kappa}{2(1+\kappa)}\right)$$

with the decay rate $(1+\kappa)/\kappa\tau$.

It's worth noting that in this case, all power-law dependences described earlier should be more explicit, because the total energy does not depend directly on the mass of the galactic gas, which should decrease the spread of the values due to differences in morphology of considered galaxies. However, the total energy would still depend on the average matter density of the chosen selection of galaxies.

We defined the characteristic time to be proportional to the period of rotation of a galaxy, which in turn depends on the characteristic velocity $v$ of a galaxy and its radius $R$, whence

$$\mathcal{E}_\Sigma = \kappa k_\tau \frac{R}{v}\dot{\mathcal{E}}_{io},$$

which by means of the virial theorem turns into

$$\mathcal{E}_\Sigma = \kappa k_\tau \frac{\dot{\mathcal{E}}_{io}}{\sqrt{GM/R^3}} = \kappa k_\tau \frac{\dot{\mathcal{E}}_{io}}{\sqrt{(4\pi/3)G\rho}}.$$

Thus, in addition to dependences on stellar mass and other parameters through energy inputs and outputs, there also exists an explicit dependence on the overall average density of a galaxy and consequently on its morphology.

### 3.2. Evolution of magnetic fields in mature galaxies

A very important special case of the evolution of magnetic fields is their development in mature galaxies, which we discuss in this section. It is of great interest for several reasons. One of the reasons is its wide applicability, because it can be applied to a majority of galaxies in intermediate and late epochs of the evolution. Another reason is its clarity and relative simplicity, which provides us not only with the understanding of behavior of considered mature galaxies but also with the understanding of some general aspects of the evolution of magnetic fields in all galaxies. Finally, it allows us to obtain the ultimate limit of the evolution of any mature galaxy.

We first clarify our definition of a mature galaxy, which we use in what follows. We consider a galaxy to be mature when

1) the rate of energy inputs and outputs in this galaxy is constant, fluctuates around a constant value or evolves very slowly, i. e.,

$$\dot{\mathcal{E}}_{io} = \text{const} \lor \dot{\mathcal{E}}_{io} \approx \text{const}; \tag{20}$$

2) the ratio $\mu = \dot{\mathcal{M}}_{-}/\mathcal{M}$ of the negative gas mass loss to the total gas mass in the disk, is constant or changes insignificantly, i. e.,

$$\mu = \text{const} \lor \mu \approx \text{const}; \tag{21}$$

3) the time scale of formation of ordered magnetic fields in the galaxy is constant or its changes are negligible, i. e.,

$$\tau = \text{const} \lor \tau \approx \text{const}. \tag{22}$$

When all three conditions are satisfied, we call the galaxy mature. It should be noted that there is no explicit requirement for such a galaxy to have a constant mass for any of its components or for each individual energy input or output to be constant. The definition does not require knowing what energy inputs and outputs are taken into account. All of the aforementioned makes the following reasoning applicable not only to the current implementation of our model but also to other possible implementations, which may include new energy inputs and outputs, new galactic components, new mass losses, etc.

After taking (20) and (21) into account, integrating Eq. (6) becomes straightforward and we obtain

$$\mathcal{E}_{\Sigma}(t) = \mathcal{E}_{\Sigma,0} \exp\left[-\mu (t - t_0)\right] + \frac{\dot{\mathcal{E}}_{io}}{\mu} \left\{1 - \exp\left[-\mu (t - t_0)\right]\right\}. \tag{23}$$

This simple analytic solution, when written in such a form, shows that there are two major influencing factors. First of all, we have the initial total energy $\mathcal{E}_{\Sigma,0}$ at a time $t_0$. Its influence decreases according to an exponential decay law with the decay rate $\mu$. On the other hand, we have the energy inputs and outputs per negative fractional gas loss rate $\dot{\mathcal{E}}_{io}/\mu$, whose influence grows with exactly the same speed. After a large enough amount of time, i. e., formally as $t \to \infty$, we obtain

$$\mathcal{E}_{\Sigma,\infty} \equiv \lim_{t\to\infty} \mathcal{E}_{\Sigma}(t) = \frac{\dot{\mathcal{E}}_{io}}{\mu},$$

which allows us to conclude that the end state of the evolution of the total energy of a system and consequently of magnetic fields does not depend on the starting point and that the starting point merely influences the way the system reaches its final state.

Using energy balance equations (1), (2), and (3) together with (5) allows obtaining an equation for the energy of random magnetic fields:

$$\dot{\mathcal{E}}_c = \begin{cases} 0, & \mathcal{E}_o = \frac{1}{2}\mathcal{E}_{\Sigma} \text{ and } \dot{\mathcal{E}}_{\Sigma} < 0, \\ \frac{1}{2}\dot{\mathcal{E}}_{io} - \left(\mu - \frac{1}{\tau}\right)\mathcal{E}_c & \text{otherwise,} \end{cases}$$

and after solving it we obtain a piecewise solution. When

$$\mathcal{E}_o = \frac{1}{2}\mathcal{E}_{\Sigma} \land \dot{\mathcal{E}}_{\Sigma} < 0,$$

we have

$$\mathcal{E}_c = 0$$

and in all other cases,

$$\mathcal{E}_c = \exp\left[-\int_{t_0}^{t}\left(\mu + \frac{1}{\tau}\right) dt'\right] \times \left(\mathcal{E}_{c,0} + \int_{t_0}^{t} \exp\left[\int_{t_0}^{t''}\left(\mu + \frac{1}{\tau}\right) dt'\right] \frac{1}{2}\dot{\mathcal{E}}_{io}\, dt''\right). \tag{24}$$

Now if we take (20), (21), and (22) into account, we obtain

$$\mathcal{E}_c(t) = \mathcal{E}_{c,0} \exp\left[-\left(\mu+\frac{1}{\tau}\right)(t-t_0)\right] + \frac{1}{2}\,\frac{\dot{\mathcal{E}}_{io}}{\mu+1/\tau} \times \times \left\{1 - \exp\left[-\left(\mu+\frac{1}{\tau}\right)(t-t_0)\right]\right\}. \quad (25)$$

This solution is very similar to the one for the total energy. But there are two essential differences. First, the random magnetic field is fueled only by a half of all energy inputs and outputs. Second, the exponential decay rate is $\mu + 1/\tau$, which is always bigger than the similar rate for the total energy, because the time scale of formation of ordered magnetic fields is always positive. Just as we did for the total energy, for the energy of random magnetic fields, we can find a limit that should allow us to describe the system after a large period of time. We obtain

$$\mathcal{E}_{c,\infty} \equiv \lim_{t\to\infty} \mathcal{E}_c(t) = \frac{1}{2}\,\frac{\dot{\mathcal{E}}_{io}}{\mu+1/\tau}.$$

The conclusion in this case is the same as we had for the total energy. The initial value of the energy of chaotic magnetic fields influences only the route towards the final state, but does not define that final value. The final value only depends on the rate of inputs and outputs, the mass rate, and the time scale of formation of ordered magnetic fields.

Lastly, we should obtain a similar solution for the energy accumulated in ordered magnetic fields. One of the ways to find it is to substitute (6) in (8) and to integrate the resulting expression assuming (20), (21), and (22); alternatively, we can use energy balance equations (1), (2), and (3) with the previously obtained solutions for the total energy and the energy of random fields. This way or another, we obtain the same result:

$$\mathcal{E}_o(t) = \frac{1}{2}\mathcal{E}_{\Sigma,0} \exp\left[-\mu(t-t_0)\right] - \left(\frac{1}{2}\mathcal{E}_{\Sigma,0} - \mathcal{E}_{o,0}\right) \times \times \exp\left[-\left(\mu+\frac{1}{\tau}\right)(t-t_0)\right] + + \frac{\dot{\mathcal{E}}_{io}}{2\mu}\left\{1-\exp\left[-\mu(t-t_0)\right]\right\} - \frac{1}{2}\,\frac{\dot{\mathcal{E}}_{io}}{\mu+1/\tau} \times \times \left\{1 - \exp\left[-\left(\mu+\frac{1}{\tau}\right)(t-t_0)\right]\right\}. \quad (26)$$

This result contains four summands and each of them corresponds to one of several processes that govern the evolution of ordered fields. The first summand shows that the influence of the initial value of the total energy of a system exponentially decays with decay rate $\mu$. Interestingly enough, the energy in ordered magnetic fields depends on the initial value of the total energy explicitly. The second component, after we apply energy balance equations, turns into $-\mathcal{E}_{c,0}\exp[-(\mu+1/\tau)(t-t_0)]$. This summand shows that the influence of the initial value of random magnetic fields also decays exponentially, albeit with a different rate $\mu + 1/\tau$. The last two summands describe the rate of formation of the limit value of ordered magnetic fields. This limiting value is

$$\mathcal{E}_{o,\infty} \equiv \lim_{t\to\infty} \mathcal{E}_o(t) = \frac{1}{2}\,\frac{\dot{\mathcal{E}}_{io}}{\mu(\mu\tau+1)}.$$

Just as previously, the limit value does not depend on the initial value of ordered magnetic fields at $t_0$. The major difference with both previous cases is in the way the ordered field evolves. The evolution path depends on both the path taken by the total energy and the path taken by the energy in chaotic fields.

Knowing the limit values for all the energy components, we can now easily obtain various relations between them, which are as follows:

$$\lim_{t\to\infty} \frac{\mathcal{E}_o(t)}{\mathcal{E}_\Sigma(t)} = \frac{1}{2(1+\tau\mu)},$$
$$\lim_{t\to\infty} \frac{\mathcal{E}_c(t)}{\mathcal{E}_\Sigma(t)} = \frac{1}{2(1+\tau^{-1}\mu^{-1})},$$
$$\lim_{t\to\infty} \frac{\mathcal{E}_o(t)}{\mathcal{E}_c(t)} = \frac{1}{\tau\mu}.$$

Most interestingly, these relations do not depend on the actual values of energy inputs and outputs. They only depend on the time scale of formation of ordered magnetic fields and on the gas mass rate per unit of the gas mass.

A very important consequence of all the above analytic solutions is that the energy of random fields does not vanish even after an infinite amount of time. From the general analytic solution for chaotic fields, we can also make the following conclusion. The energy of chaotic magnetic fields can tend to be very small, especially if the time scale of formation of ordered magnetic fields is very short. However, as long as the component describing energy inputs and outputs is not zero, chaotic fields do not vanish completely.

### 3.3. Possible applications of analytic solutions to observations

The analytic solutions obtained above can be used to obtain various properties of galaxies with known dependences of the strength of magnetic fields on time as well as to test the applicability of the assumptions used in the analysis to the real observed galaxies. For this, we rewrite Eqs. (23), (25), and (26) as

$$\mathcal{E}_\Sigma = \alpha_\Sigma + \beta_\Sigma \exp(-\gamma_\Sigma \Delta t),$$
$$\mathcal{E}_c = \alpha_c + \beta_c \exp(-\gamma_c \Delta t),$$
$$\mathcal{E}_o = \alpha_o + \beta_{o,\,1} \exp(-\gamma_{o,\,1}\Delta t) + \beta_{o,\,2}\exp(-\gamma_{o,\,2}\Delta t),$$

where

$$\alpha_\Sigma \equiv \mathcal{E}_{\Sigma,\,\infty} \equiv \frac{\dot{\mathcal{E}}_{io}}{\mu}, \quad \beta_\Sigma \equiv \mathcal{E}_{\Sigma,\,0} - \mathcal{E}_{\Sigma,\,\infty}, \quad \gamma_\Sigma \equiv \mu,$$

$$\alpha_c \equiv \mathcal{E}_{c,\,\infty} \equiv \frac{1}{2}\frac{\dot{\mathcal{E}}_{io}}{\mu + 1/\tau}, \quad \beta_c \equiv \mathcal{E}_{c,\,0} - \mathcal{E}_{c,\,\infty},$$

$$\gamma_c \equiv \mu + \frac{1}{\tau}, \quad \alpha_o \equiv \mathcal{E}_{o,\,\infty} \equiv \frac{1}{2}\frac{\dot{\mathcal{E}}_{io}}{\mu\,(\mu\tau + 1)},$$

$$\beta_{o,\,1} \equiv \frac{1}{2}\left(\mathcal{E}_{\Sigma,\,0} - \mathcal{E}_{\Sigma,\,\infty}\right), \quad \gamma_{o,\,1} \equiv \mu,$$

$$\beta_{o,\,2} \equiv -\left(\mathcal{E}_{c,\,0} - \mathcal{E}_{c,\,\infty}\right), \quad \gamma_{o,\,2} \equiv \mu + \frac{1}{\tau}, \quad \Delta t \equiv t - t_0.$$

In this form, the previously obtained dependences are much easier to fit with data. Moreover, all the coefficients have their physical meanings. All $\alpha$ coefficients define ultimate limits of the evolution, the $\beta$ coefficients describe differences between initial and final states, which affect absolute rates of the evolution processes, and the $\gamma$ coefficients all correspond to the rates of the system evolution.

## 4. ANALYSIS OF THE ANALYTICAL GENERAL CASE SOLUTION

General solutions (6) and (8) can be analyzed in more detail even without substituting actual values and functions. We first concentrate on the total energy and then apply the same reasoning to the energy of ordered magnetic fields.

Solution (6) consists of two summands. The first represents the influence of the initial state of the system and the second describes either corrections to the initial state or the final state of the system. How fast the influence of the initial state decays depends on the integral

$$\int\limits_{t_0}^{t} \frac{\dot{\mathcal{M}}_-}{\mathcal{M}}\, dt'.$$

If it tends to $\infty$ as $t \to \infty$, then the right summand represents the final state of a system. If the integral converges as $t \to \infty$, then

$$\exp\left(-\int\limits_{t_0}^{t} \frac{\dot{\mathcal{M}}_-}{\mathcal{M}} dt'\right)$$

defines how strong the influence of the initial state is in the final state of the system. The limit value of the integral depends on the way the ratio $\dot{\mathcal{M}}_-/\mathcal{M}$ behaves. From mathematical analysis we know that if this function decreases slower than $1/t$, then the integral always diverges as $t \to \infty$. Even if the function decreases more rapidly, in most scenarios this integral is large enough to make the summand corresponding to the intial state negligible.

Similar reasoning is applicable to Eq. (8). Here, the behavior depends on $1/\tau + \dot{\mathcal{M}}_-/\mathcal{M}$. Each part of this expression can be integrated separately and as a result we conclude that if any individual summand diverges, then the whole result diverges as well. The second summand is exactly the same as in the case of the total energy. Therefore, as $t \to \infty$, the energy of ordered fields is independent of its initial value in all cases where the same is true for the total energy. However, even when the total energy depends on its initial value, the energy of ordered magnetic fields may be independent if $1/\tau$ decreases slower than $1/t$. Even if initially it decreases faster, then by the following physical reasoning we may conclude that in all scenarios it should decrease slower or even be constant on the late stages of the evolution. This reasoning is based on the idea that $\tau$ is related to the mass, size, and other properties of a galaxy. They all eventually begin evolving slower. Even the most simplistic reasoning would give $\tau \propto \rho^{-1/2}$, and hence, even if the density decreases at a constant rate, then the characteristic time scale increases relatively slowly. In reality, during the late epochs of the evolution, properties of galaxies evolve very slowly compared to the early epochs and therefore $\tau$ may be approximated by a constant value and the integral

$$\int\limits_{t_0}^{t} \left[\frac{1}{\tau} + \frac{\dot{\mathcal{M}}_-}{\mathcal{M}}\right] dt'$$

diverges. The energy of ordered magnetic fields for $t \to \infty$, however, still may be indirectly influenced by the inital state of the total energy through the second summand in Eq. (8).

## 5. CONCLUSION

In this paper, we have analyzed the results of numerical simulations and obtained and explored various analytic solutions. We have discovered

• tendencies that cause the average values of energies of magnetic fields, densities of energies, and their ratios for collectives of galaxies to evolve in the direction of certain limit values;

• limit values of energies of magnetic fields for individual galaxies in several cases and analytic expressions to describe their evolution in such scenarios;

• underlying reasons for various power-law dependences of energies of magnetic fields on stellar masses of galaxies;

• independence of the final state of the galactic magnetic field evolution from their initial state in a majority of scenarios.

We came to a conclusion that even after a considerable amount of time, chaotic fields still possess some energy. Moreover, the energy of any component of magnetic fields depends only on the overall energy inputs and outputs rate, the gas mass loss per unit of gas mass and the time scale of formation of ordered magnetic fields, and the ratios between any two energy components depend only on the gas mass loss rate per unit of gas mass and the time scale of formation of ordered fields.

The independence of the final state of evolution from previous states also means that the accuracy of the end results of numerical simulations does not depend on the overall accuracy of integration techniques. The most significant numerical errors should present themselves only when systems undergo rapid evolution. However, shortly after they satisfy the conditions that lead to the independence of the final state, all computational errors should rapidly decline.

## REFERENCES


1. D. Yar-Mukhamedov, Zh. Eksp. Teor. Fiz. **147**, 702 (2015).
2. S. S. Shabala, J. M. G. Mead, and P. Alexander, Month. Not. Roy. Astron. Soc. **405**, 1960 (2010).
3. A. J. Benson, GALACTICUS: A semi-analytic model of galaxy formation. New Astron. **17**, 175 (2012).
4. K. N. Abazajian, J. K. Adelman-McCarthy, M. A. Agüeros et al., Astrophys. J. Suppl. Ser. **182**, 543 (2009).
5. A. J. Fitt and P. Alexander, Month. Not. Roy. Astron. Soc. **261**, 445 (1993).
6. A. M. Hopkins, Astrophys. J. **615**, 209 (2004).
7. M. D. Kistler, H. Yüksel, J. F. Beacom et al., Astrophys. J. **705**, L104 (2009).