The paper of L.P. Gor'kov "About the energy spectrum of superconductors"

L. P. Pitaevskii

The presented paper of L.P. Gor'kov [1] ["About the energy spectrum of superconductors", Soviet Phys. JETP, **7**, 505 (1958)] is one of the most important contributions to the theory of superconductivity after the papers, of Bardeen–Cooper, and Shrieffer (BCS) [2]. In these papers followed by the work of Bogoliubov [3], a microscopic theory of superconductivity was developed, but important conceptual questions remained not clarified.

The theory predicted a second-order phase transition to the superconducting phase, but the order parameter remained unknown. The physical meaning of the transition was not clarified. The relation to the very successful semi-phenomenological theory of Ginzburg and Landau was also not clear.

The methods used by BCS and by Bogoliubov apply to 'pure' superconductors. In the original form, they did not allow a direct generalization to problems related to the behavior of superconductors in strong magnetic fields and were restricted by the weak-coupling approximation.

The work of Gor'kov [1] is based on a deep physical idea. The author realized that the transition to the superconducting state can be considered in a certain sense as the Bose–Einstein condensation of Cooper pairs. Instability with respect to formation of these pairs was discovered in the paper of Cooper [4] and the existence of the pairs is a basis of the theories of Bardeen–Cooper–Shrieffer [2] and Bogoliubov [3].

Go'rkov has shown [1] that the condensation manifest itself analytically as the appearance of anomalous Green's functions of the type $\langle T(\psi(x_1))\psi(x_2)\rangle$ and $\langle T(\psi^\dagger(x_1))\psi^\dagger(x_2)\langle$ (cf. Eq.(5)) in the expansion of the two-particle Green's function. As follows from this and the subsequent Gor'kov's papers, the superconducting transition is characterized by the onset of an off-diagonal long-range order. The complex order parameter for this transition is the anomalous Green's function, more precisely, its value at coincident temporal arguments, which has the meaning of the wave function of a Cooper pair. In the weak-coupling model considered in the paper, the gap in the energy spectrum is proportional to the absolute value of this quantity. Gor'kov's equations are explicitly gradient invariant. In the previous formulations, this property was not obvious.

Subsequently Gor'kov has sown that close to the transition temperature, his order parameter satisfies the Ginzburg–Landau equations, but the electron charge e has to be substituted by the Cooper pair charge $2e$ [5]. This substitution improved agreement with experiment.

Gor'kov's paper [1] was of great practical interest for researchers in the field. Calculations in Gor'kov's formalism are much simpler than within the previous methods. In this three-page paper, not only an essentially new method is formulated but the energy spectrum and the specific heat are also found. Most important is that Gor'kov's technique allows deciding where the first approxmation is not sufficient and summation of an infinite number of terms is necessary. Gor'kov's equations have a convenient diagram interpretation. This allows selecting the principle terms graphically and summing them.

A good example of the application of these possibilities is given by the famous joint papers of Abricosov and Gor'kov on the theory of superconducting alloys [6]. It is impossible to list even the most important results obtained by this method.

It is worth mentioning that the paper of Gor'kov [1] is very well written and it is a pleasure to read it. Even now, after so many years, no single word in the paper has to be corrected; this is a generic property of classic works.

[1] L. P. Gor'kov, Soviet Phys. JETP **7**, 505 (1958).

[2] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957); **108**, 1175 (1957).

[3] N. N. Bogoliubov, Soviet Phys. JETP **7**, 41 (1958).

[4] L. N. Cooper, Phys. Rev. **104**, 1189 (1956).

[5] L. P. Gor'kov, Soviet Phys. JETP **9**, 636 (1959).

[6] A. A. Abrikosov, L. P. Gor'kov, Soviet Phys. JETP **8**, 1090 (1959); **9**, 220 (1959).

L. P. Pitaevsky